HW6 Solutions

Problem 1

Part A.

1. $\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu}{\rho}\frac{\partial E_y}{\partial t} = -\frac{E_0 \mu \omega}{\rho} e^{-k_c x} \cos(t\omega - k_c x)$.

2. wave energy is transfered to heat.

3. $d = \frac{1}{k_c} = \sqrt{\frac{2\rho}{2\pi \nu \mu_0 \mu_r}} = 6.601 \times 10^{-5}\ m$.

Problem 2.

Part A.

1. $m_e \frac{d\vec{v}_e}{dt} = -e(\vec{E} + \vec{v}_e \times \vec{B}) \approx -e\vec{E}$

$\vec{j} = -\rho_e e \vec{v}_e = i\frac{\rho_e e^2 \vec{E}}{m_e \omega}$.

2. $\nabla^2 \vec{E} - \frac{1}{c^2}\cancel{\left(1 - \frac{\omega_p^2}{\omega^2}\right)}\left(1 - \frac{\omega_p^2}{\omega^2}\right)\frac{\partial^2 \vec{E}}{\partial t^2} = 0$. $\omega_p^2 = \frac{\rho_e e^2}{m_e \varepsilon_0}$

$\vec{B}$ is the same.

3. $\omega^2 > \omega_p^2 \Rightarrow \rho_e < \frac{\varepsilon_0 m_e \omega^2}{e^2}$.

Part B.

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, $\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$, $\nabla \cdot \vec{D} = \nabla \cdot \vec{B} = 0$.

$\Rightarrow \nabla^2 \vec{E} + (\omega^2 \varepsilon \mu + i\omega \mu \sigma)\vec{E} = 0$.

let $k'^2 = \omega^2 \mu \varepsilon'$, $\varepsilon' = \varepsilon + i\frac{\sigma}{\omega}$.

for $k' = \beta + i\alpha$. $\beta^2 - \alpha^2 = \omega^2 \mu \varepsilon$, $\beta\alpha = \frac{1}{2}\omega\mu\sigma$.

$$\Rightarrow \begin{cases} \beta = \omega\sqrt{\mu\varepsilon}\left[\frac{1}{2}\left(1 + \sqrt{1 + \frac{\sigma^2}{\varepsilon^2 \omega^2}}\right)\right]^{1/2} \\ \alpha = \omega\sqrt{\mu\varepsilon}\left[\frac{1}{2}\left(-1 + \sqrt{1 + \frac{\sigma^2}{\varepsilon^2 \omega^2}}\right)\right]^{1/2} \end{cases}$$

1. $\frac{\sigma}{\varepsilon\omega} \gg 1$. $\beta = \alpha \approx \sqrt{\frac{\omega\mu\sigma}{2}}$ $\vec{E}'' = E''_{y0} e^{-\alpha z} e^{i(\beta z - \omega t)} \hat{e}_y$

Here, $E''_{y0} = \frac{2E_{y0}}{1 + \sqrt{\frac{\mu\sigma}{2\varepsilon_0 \omega \mu}} + i\sqrt{\frac{\mu\sigma}{2\varepsilon_0 \omega \mu}}}$

2. $\sigma = i\frac{ne^2}{m\omega}$. $\omega_p^2 = \frac{ne^2}{m\varepsilon_0}$ $k'^2 = \frac{\omega^2 - \omega_p^2}{c^2}$

for $\omega_p^2 < \omega^2$. $\vec{E}'' = E''_{y0} e^{i(kz - \omega t)} \hat{e}_y$. $E''_{y0} = \frac{2E_{y0}}{1 + (1 - \omega_p^2/\omega^2)^{1/2}}$

for $\omega_p^2 > \omega^2$ $\vec{E}'' = E''_{y0} e^{-|k'|z} e^{i\omega t} \hat{e}_y$. $E''_{y0} = \frac{2E_{y0}}{1 + i(\omega_p/\omega^2 - 1)^{1/2}}$

3. $\omega_p = 0.56 \times 10^{16}\ Hz$ for $n \approx 10^{22}\ cm^{-3} \Rightarrow \omega_p^2 < \omega^2$ ✓

Problem 3.

Part A

1. $\vec{E} = \rho \vec{J} = \frac{\rho I}{\pi a^2} \vec{n_z}$, $\vec{B} = \frac{\mu_0 I}{2\pi a} \vec{n_\theta}$

2. $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = -\frac{\rho I^2}{2\pi^2 a^3} \vec{n_r}$

3. $P_{in} = 2\pi a L \frac{\rho I^2}{2\pi^2 a^3} = \frac{L \rho I^2}{\pi a^2}$

4. $P_{heat} = I^2 R = I^2 \frac{\rho L}{A} = P_{in}$.

The direction means the energy goes into conductor and become heat.