# Due: 11:59 pm on Oct 24, 2024

## Problem 1

### Part A

A particle with mass m and charge q moves in mutually perpendicular electric and magnetic fields $\mathbf{E} = (0, 0, E_0)$ and $\mathbf{B} = (B_0, 0, 0)$, where $E_0$ and $B_0$ are positive constants. Find and sketch the trajectory (use the computer software) of the particle if it starts out at the origin with velocity

1. $\mathbf{v}(0) = (E/B)\hat{n}_y$,
2. $\mathbf{v}(0) = (E/2B)\hat{n}_y$,
3. $\mathbf{v}(0) = (E/B)(\hat{n}_y + \hat{n}_z)$.

### Part B

A particle with mass $m$ and positive charge $q$ moves in antiparallel electric and magnetic fields $\mathbf{E} = (-E_0, 0, 0)$ and $\mathbf{B} = (B_0, 0, 0)$, where $E_0$ and $B_0$ are positive constants. Assuming the initial conditions: $\mathbf{v}(0) = (v_{0x}, v_{0y}, 0)$ and $\mathbf{r}(0) = (0, 0, 0)$, find the velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ for $t > 0$.

## Problem 2

### Part A

A circular loop of radius $R$ carries a clockwise electric current $I$. The loop is placed in a uniform magnetic field $\mathbf{B}$ (see the figure).

1. What is the net force on the current loop?
2. Find the torque on the current loop with respect to the axis of symmetry of the loop perpendicular to the vector $\mathbf{B}$.

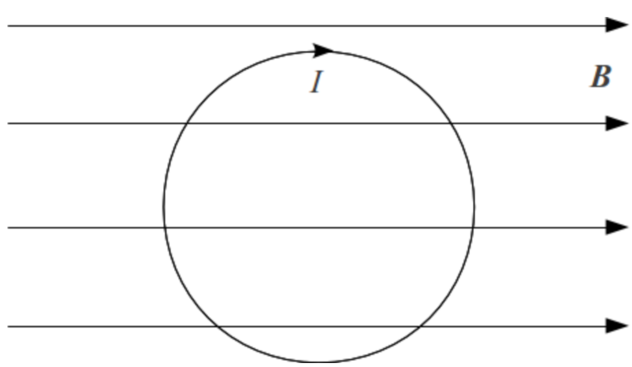


Figure 1: 2A

### Part B

In class we derived an expression for the torque on a current loop assuming that the magnetic field $\mathbf{B}$ was uniform. But what if $\mathbf{B}$ is not uniform?
Assume we have a square loop of wire that lies in the $xy$-plane. The loop has corners at (0, 0), (0, $L$), ($L$, $L$) and ($L$, 0) and carries a constant current $I$ in the clockwise direction. The magnetic field $\mathbf{B} = (B_0 y/L, B_0 x/L, 0)$, where $B_0$ is a positive constant.

1. Sketch the magnetic field lines in the $xy$-plane.
2. Find the magnitude and direction of the magnetic force exerted on each of the sides of the loop.
3. If the loop is free to rotate about the $x$-axis, find the magnitude and direction of the magnetic torque on the loop.
4. Repeat part (c) for the case in which the loop is free to rotate about the $y$-axis.
5. Is equation $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ an appropriate description of the torque on this loop. Why or why not?

### Part C

In a certain region of space, the magnetic field $\mathbf{B}$ is not uniform: it has both a $z$-component and a component that points radially away from or towards the $z$-axis. The $z$-component is given by $B_z(z) = \beta z$, where $\beta$ is a positive constant. The radial component $B_r$ depends only on $r$, the radial distance from the $z$-axis.

1. Use Gauss's law for magnetism, to find $B_r$ as a function of $r$.
2. Sketch the magnetic field lines.

## Problem 3

### Part A

What is the force on the equilateral triangle loop with side $a$ placed at distance $s$ from a long, straight-line wire? The electric current in both is $I$.

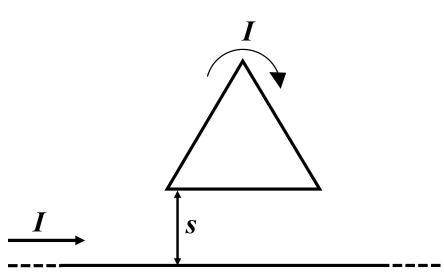


Figure 2: 3A

**Part B**

A large parallel-plate capacitor with uniform surface charge of density $\sigma$ on the upper plate and $-\sigma$ on the lower is moving with a constant speed $v$, as shown in the figure below.

1. Find the magnetic field between the plates and also above and below them.
2. Find the magnetic force per unit area on the upper plate, including its direction.
3. At what speed $v$ would the magnetic force balance the electric force?

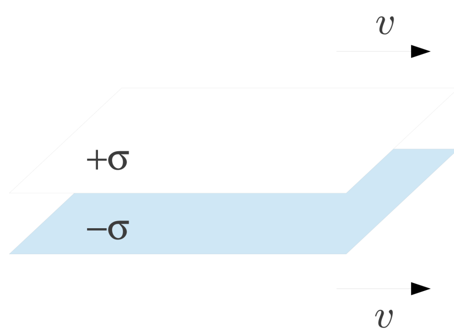


Figure 3: 3B

**Part C**

A thin disk made of dielectric material with radius $a$ has total charge $Q > 0$ distributed uniformly over its surface. It rotates $n$ times per second about the axis perpendicular to the surface of the disk and passing through its center. Find the magnetic field at the center of the disk.

**Part D | Optional**

An infinitely long conducting tape of width $L$ and negligible thickness lies in a horizontal plane and carries a uniform current $I$ (in the direction of the long dimension).

1. Show that at on the axis of symmetry of the tape, at a distance $y$ from its surface, the magnitude of the magnetic field is equal to $B(y) = (\mu_0 I/\pi L)\arctan(L/2y)$.
2. Discuss the result in the limit $y \gg L$.