Solutions for HW 2.

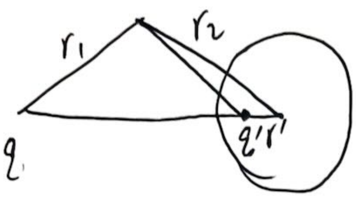
Problem 1.

Part A. $E_{plane} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{d^2+r^2}\frac{d}{\sqrt{d^2+r^2}} = \frac{\sigma(r)}{2\varepsilon_0}$

Then $\sigma(r) = \frac{-qd}{2\pi}(d^2+r^2)^{-3/2}$.

$Q = \int_0^\infty \sigma(r)\cdot 2\pi r\,dr = -q$.

Part B.



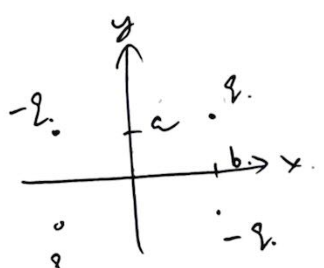
1. $$\begin{cases} \frac{q}{d-R} + \frac{q'}{R-r'} = 0. \\ \frac{q}{d+R} + \frac{q'}{R+r'} = 0 \end{cases} \Rightarrow \begin{cases} q' = -\frac{R}{d}q. \\ r' = \frac{R^2}{d}. \end{cases}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{R}{dr_2}\right).$$

2. put $-q'-Q$ at the center

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{Rq}{dr_2} + \frac{Rq-dQ}{dr}\right).$$

Part C.



$$\vec{F} = \frac{q^2}{4\pi\varepsilon_0}\left[\left(\frac{1}{4a^2+4b^2}\frac{b}{\sqrt{a^2+b^2}} - \frac{1}{4b^2}\right)\hat{n}_x + \left(\frac{1}{4a^2+4b^2}\frac{a}{\sqrt{a^2+b^2}} - \frac{1}{4a^2}\right)\hat{n}_y\right]$$

$$W = \frac{1}{4}U_{config} = \frac{1}{8}\sum q_i V(\vec{r}_i) = \frac{1}{2}qV(b,a)$$
$$= \frac{q^2}{16\pi\varepsilon_0}\left(\frac{1}{\sqrt{a^2+b^2}} - \frac{1}{a} - \frac{1}{b}\right).$$

No. π/k. $k = N^*$.

Problem 2.

Part A. $C = \varepsilon_0\frac{A}{x}$. $U = \frac{Q^2}{2C} = \frac{Q^2x}{2\varepsilon_0 A}$.

$\delta U = \frac{Q^2 dx}{2\varepsilon_0 A}$

$\frac{Q^2dx}{2\varepsilon_0 A} = F\,dx \Rightarrow F = \frac{Q^2}{2\varepsilon_0 A}$

$QE = \frac{Q^2}{A\varepsilon_0} = 2F$.

plate doesn't apply force to itself.

Part B. $C = \varepsilon_0 \frac{(L-x)L}{D} + \varepsilon_r \varepsilon_0 \frac{xL}{D}$.

$U = \frac{1}{2}CV^2 \Rightarrow dU = \frac{V^2\varepsilon_0 L(\varepsilon_r - 1)}{2D} dx$.

$Q = CV = \frac{\varepsilon_0 LV}{D}(L - x + \varepsilon_r x)$

for $U = \frac{Q^2}{2C}$. $\Rightarrow \frac{du}{dx} = -\frac{Q^2}{2C^2}\frac{dc}{dx} = -\frac{V^2\varepsilon_0 L(\varepsilon_r - 1)}{2D}$.

$du = -\frac{V^2\varepsilon_0 L(\varepsilon_r - 1)}{2D} dx$.

2. $F = -\frac{V^2\varepsilon_0 L(\varepsilon_r - 1)}{2D} < 0 \Rightarrow -x$ push. out.

3. $F > 0$. pull into

There is charge motion. $|F| = \frac{V^2\varepsilon_0 L(\varepsilon_r - 1)}{2D}$.

Problem 3.

Part A. $J(x) = \frac{I}{a \cdot 2\pi x} + \frac{I}{a \cdot 2\pi(b-x)}$

$V_{0\to b} = \int_{r_0}^{b-r_0} E(x)\,dx = \int_{r_0}^{b-r_0} \rho J(x)\,dx = \frac{\rho I}{2\pi a} \ln\left(\frac{b-r_0}{r_0}\right)^2$

$R = \frac{V}{I} = \frac{1}{\sigma\pi a}\ln\left(\frac{b}{r_0} - 1\right) \approx \frac{1}{\sigma\pi a}\ln\frac{b}{r_0}$.

Part B. $I = 0.4A$ anti-clockwise.

$P_R = 1.28W$. $P_r = 0.32W$ or $0.16W$ (one or two).

$P_1 = 4.8W$. $P_2 = 3.2W$.

$P_e = P_1 - P_2 = 1.6W = P_{consumption} = P_R + P_r = 1.6W$.

Part C. $q(t_d) = 1.6 \times 10^{-19} C \Rightarrow t_d > 19.36s$.

let $q(t_d) = e \Rightarrow t_d = -RC \ln\frac{e}{Q_{max}}$

RC is the time constant

If RC is fixed. charge of R or C will not influence t_d.