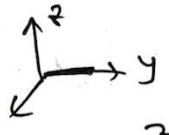
Solutions for HW 3.

Problem 1.

Part A.
$$\begin{cases} y(t) = C_1 \sin\omega t - C_2 \cos\omega t + \frac{E}{B} t + C_3 . \\ z(t) = C_1 \cos\omega t + C_2 \sin\omega t + C_4 . \end{cases}$$
$$\begin{cases} v_y(t) = C_1 \omega \cos\omega t + C_2 \omega \sin\omega t + \frac{E}{B} . \\ v_z(t) = -C_1 \omega \sin\omega t + C_2 \omega \cos\omega t . \end{cases}$$

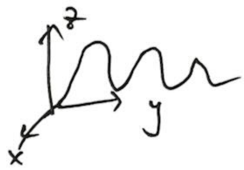
1. $y(0) = z(0) = v_z(0) = 0 .\quad v_y(0) = \frac{E}{B} \Rightarrow C_1 = C_2 = C_3 = C_4 = 0 .$

$\vec{r} = (0, \frac{E}{B} t, 0) .$



2. $C_1 = -\frac{E}{2B\omega} .\quad C_2 = C_3 = 0 .\quad C_4 = \frac{E}{2B\omega} .$

$\vec{r} = (0, -\frac{E}{2B\omega} \sin\omega t + \frac{E}{B} t, -\frac{E}{2B\omega} (\cos\omega t + \frac{E}{2B\omega})) .$



Part B. $\vec{r}(t) = \left(-\frac{1}{2} \frac{E_0 q t^2}{m} + V_{0x} t, \frac{m V_{0y}}{q B_0} \sin\omega t, \frac{m V_{0y}}{q B_0} (\cos\omega t - 1)\right) .$

$\vec{v}(t) = \left(-\frac{E_0 q t}{m} + V_{0x}, V_{0y} \cos\omega t, -V_{0y} \sin\omega t\right)$

Problem 2.

Part A. 1. 0 . 2. $T = \int_0^{2\pi} IB \sin\theta \cdot R d\theta \cdot R \sin\theta = BI\pi R^2 .$

Part B.

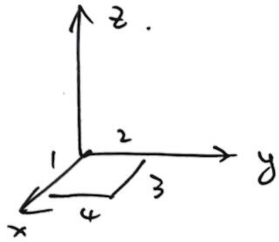
1.

2. $\vec{F_1} = -\frac{IB_0L}{2} \vec{n_z}$

$\vec{F_2} = -\frac{IB_0L}{2} \vec{n_z} .$

$\vec{F_3} = \frac{IB_0L}{2} \vec{n_z} .$

$\vec{F_4} = \frac{IB_0L}{2} \vec{n_z} .$



3. $T = \frac{IB_0L^2}{2} \vec{n_x} .$

4. $T = -\frac{IB_0L^2}{2} \vec{n_y} .$

5. No. It's appropriate for the integral case.

Part C.

1. $\nabla\cdot\vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0.$

$\frac{\partial}{\partial r}(rB_r) = -r\beta . \Rightarrow B_r = -\frac{1}{2}\beta r + \frac{c}{r}.$

$B_r(0) = 0 \Rightarrow B_r = -\frac{1}{2}\beta r .$

2. 

Problem 3.

Part A. $\vec{B} = \frac{\mu_0 I}{2\pi l}\hat{k} \cdot (l>0).$

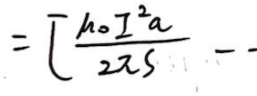
$\vec{F}_1 = I a B(s)\hat{j} = \frac{\mu_0 I^2 a}{2\pi s}\hat{j}$

$\vec{F}_2 = \int_s^{s+\frac{\sqrt{3}}{2}a} IB\frac{2\sqrt{3}}{3}dl\cdot\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \int_s^{s+\frac{\sqrt{3}}{2}a} IB\frac{2\sqrt{3}}{3}dl\left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right).$

$= -\frac{\mu_0 I^2}{\sqrt{3}\pi}\ln\left(\frac{\sqrt{3}a+2s}{2s}\right)\hat{j}.$

$\vec{F} = \left[\frac{\mu_0 I^2 a}{2\pi s} - \frac{\mu_0 I^2}{\sqrt{3}\pi}\ln\left(\frac{\sqrt{3}a}{2s}+1\right)\right]\hat{j}.$

Part B. 1.



$\oint B_1 dl = 2B_1 S = \mu_0 I_{end} = \mu_0 S\sigma v.$

$\Rightarrow B_1 = \frac{\mu_0\sigma v}{2} \Rightarrow B = \begin{cases} \mu_0\sigma v\,\hat{i} & \text{(between)} \\ 0 & \text{(other)} \end{cases}$

2. $\vec{F}_B = Bl I_{end}\hat{j} = \frac{\mu_0\sigma^2 v^2}{2}\hat{j}.$

3. $F_E = F_B \Rightarrow \frac{\sigma^2}{2\varepsilon_0} = \frac{\mu_0\sigma^2 v^2}{2} \Rightarrow v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Part C. $\sigma = \frac{Q}{\pi a^2}.$ $B = \int_0^a \frac{\mu_0}{4\pi}\left[\int_l \frac{d\vec{l}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right]\cdot n\sigma\cdot 2\pi r\,dr = \frac{n\mu_0 Q}{a}.$

Part D. 1. $\vec{B} = \hat{i}\cdot 2\int_0^{\arctan(\frac{L}{2y})} \frac{\mu_0 I}{2\pi L}\frac{1}{y\cos\theta}\,y\cos\theta\,d\theta = \frac{\mu_0 I}{\pi L}\arctan\left(\frac{L}{2y}\right)\hat{i}$

2. $\frac{L}{y} \ll 1 \Rightarrow \arctan\left(\frac{L}{2y}\right) \approx \frac{L}{2y} \Rightarrow \vec{B} \approx \frac{\mu_0 I}{2\pi y}\hat{i}.$ (Wire).