Additional Homework #3

JOINT INSTITUTE 交大密西根学院

Due: 11:59 pm on Nov 26, 2024

Problem 1 (40 pts)

A plane wave of angular frequency ω and wave number $|\mathbf{K}|$ propagates in a neutral, homogeneous, anisotropic, non-conducting medium with $\mu = 1$.

1. Show that $\mathbf{H}$ is orthogonal to $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{K}$, and also that $\mathbf{D}$ and $\mathbf{H}$ are transverse but $\mathbf{E}$ is not.

2. Let $D_k = \sum_{l=1}^{3} \varepsilon_{kl} E_l$, where ε_{kl} is a real symmetric tensor. Choose the principal axes of ε_{kl} as a coordinate system ($D_k = \varepsilon_k E_k; k = 1, 2, 3$). Define $\mathbf{K} = K\hat{S}$, where the components of the unit vector $\hat{S}$ along the principal axes are S_1, S_2, and S_3. If $V = \omega/K$ and $V_j = c/\sqrt{\varepsilon_j}$, show that the components of $\mathbf{E}$ satisfy

$$S_j \sum_{i=1}^{3} S_i E_i + (\frac{V^2}{V_j^2} - 1) E_j = 0.$$

   Write down the equation for the phase velocity V in terms of $\hat{S}$ and V_j. Show that this equation has two finite roots for V^2, corresponding to two distinct modes of propagation in the direction $\hat{S}$.

Problem 2 (30 pts)

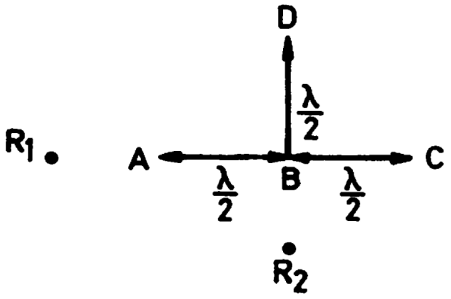
Four identical coherent monochromatic wave sources A, B, C and D produce waves of the same wavelength λ. Two receivers R_1, and R_2 are at great (but equal) distances from B.

1. Which receiver picks up the greater signal?

2. Which receiver, if any, picks up the greater signal if source B is turned off?

3. if source D is turned off?

4. Which receiver can tell which source, B or D, has been turned off?



Problem 3 (30 pts)

In a region of empty space, the magnetic field (in Gaussian units) is described by

$$\mathbf{B} = B_0 e^{ax} \hat{\mathbf{e}}_\mathbf{z} \sin \omega,$$

where $\omega = ky - \omega t$.

1. Calculate $\mathbf{E}$.
2. Find the speed of propagation v of this field.
3. Is it possible to generate such a field? If so, how?