Homework #8

JOINT INSTITUTE 交大密西根学院

Due: 11:59 pm on December 12, 2024

Problem 1

Part A

Discuss the normalization problem of the following wave functions:

1. Particle moves in a one-dimensional infinite potential well, let $\psi(x) = A\sin\frac{\pi x}{a}(0 \leq x \leq a)$, find A to normalize the wave function.
2. Let $\psi(x) = A\exp\left(-\frac{1}{2}\alpha^2 x^2\right)$, where α is a known constant. Find A to normalize the function.
3. Let $\psi(x) = \exp(ikx)$. What is the probability distribution of the particle's position? Can the function be normalized?
4. Let $\psi(x) = \delta(x)$. What is the probability distribution of the particle's position? Can the function be normalized?

Part B

Let the particle wave function in the spherical coordinates to be $\psi(r, \theta, \varphi)$. Show

1. The probability to find the particle within the spherical shell$(r, r + dr)$;
2. The probability to find the particle within the solid angle $d\Omega$ in the (θ, φ) direction.

Problem 2

Part A

Use the uncertainty principle to estimate the ground-state energy of a particle in an infinite square potential well with width a.

Part B

Assume the state of a particle at $t = 0$ is $\psi(x) = A(\sin^2 kx + \frac{1}{2}\cos kx)$, calculate the particle's average momentum and average kinetic energy in this case.

Part C

$\psi(x, t)$ is the solution to the one-dimensional Schrödinger equation for a free particle with mass m. $\psi(x, 0) = Ae^{-x^2/a^2}$.

1. Calculate the probability amplitude of the momentum space at $t = 0$.
2. Calculate the $\psi(x, t)$.

Problem 3

Part A

A particle moves in a one-dimensional infinite potential well ($0 \leq x \leq a$), the initial state wave function at time $t = 0$ is

$$\psi(x,0) = \sqrt{\frac{8}{5a}}(1+\cos\frac{\pi x}{a})\sin\frac{\pi x}{a}.$$

1. What is the wave function at time $t_0 > 0$?
2. What the average energy of the system at time $t = 0$ and $t = t_0$?
3. At time $t = t_0$, what the probability to find the particle within the left part of the potential well ($0 \leq x \leq \frac{a}{2}$)

Part B

A particle with mass m is confined in a one-dimensional box of length l

$$\begin{cases} V = \infty, & x < 0 \\ V = 0, & 0 < x < l \\ V = \infty, & x > l \end{cases}$$

At time $t = 0$, the particle wave function is

$$\begin{cases} \psi = \sqrt{\frac{30}{l^5}}x(l-x), & 0 < x < l \\ \psi = 0, & x > l \text{ or } x < 0 \end{cases}$$

Find the series representation and expression for the series coefficients of $\psi(x, t > 0)$.

Congratulations on completing all Homeworks of PHYS2600J 24FA!