Due: 11:59 pm on Oct 17, 2024

Problem 1

Part A

A point charge q is placed at a distance d from a grounded conducting plane.

1. Find the surface density of charge induced on the plane as a function of the position on the plane and plot its graph.
2. Check that the total charge induced is equal to $-q$.

Part B

Use the method of images to find the potential due to a point charge q placed at a distance d from the center of

1. a grounded conducting ball with radius $R < d$.
2. an ungrounded conducting ball with radius $R < d$ charged with charge $-Q$.

Part C

Two semi-infinite grounded conducting planes meet at the right angle. In the region between them, there is a point charge q, situated as shown in the cross-sectional figure below.

1. What is the force on q?
2. How much work did it take to bring the charge q from infinity?
3. Suppose the planes met as some angle other than $\pi/2$. Would you still be able to solve the problem by the method of images? If not, for what particular angles does the method work?

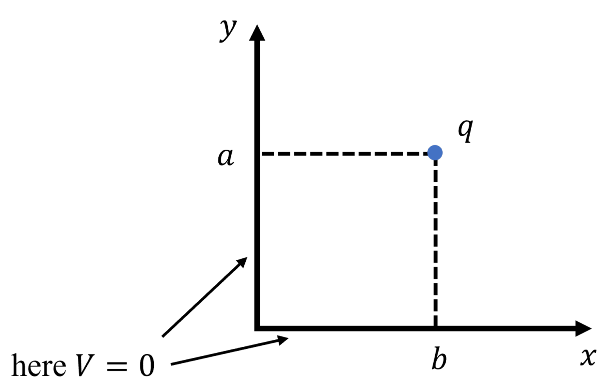

Figure 1: 1C

Problem 2

Part A | Optional

A parallel-plate vacuum capacitor with plate area A and separation x has charges Q and $-Q$ on its plates. The capacitor is disconnected from the source of charge.

1. What is the total energy stored in the capacitor?
2. The plates are pulled apart an additional distance dx. What is the change in value of the stored energy?
3. If F is the force with which the plates attract each other, then the change in the stored energy must be equal to the work $\delta W = Fdx$ done in pulling the plates apart. Find an expression for F.
4. Explain why F is not equal to QE, where E is the electric field between the plates.

Part B

Two square conducting plates with sides of length L are separated by a distance D. A dielectric slab with relative permittivity ε_r and dimensions $L \times L \times D$ is inserted a distance x into the space between the plates.

1. Find the capacitance of this system.
2. Suppose that the capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the dielectric slab is inserted an additional distance dx into the space between the plates, show that the change in stored energy is

$$dU = \frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D} dx.$$

3. Suppose that before the slab is moved by dx, the plates are disconnected from the battery, so that the charges on the plates remain constant. Determine the magnitude of the charge on each plate, and then show that when the slab is moved dx farther into the space between the plates, the stored energy changes by an amount that is the negative of the expression for dU given in *question 2*.
4. If F is the force exerted on the slab by the charges on the plates, then dU should equal the work done against this force to move the slab a distance dx. Thus $dU = -Fdx$. Show that applying this expression to the result of *question 2* suggests that the electric force on the slab pushes it out of the capacitor, while the result of *question 3* suggests that the force pulls the slab into the capacitor.
5. As we discussed in class, the force in fact pulls the slab into the capacitor. Explain why the result of *question 2* gives an incorrect answer for the direction of this force, and calculate the magnitude of the force.

Problem 3

Part A

Two thin conducting wires shaped as cylinders with radii r_0, separated by distance b from each other, are attached to a large conducting slab of thickness a, where $a \ll r_0 \ll b$. Estimate the resistance between the wires, assuming that the conductivity of the wires σ_0 is much larger than the conductivity of the slab σ.

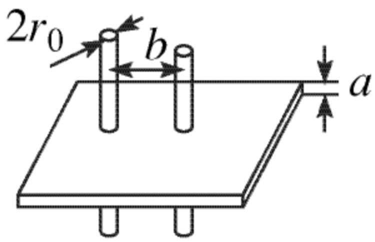

Figure 2: 3A

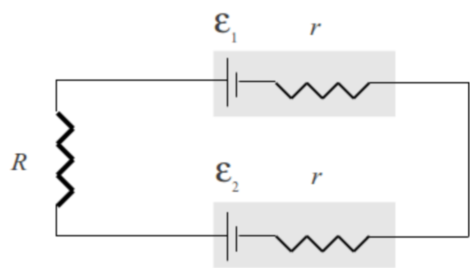

Figure 3: 3B

Part B

Consider the circuit shown in the figure below ($\varepsilon_1 = 12\ V$, $\varepsilon_2 = 8\ V$, $r = 1\ \Omega$, $R = 8\ \Omega$).

1. Find the current through the resistor R,
2. and the total rate of dissipation of electrical energy in the resistor R and in the internal resistance of the batteries.
3. In one of the batteries, chemical energy is being converted into electrical energy. In which one it is happening, and at what rate?
4. In one of the batteries, electrical energy is being converted into chemical energy. In which one it is happening, and at what rate?
5. Show that the overall rate of production of electrical energy is equal to the overall rate of consumption of electrical energy in the circuit.

Part C

Strictly speaking, the formula $q(t) = Q_{\max}e^{-t/RC}$ implies that an infinite amount of time is required to discharge a capacitor in a $R-C$ circuit completely. Yet for practical purposes, a capacitor may be considered to be fully discharged after a finite time t_d, defined as the time when the charge on the capacitor $q(t_d)$ differs from zero by no more than the charge of one electron.

1. Find t_d if $C = 0.92\ \mu$F, $R = 670$ kΩ, and $Q_{\max} = 7\ \mu$C.
2. For a given $Q_{\max}$ is the time required to reach this state always the same number of time constants, independent of R and C. Why or why not?