Ans for HW1.

Problem 1.

Part A. $|E_C| > |E_B| > |E_A| > |E_D|$.

$E_A = \frac{1}{3} E_C$. $E_B = \frac{\sqrt{2}}{2} E_C$. $E_D = 0$.

Part B.

$$\vec{E} = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)\hat{n}_x .$$

For $0 < x \ll R_1 < R_2$. $\vec{E} = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\hat{n}_x$.

$$m\frac{d^2x}{dt^2} = -Eq \Rightarrow \ddot{x} + \frac{\sigma q}{2\varepsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x = 0 .$$

$$T = 2\pi\sqrt{\frac{2\varepsilon_0 m R_1 R_2}{\sigma q (R_2 - R_1)}}$$

Part C.

$$F = \int_{-\frac{a}{2}}^{-\frac{a}{2}-l}\int_{\frac{a}{2}}^{\frac{a}{2}+l} \frac{1}{4\pi\varepsilon_0}\frac{(Q/l)^2}{(y-x)^2}\,dy\,dx$$

$$= \frac{Q^2}{4\pi\varepsilon_0 l^2}\ln\frac{(a+l)^2}{a(a+2l)}$$

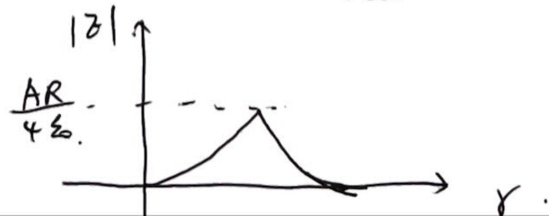
$$\approx \frac{Q^2}{4\pi\varepsilon_0 a(a+l)} \approx \frac{Q^2}{4\pi\varepsilon_0 a^2}.$$

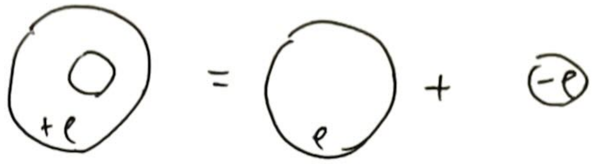
Problem 2.

Part A. $Q = \int_0^R \frac{Ar}{R}\cdot 4\pi r^2 dr = A\pi R^3$.

Inside: $Q(r) = \frac{A\pi r^4}{R} \Rightarrow \vec{E}(r) = \frac{Ar^2}{4\varepsilon_0 R}\hat{n}_r$.

Outside: $\vec{E}(r) = \frac{AR^3}{4\varepsilon_0 r^2}\hat{n}_r$.



Part B. 

$$\vec{E} = \frac{\pi r^2 \rho}{\varepsilon_0 \cdot 2\pi r}\hat{r} - \frac{\pi r_a^2 \rho}{\varepsilon_0 \cdot 2\pi r_a}\hat{r}_a = \frac{\rho}{2\varepsilon_0}\vec{r}_b .$$

It's uniform.

Part C. $\sigma_a = -\frac{q_a}{4\pi r_a^2}$. $\sigma_b = -\frac{q_b}{4\pi r_b^2}$.

$$\sigma_R = \frac{q_a + q_b}{4\pi R^2}.$$

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_a + q_b}{r^2}\hat{r}.$$

$$\vec{E}_a = \frac{1}{4\pi\varepsilon_0}\frac{q_a}{r_a^2}\hat{r}_a \quad \vec{E}_b = \frac{1}{4\pi\varepsilon_0}\frac{q_b}{r_b^2}\hat{r}_b .$$

$$\vec{F} = 0 .$$

σ_R and the electric field outside will change.

Problem 3.

Part A

$$\vec{E} = \left(1+\frac{\sqrt{2}}{4}\right)\frac{Q}{4\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}).$$

$$V = \frac{Q}{4\pi\varepsilon_0 a}\left(2+\frac{\sqrt{2}}{2}\right).$$

$$W = QV = \left(2+\frac{\sqrt{2}}{2}\right)\frac{Q^2}{4\pi\varepsilon_0 a}.$$

$$U_{config} = \frac{1}{2}\sum_{i=1}^{3} q_i V(\vec{r}_i) = \left(2+\frac{\sqrt{2}}{2}\right)\frac{Q^2}{4\pi\varepsilon_0 a}.$$

Part C. All are $U_{config} = \frac{3q^2}{20\pi\varepsilon_0 R}$.