Due: 11:59 pm on Oct 10, 2024

Problem 1

Part A

Four circular plastic rods are charged uniformly, each with charge $Q < 0$. Rank the four arrangements according to the magnitude of the electric field at the origin. Explain your answer.

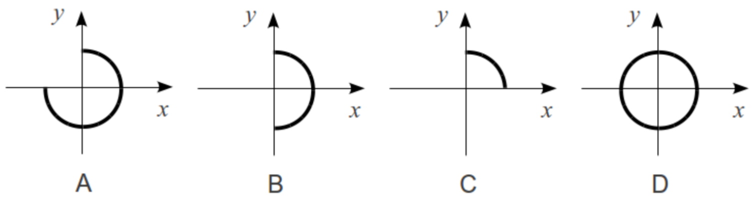


Figure 1: 1A

Part B

A thin disk with a circular hole in its center has its inner radius R_1, and the outer radius R_2. The disk is uniformly charged, with surface charge density $\sigma > 0$.

1. Find the electric field on the axis of the symmetry of the disk perpendicular to its surface.
   Rather than solving this problem by direct integration, use results and formulas derived in the lecture.

2. Show that at points on this axis, sufficiently close to the geometric center of the disk, the magnitude of the electric field is approximately proportional to the distance from the center. Then consider a particle with mass m and negative charge $-q$, which is placed on this axis, at distance $0.01R_1$ from the center of the disk and is free to move along the axis. Show that, after the particle is released at $t = 0$, its motion may be treated as harmonic. Find the period of oscillations.

Part C

Two thin rods of length l lie along the x-axis, one between $x = a/2$ and $x = a/2 + l$, and the other between $x = -a/2$ and $x = -a/2 - l$. Each rod has positive charge Q distributed uniformly along its length.

1. Show that the magnitude of the force that one rod exerts on the other is
$$F = \frac{Q^2}{4\pi\varepsilon_0 l^2} \ln\left[\frac{(a+l)^2}{a(a+2l)}\right].$$

2. Show that if $a \gg l$, the magnitude of this force reduces to $F = Q^2/4\pi\varepsilon_0 a^2$. What is the interpretation of this result?
   Hint. For $|u| \ll 1$, you may find the expansion $\ln(1+u) = u - u^2/2 + u^3/3 - \ldots$ useful.

Problem 2

Part A

A solid insulating ball with radius R is charged non-uniformly: the volume charge density $\rho = Ar/R$, where A is a positive constant, and r is the distance from the center of the ball

1. Show that the total charge of the ball is $Q = \pi A R^3$.

2. Find the electric field $\mathbf{E}(\mathbf{r})$ both inside and outside of the ball. Sketch $|\mathbf{E}|$ as a function of r.

Part B | Optional

Consider an infinite solid insulating cylinder with radius R uniform charged with constant density $\rho < 0$. Imagine that a cylindrical hole with radius a has been bored along the entire length of the cylinder. The axis of the cavity is a distance b from the axis of the cylinder, where $a < b < R$.
Find the magnitude and the direction of the electric field $\mathbf{E}$ inside the cavity to show that $\mathbf{E}$ is uniform over the entire cavity.

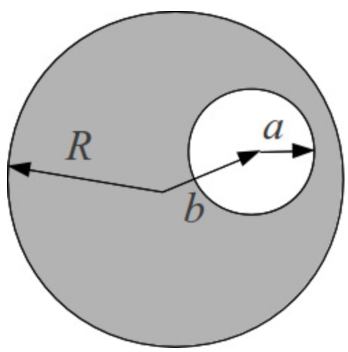


Figure 2: 2B

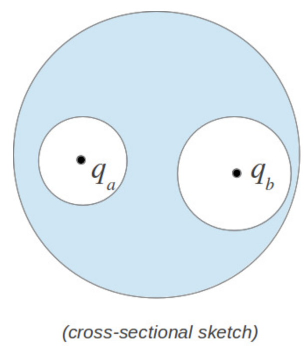


Figure 3: 2C

Part C

Two spherical cavities, of radii r_a and r_b, are hollowed out from the interior of a neutral conducting ball of radius R. At the center of each cavity a point charge is placed: q_a and q_b, respectively.

1. Find the surface densities of charge σ_a, σ_b on the walls of the cavities as well as on the surface of the ball σ_R.

2. What is the electric field outside of the conductor?

3. What is the electric field within each cavity?

4. What is the force on q_a and q_b?

5. Which of these answers would change if a third charge q_c were brought near the conductor?

Explain your answers.

Problem 3

Part A

Three identical charges $+Q$ are placed at the corners of a square of side a.

1. What is the electric field at the fourth corner (the one missing a charge) due to the first three charges?

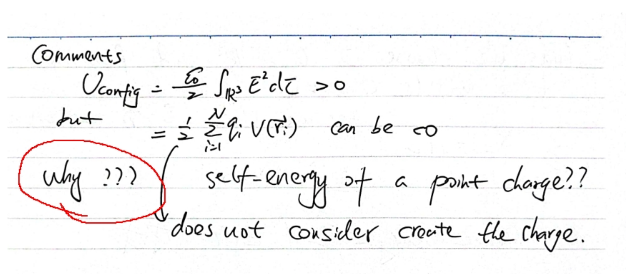
2. What is the electric potential at that corner?

3. How much work does it take to bring another charge, $+Q$, from infinity and place it at that corner?

4. How much energy did it take to assemble the configuration of three charges mentioned in the first sentence, starting with charges infinitely far apart from each other?

Part B

Read `https://farside.ph.utexas.edu/teaching/em/lectures/node56.html`.
You need to understand what is discussed in this image.


Comments

$U_{config} = \frac{\varepsilon_0}{2}\int_{\mathbb{R}^3} E^2 d\tau > 0$

but $= \frac{1}{2}\sum_{i=1}^{N} q_i V(\vec{r}_i)$ can be ∞

why ??? self-energy of a point charge??

does not consider create the charge.

Figure 4: 3B

Part C

Find the energy stored in a uniformly charged solid ball of radius R and charge q. This energy is also often called the "self-energy" of the charge distribution.

1. Use $U_{\text{conf}} = \frac{1}{2}\int_{\Omega} \rho V \, d\tau$ (specify ρ and the integration region Ω).
   Hint. To find the potential use the integral relation between the potential and the electric field. We have found the electric field due to this charge distribution in class; you may use this result without deriving it again.

2. Use $U_{\text{conf}} = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 d\tau$.

3. Use $U_{\text{conf}} = \frac{\varepsilon_0}{2}\left(\int_{\Omega} E^2 d\tau + \oint_{\Sigma} V\mathbf{E} \circ d\mathbf{A}\right)$ taking Σ as a sphere of radius $a > R$ centered at the center of the ball. Comment on what happens as $a \to \infty$.

4. Find the amount of work needed to be done to assemble the ball by bringing infinitesimal charges from far away.
   Hint. Use symmetry to choose the infinitesimal charges in a smart way.