Additional Homework #5

Due: 11:59 pm on Dec 17, 2024

Problem 1 (25 pts)

At the initial instant of time $t = 0$, the wave function of a particle in an infinite potential well with walls at $x = 0$ and $x = a$ is $\Psi(x, 0) = \sqrt{\frac{1}{3}}\psi_3 + \sqrt{\frac{2}{3}}\psi_5$, where ψ_3 and ψ_5 are normalized solutions of the stationary Schrödinger equation.

1. Is Ψ normalized? Explain.
2. Write down the explicit form of $\Psi(x, 0)$. What is the probability that the particle is in the region $0 < x < a/2$ (use a computer to calculate the integral).
3. What are possible outcomes of a measurement of energy in the state described by $\Psi(x, 0)$? What are their probabilities?
4. What is the average value of the particle's total energy in this state?
5. What is the wave function of the particle for $t > 0$?

Problem 2 (35 pts)

A particle moving in an infinite potential well with walls at $x = 0$ and $x = a$ is in its ground state. At $t = 0$, the right wall of the well is suddenly moved to $x = 2a$.

1. Find the wave function of the particle $\Psi(x, t)$ for $t > 0$. Use a computer to calculate the integrals in the expansion coefficients and list the values of the first five coefficients $c_1, c_2, \ldots, c_5$. Plot $|\Psi(x, t)|^2$ for $t = 0$ and three other instants $t > 0$. *Hint.* Is the ground-state wave function in the narrow well an eigenstate in the wide well?
2. What is the probability that a measurement of the particle's energy at $t = 0^+$ finds its value unchanged?

Problem 3 (40 pts)

1. Find the energy levels of a particle moving in the potential field with

$$V(x) = \begin{cases} \infty & \text{for } x < 0, \\ -V_0 & \text{for } 0 \le x \le a, \\ 0 & \text{for } x > a \end{cases}$$

   where a, V_0 are positive constants. For the energy levels it is enough to write down a system of transcendental equations.

2. Discuss the solution of the system of transcendental equations from part 1 graphically. Is it possible to set the width and the depth of the well so that there are no bound states? One bound state?
3. For $a = 10^{-10}$ m and m equal to the mass of the electron, find the minimum depth of the well with exactly one bound state.