Solution for HW4.

Problem 1.

Part A.

1. $I_0 = \int_0^a \frac{b}{r} \exp\left(\frac{r-a}{\delta}\right) \cdot 2\pi r dr = 2\pi b\delta e^{-\frac{a}{\delta}}(e^{\frac{a}{\delta}}-1)$.

2. $B = \frac{\mu_0 I_0}{2\pi r}$

3. $I = \int_0^R \frac{b}{r} \exp\left(\frac{r-a}{\delta}\right) \cdot 2\pi r dr = 2\pi b\delta e^{-\frac{a}{\delta}}(e^{\frac{R}{\delta}}-1)$

   $I/I_0 = \frac{e^{R/\delta}-1}{e^{a/\delta}-1}$.

4. $B = \frac{\mu_0 I}{2\pi r} = \frac{e^{R/\delta}-1}{e^{a/\delta}-1} \frac{\mu_0 I_0}{2\pi r}$.

Part B.

$\int_l \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow \int_A \nabla \times \vec{B} \cdot d\vec{A} = \int_A \mu_0 \vec{J}_{encl} \cdot d\vec{A} \Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J}_{encl}$

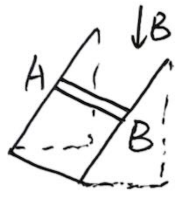
Then, $\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \nabla \cdot \vec{J}_{encl} = 0 \Rightarrow \nabla \cdot \vec{J}_{encl} = 0$.

Since $\nabla \cdot \vec{J}_{encl} + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial \rho}{\partial t} = 0. \Rightarrow \frac{\partial \vec{E}}{\partial t} = 0$.

If $\nabla \cdot \vec{J} \neq 0$. $\nabla \cdot (\nabla \times \vec{B}) \neq 0$. conflicts.

Therefore, Ampère's law shouldn't be valid outside magnetostatics.

Problem 2.



Part A.

1. $mg\sin\varphi = \frac{B^2L^2 v\cos^2\varphi}{R} \Rightarrow v = \frac{mgR\sin\varphi}{B^2L^2\cos^2\varphi}$

2. $A \to B$.

3. $I = \frac{BLv\cos\varphi}{R} = \frac{mg\sin\varphi}{BL\cos\varphi}$

4. $P_e = I^2 R = \frac{Rm^2g^2\sin^2\varphi}{B^2L^2\cos^2\varphi}$.

5. $P_g = v \cdot mg\sin\varphi = \frac{Rm^2g^2\sin^2\varphi}{B^2L^2\cos^2\varphi} = P_e$.

Part B.

$\Phi_B = a\int_0^a 4t^2 y dy = 2t^2a^3$.

$\mathcal{E} = -\frac{d\Phi_B}{dt} = -4ta^3$.

Part C. 1. $\phi_B = a\int_{d+vt}^{b+d+vt} \frac{\mu_0 I}{2\pi s}\, ds = \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{b}{d+vt}\right)$.

2. $\mathcal{E} = -\frac{d\phi_B}{dt} = \frac{\mu_0 I a b v}{2\pi(d+tv)(b+d+tv)}$

$I_{loop} = \frac{\mathcal{E}}{R} = \frac{\mu_0 I a b v}{2\pi R(d+tv)(b+d+tv)}$

Part D. direction: $\circlearrowright$

$\phi_B = a\int_{s}^{s+a} \frac{\mu_0 I}{2\pi s'}\, ds' = \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{s}\right)$.

$Q = I\Delta t = -\frac{\Delta\phi_B}{\Delta t}\frac{\Delta t}{R} = \frac{\phi_B}{R} = \frac{\mu_0 I a}{2\pi R}\ln\left(1+\frac{a}{s}\right)$.

Problem 3.

Part A 1. $V = \frac{Q}{C} = \frac{Qd}{\varepsilon_r\varepsilon_0 A}$ $\quad I = \frac{V}{R} = \frac{VA}{\rho d}$.

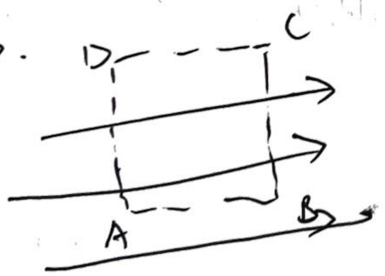
$dQ = -I\,dt \Rightarrow Q = Q_0 e^{-\frac{t}{\varepsilon_r\varepsilon_0\rho}}$.

$I = -\frac{dQ}{dt} = \frac{Q_0}{\varepsilon_r\varepsilon_0\rho} e^{-\frac{t}{\varepsilon_r\varepsilon_0\rho}}$.

$J_c(t) = \frac{Q_0}{\varepsilon_r\varepsilon_0\rho A} e^{-\frac{t}{\varepsilon_r\varepsilon_0\rho}}$.

2. $E = \frac{Q_0}{\varepsilon_r\varepsilon_0 A} e^{-\frac{t}{\varepsilon_r\varepsilon_0\rho}}$ $\quad J_D(t) = \frac{\varepsilon_r\varepsilon_0}{A}\frac{d\phi_E}{dt} = -J_c(t)$.

Part B.



$\oint_l \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} \neq 0$.

conflict with $\frac{d\phi_B}{dt} = 0$.