## Special relativity

For history and motivation, see History of special relativity.

In physics, **special relativity** (**SR**, also known as the **special theory of relativity** or **STR**) is the generally accepted and experimentally well confirmed physical theory regarding the relationship between space and time. In Einstein's original pedagogical treatment, it is based on two postulates:

- 1. that the laws of physics are invariant (i.e. identical) in all inertial systems (non-accelerating frames of reference).
- that the speed of light in a vacuum is the same for all observers, regardless of the motion of the light source.

It was originally proposed in 1905 by Albert Einstein in the paper "On the Electrodynamics of Moving Bodies". [1] The inconsistency of Newtonian mechanics with Maxwell's equations of electromagnetism and the lack of experimental confirmation for a hypothesized luminiferous aether led to the development of special relativity, which corrects mechanics to handle situations involving motions nearing the speed of light. As of today, special relativity is the most accurate model of motion at any speed. Even so, the Newtonian mechanics model is still useful (due to its simplicity and high accuracy) as an approximation at small velocities relative to the speed of light.

Special relativity implies a wide range of consequences, which have been experimentally verified, [2] including length contraction, time dilation, relativistic mass, massenergy equivalence, a universal speed limit and relativity of simultaneity. It has replaced the conventional notion of an absolute universal time with the notion of a time that is dependent on reference frame and spatial position. Rather than an invariant time interval between two events, there is an invariant spacetime interval. Combined with other laws of physics, the two postulates of special relativity predict the equivalence of mass and energy, as expressed in the mass—energy equivalence formula  $E = mc^2$ , where c is the speed of light in a vacuum. [3][4]

A defining feature of special relativity is the replacement of the Galilean transformations of Newtonian mechanics with the Lorentz transformations. Time and space cannot be defined separately from each other. Rather space and time are interwoven into a single continuum known as spacetime. Events that occur at the same time for one observer can occur at different times for another.

The theory is "special" in that it only applies in the special case where the curvature of spacetime due to gravity is negligible. [5][6] In order to include gravity, Einstein formulated general relativity in 1915. Special relativity, contrary to some outdated descriptions, is capable of handling accelerated frames of reference. [7]

As Galilean relativity is now considered an approximation of special relativity that is valid for low speeds, special relativity is considered an approximation of general relativity that is valid for weak gravitational fields, i.e. at a sufficiently small scale and in conditions of free fall. Whereas general relativity incorporates noneuclidean geometry in order to represent gravitational effects as the geometric curvature of spacetime, special relativity is restricted to the flat spacetime known as Minkowski space. A locally Lorentz-invariant frame that abides by special relativity can be defined at sufficiently small scales, even in curved spacetime.

Galileo Galilei had already postulated that there is no absolute and well-defined state of rest (no privileged reference frames), a principle now called Galileo's principle of relativity. Einstein extended this principle so that it accounted for the constant speed of light, [8] a phenomenon that had been recently observed in the Michelson–Morley experiment. He also postulated that it holds for all the laws of physics, including both the laws of mechanics and of electrodynamics. [9]

#### 1 Postulates

Einstein discerned two fundamental propositions that seemed to be the most assured, regardless of the exact validity of the (then) known laws of either mechanics or electrodynamics. These propositions were the constancy of the speed of light and the independence of physical laws (especially the constancy of the speed of light) from the choice of inertial system. In his initial presentation of special relativity in 1905 he expressed these postulates as:<sup>[1]</sup>

• The Principle of Relativity – The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems in uniform translatory motion relative to each other.<sup>[1]</sup>

2 1 POSTULATES



Albert Einstein around 1905, the year his "Annus Mirabilis papers" – which included Zur Elektrodynamik bewegter Körper, the paper founding special relativity – were published.

• The Principle of Invariant Light Speed – "... light is always propagated in empty space with a definite velocity [speed] c which is independent of the state of motion of the emitting body" (from the preface). That is, light in vacuum propagates with the speed c (a fixed constant, independent of direction) in at least one system of inertial coordinates (the "stationary system"), regardless of the state of motion of the light source.

The derivation of special relativity depends not only on these two explicit postulates, but also on several tacit assumptions (made in almost all theories of physics), including the isotropy and homogeneity of space and the independence of measuring rods and clocks from their past history.<sup>[11]</sup>

Following Einstein's original presentation of special relativity in 1905, many different sets of postulates have been proposed in various alternative derivations. [12] However, the most common set of postulates remains those employed by Einstein in his original paper. A more mathematical statement of the Principle of Relativity made later by Einstein, which introduces the concept of simplicity not mentioned above is:

Special principle of relativity: If a system of coordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form, the *same* laws hold good in relation to

any other system of coordinates K' moving in uniform translation relatively to K.<sup>[13]</sup>

Henri Poincaré provided the mathematical framework for relativity theory by proving that Lorentz transformations are a subset of his Poincaré group of symmetry transformations. Einstein later derived these transformations from his axioms.

Many of Einstein's papers present derivations of the Lorentz transformation based upon these two principles.<sup>[14]</sup>

Einstein consistently based the derivation of Lorentz invariance (the essential core of special relativity) on just the two basic principles of relativity and light-speed invariance. He wrote:

The insight fundamental for the special theory of relativity is this: The assumptions relativity and light speed invariance are compatible if relations of a new type ("Lorentz transformation") are postulated for the conversion of coordinates and times of events... The universal principle of the special theory of relativity is contained in the postulate: The laws of physics are invariant with respect to Lorentz transformations (for the transition from one inertial system to any other arbitrarily chosen inertial system). This is a restricting principle for natural laws...<sup>[10]</sup>

Thus many modern treatments of special relativity base it on the single postulate of universal Lorentz covariance, or, equivalently, on the single postulate of Minkowski spacetime. [15][16]

From the principle of relativity alone without assuming the constancy of the speed of light (i.e. using the isotropy of space and the symmetry implied by the principle of special relativity) one can show that the spacetime transformations between inertial frames are either Euclidean, Galilean, or Lorentzian. In the Lorentzian case, one can then obtain relativistic interval conservation and a certain finite limiting speed. Experiments suggest that this speed is the speed of light in vacuum. [17][18]

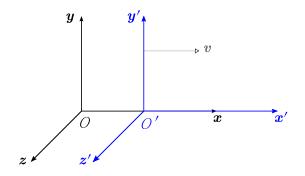
The constancy of the speed of light was motivated by Maxwell's theory of electromagnetism and the lack of evidence for the luminiferous ether. There is conflicting evidence on the extent to which Einstein was influenced by the null result of the Michelson–Morley experiment. [19][20] In any case, the null result of the Michelson–Morley experiment helped the notion of the constancy of the speed of light gain widespread and rapid acceptance.

# 2 Lack of an absolute reference perspective in space which is not undergoing any change in motion (acceleration), from which a position can be

The principle of relativity, which states that there is no preferred inertial reference frame, dates back to Galileo, and was incorporated into Newtonian physics. However, in the late 19th century, the existence of electromagnetic waves led physicists to suggest that the universe was filled with a substance that they called "aether", which would act as the medium through which these waves, or vibrations travelled. The aether was thought to constitute an absolute reference frame against which speeds could be measured, and could be considered fixed and motionless. Aether supposedly possessed some wonderful properties: it was sufficiently elastic to support electromagnetic waves, and those waves could interact with matter, yet it offered no resistance to bodies passing through it. The results of various experiments, including the Michelson-Morley experiment, led to the theory of special relativity, by showing that there was no aether.<sup>[21]</sup> Einstein's solution was to discard the notion of an aether and the absolute state of rest. In relativity, any reference frame moving with uniform motion will observe the same laws of physics. In particular, the speed of light in vacuum is always measured to be c, even when measured by multiple systems that are moving at different (but constant) velocities.

# 3 Reference frames, coordinates and the Lorentz transformation

Main article: Lorentz transformation Reference frames play a crucial role in relativity theory.



The primed system is in motion relative to the unprimed system with constant velocity v only along the x-axis, from the perspective of an observer stationary in the unprimed system. By the principle of relativity, an observer stationary in the primed system will view a likewise construction except that the velocity they record will be -v. The changing of the speed of propagation of interaction from infinite in non-relativistic mechanics to a finite value will require a modification of the transformation equations mapping events in one frame to another.

The term reference frame as used here is an observational

perspective in space which is not undergoing any change in motion (acceleration), from which a position can be measured along 3 spatial axes. In addition, a reference frame has the ability to determine measurements of the time of events using a 'clock' (any reference device with uniform periodicity).

An event is an occurrence that can be assigned a single unique time and location in space relative to a reference frame: it is a "point" in spacetime. Since the speed of light is constant in relativity in each and every reference frame, pulses of light can be used to unambiguously measure distances and refer back the times that events occurred to the clock, even though light takes time to reach the clock after the event has transpired.

For example, the explosion of a firecracker may be considered to be an "event". We can completely specify an event by its four spacetime coordinates: The time of occurrence and its 3-dimensional spatial location define a reference point. Let's call this reference frame *S*.

In relativity theory we often want to calculate the position of a point from a different reference point.

Suppose we have a second reference frame S', whose spatial axes and clock exactly coincide with that of S at time zero, but it is moving at a constant velocity v with respect to S along the x-axis.

Since there is no absolute reference frame in relativity theory, a concept of 'moving' doesn't strictly exist, as everything is always moving with respect to some other reference frame. Instead, any two frames that move at the same speed in the same direction are said to be *comoving*. Therefore, *S* and *S'* are not *comoving*.

Define the event to have spacetime coordinates (t,x,y,z) in system S and (t',x',y',z') in S'. Then the Lorentz transformation specifies that these coordinates are related in the following way:

$$t' = \gamma (t - vx/c^{2})$$

$$x' = \gamma (x - vt)$$

$$y' = y$$

$$z' = z,$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is the Lorentz factor and c is the speed of light in vacuum, and the velocity v of S' is parallel to the x-axis. The y and z coordinates are unaffected; only the x and t coordinates are transformed. These Lorentz transformations form a one-parameter group of linear mappings, that parameter being called rapidity.

There is nothing special about the x-axis, the transformation can apply to the y or z axes, or indeed in any di-

rection, which can be done by directions parallel to the motion (which are warped by the  $\gamma$  factor) and perpendicular; see main article for details.

A quantity invariant under Lorentz transformations is known as a Lorentz scalar.

Writing the Lorentz transformation and its inverse in terms of coordinate differences, where for instance one event has coordinates  $(x_1, t_1)$  and  $(x'_1, t'_1)$ , another event has coordinates  $(x_2, t_2)$  and  $(x'_2, t'_2)$ , and the differences are defined as

$$\Delta x' = x'_2 - x'_1$$
,  $\Delta x = x_2 - x_1$ ,  
 $\Delta t' = t'_2 - t'_1$ ,  $\Delta t = t_2 - t_1$ ,

we get

$$\Delta x' = \gamma \left( \Delta x - v \, \Delta t \right), \qquad \Delta x = \gamma \left( \Delta x' + v \, \Delta t' \right),$$
  
$$\Delta t' = \gamma \left( \Delta t - \frac{v \, \Delta x}{c^2} \right), \quad \Delta t = \gamma \left( \Delta t' + \frac{v \, \Delta x'}{c^2} \right)$$

These effects are not merely appearances; they are explicitly related to our way of measuring time intervals between events which occur at the same place in a given coordinate system (called "co-local" events). These time intervals will be different in another coordinate system moving with respect to the first, unless the events are also simultaneous. Similarly, these effects also relate to our measured distances between separated but simultaneous events in a given coordinate system of choice. If these events are not co-local, but are separated by distance (space), they will not occur at the same spatial distance from each other when seen from another moving coordinate system. However, the spacetime interval will be the same for all observers. The underlying reality remains the same. Only our perspective changes.

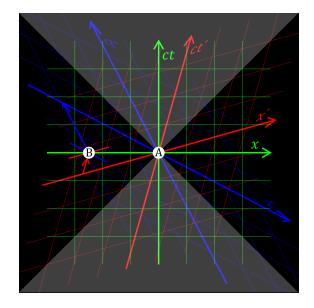
### 4 Consequences derived from the Lorentz transformation

See also: Twin paradox and Relativistic mechanics

The consequences of special relativity can be derived from the Lorentz transformation equations. [22] These transformations, and hence special relativity, lead to different physical predictions than those of Newtonian mechanics when relative velocities become comparable to the speed of light. The speed of light is so much larger than anything humans encounter that some of the effects predicted by relativity are initially counterintuitive.

#### 4.1 Relativity of simultaneity

See also: Relativity of simultaneity and Ladder paradox Two events happening in two different locations that oc-



Event B is simultaneous with A in the green reference frame, but it occurs before A in the blue frame, and occurs after A in the red frame.

cur simultaneously in the reference frame of one inertial observer, may occur non-simultaneously in the reference frame of another inertial observer (lack of absolute simultaneity).

From the first equation of the Lorentz transformation in terms of coordinate differences

$$\Delta t' = \gamma \left( \Delta t - \frac{v \, \Delta x}{c^2} \right)$$

it is clear that two events that are simultaneous in frame S (satisfying  $\Delta t = 0$ ), are not necessarily simultaneous in another inertial frame S' (satisfying  $\Delta t' = 0$ ). Only if these events are additionally co-local in frame S (satisfying  $\Delta x = 0$ ), will they be simultaneous in another frame S'.

#### 4.2 Time dilation

See also: Time dilation

The time lapse between two events is not invariant from one observer to another, but is dependent on the relative speeds of the observers' reference frames (e.g., the twin paradox which concerns a twin who flies off in a spaceship traveling near the speed of light and returns to discover that his or her twin sibling has aged much more).

Suppose a clock is at rest in the unprimed system S. The location of the clock on two different ticks is then characterized by  $\Delta x = 0$ . To find the relation between the times between these ticks as measured in both systems, the first equation can be used to find:

$$\Delta t' = \gamma \, \Delta t$$
 for events satisfying  $\Delta x = 0$ .

This shows that the time ( $\Delta t'$ ) between the two ticks as seen in the frame in which the clock is moving (S'), is *longer* than the time ( $\Delta t$ ) between these ticks as measured in the rest frame of the clock (S). Time dilation explains a number of physical phenomena; for example, the lifetime of muons produced by cosmic rays impinging on the Earth's atmosphere is measured to be greater than the lifetimes of muons measured in the laboratory. <sup>[23]</sup>

#### 4.3 Length contraction

See also: Lorentz contraction

The dimensions (e.g., length) of an object as measured by one observer may be smaller than the results of measurements of the same object made by another observer (e.g., the ladder paradox involves a long ladder traveling near the speed of light and being contained within a smaller garage).

Similarly, suppose a measuring rod is at rest and aligned along the x-axis in the unprimed system S. In this system, the length of this rod is written as  $\Delta x$ . To measure the length of this rod in the system S', in which the clock is moving, the distances x' to the end points of the rod must be measured simultaneously in that system S'. In other words, the measurement is characterized by  $\Delta t' = 0$ , which can be combined with the fourth equation to find the relation between the lengths  $\Delta x$  and  $\Delta x'$ :

$$\Delta x' = \frac{\Delta x}{\gamma}$$
 for events satisfying  $\Delta t' = 0$  .

This shows that the length  $(\Delta x')$  of the rod as measured in the frame in which it is moving (S'), is *shorter* than its length  $(\Delta x)$  in its own rest frame (S).

#### 4.4 Composition of velocities

See also: Velocity-addition formula

Velocities (speeds) do not simply add. If the observer in S measures an object moving along the x axis at velocity u, then the observer in the S' system, a frame of reference moving at velocity v in the x direction with respect to S, will measure the object moving with velocity u' where (from the Lorentz transformations above):

$$u' = \frac{dx'}{dt'} = \frac{\gamma (dx - vdt)}{\gamma (dt - vdx/c^2)} = \frac{(dx/dt) - v}{1 - (v/c^2)(dx/dt)} = \frac{\frac{u}{c^2 \ln b^2} \text{increased by taking in kinetic energies.}}{1 - uv/c^2}$$

The other frame *S* will measure:

$$u = \frac{dx}{dt} = \frac{\gamma \left( dx' + vdt' \right)}{\gamma \left( dt' + vdx'/c^2 \right)} = \frac{\left( dx'/dt' \right) + v}{1 + \left( v/c^2 \right) \left( dx'/dt' \right)} =$$

Notice that if the object were moving at the speed of light in the S system (i.e. u = c), then it would also be moving at the speed of light in the S' system. Also, if both u and v are small with respect to the speed of light, we will recover the intuitive Galilean transformation of velocities

$$u' \approx u - v$$
.

The usual example given is that of a train (frame S' above) traveling due east with a velocity v with respect to the tracks (frame S). A child inside the train throws a baseball due east with a velocity u' with respect to the train. In nonrelativistic physics, an observer at rest on the tracks will measure the velocity of the baseball (due east) as u = u' + v, while in special relativity this is no longer true; instead the velocity of the baseball (due east) is given by the second equation:  $u = (u' + v)/(1 + u'v/c^2)$ . Again, there is nothing special about the x or east directions. This formalism applies to any direction by considering parallel and perpendicular components of motion to the direction of relative velocity v, see main article for details.

### 5 Other consequences

#### 5.1 Thomas rotation

See also: Thomas rotation

The orientation of an object (i.e. the alignment of its axes with the observer's axes) may be different for different observers. Unlike other relativistic effects, this effect becomes quite significant at fairly low velocities as can be seen in the spin of moving particles.

#### 5.2 Equivalence of mass and energy

Main article: Mass-energy equivalence

As an object's speed approaches the speed of light from an observer's point of view, its relativistic mass increases thereby making it more and more difficult to accelerate it from within the observer's frame of reference.

The energy content of an object at rest with mass m equals  $mc^2$ . Conservation of energy implies that, in any reaction, a decrease of the sum of the masses of particles must be accompanied by an increase in kinetic energies of the particles after the reaction. Similarly, the mass of an object  $can De^{U}$  increased by taking in kinetic energies.

In addition to the papers referenced above—which give derivations of the Lorentz transformation and describe the foundations of special relativity—Einstein also wrote at least four papers giving heuristic arguments for the equivalence (and transmutability) of mass and energy, for  $\frac{1}{4} = \frac{mc_v^2}{\sqrt{c^2}}$ .

Mass–energy equivalence is a consequence of special relativity. The energy and momentum, which are separate in Newtonian mechanics, form a four-vector in relativity, and this relates the time component (the energy) to the space components (the momentum) in a non-trivial way. For an object at rest, the energy–momentum four-vector is (E/c, 0, 0, 0): it has a time component which is the energy, and three space components which are zero. By changing frames with a Lorentz transformation in the x direction with a small value of the velocity v, the energy momentum four-vector becomes  $(E/c, Ev/c^2, 0, 0)$ . The momentum is equal to the energy multiplied by the velocity divided by  $c^2$ . As such, the Newtonian mass of an object, which is the ratio of the momentum to the velocity for slow velocities, is equal to  $E/c^2$ .

The energy and momentum are properties of matter and radiation, and it is impossible to deduce that they form a four-vector just from the two basic postulates of special relativity by themselves, because these don't talk about matter or radiation, they only talk about space and time. The derivation therefore requires some additional physical reasoning. In his 1905 paper, Einstein used the additional principles that Newtonian mechanics should hold for slow velocities, so that there is one energy scalar and one three-vector momentum at slow velocities, and that the conservation law for energy and momentum is exactly true in relativity. Furthermore, he assumed that the energy of light is transformed by the same Doppler-shift factor as its frequency, which he had previously shown to be true based on Maxwell's equations.[1] The first of Einstein's papers on this subject was "Does the Inertia of a Body Depend upon its Energy Content?" in 1905. [24] Although Einstein's argument in this paper is nearly universally accepted by physicists as correct, even self-evident, many authors over the years have suggested that it is wrong.<sup>[25]</sup> Other authors suggest that the argument was merely inconclusive because it relied on some implicit assumptions.[26]

Einstein acknowledged the controversy over his derivation in his 1907 survey paper on special relativity. There he notes that it is problematic to rely on Maxwell's equations for the heuristic mass—energy argument. The argument in his 1905 paper can be carried out with the emission of any massless particles, but the Maxwell equations are implicitly used to make it obvious that the emission of light in particular can be achieved only by doing work. To emit electromagnetic waves, all you have to do is shake a charged particle, and this is clearly doing work, so that the emission is of energy. [27][28]

#### 5.3 How far can one travel from the Earth?

See also: Space travel using constant acceleration

Since one can not travel faster than light, one might conclude that a human can never travel farther from Earth than 40 light years if the traveller is active between the ages of 20 and 60. One would easily think that a traveller would never be able to reach more than the very few solar systems which exist within the limit of 20–40 light years from the earth. But that would be a mistaken conclusion. Because of time dilation, a hypothetical spaceship can travel thousands of light years during the pilot's 40 active years. If a spaceship could be built that accelerates at a constant 1 g, it will, after a little less than a year, be travelling at almost the speed of light as seen from Earth. This is described by:

$$v(t) = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}}$$

where v(t) it the velocity at a time, t, a is the acceleration of 1g and t is the time as measured by people on Earth. [29] Therefore, after 1 year of accelerating at 9.81m/s<sup>2</sup>, the spaceship will be travelling at v = 0.77c relative to Earth. Time dilation will increase the travellers life span as seen from the reference frame of the Earth to 2.7 years, but his lifespan measured by a clock travelling with him will not change. During his journey, people on Earth will experience more time than he does. A 5-year round trip for him will take 6½ Earth years and cover a distance of over 6 light-years. A 20-year round trip for him (5 years accelerating, 5 decelerating, twice each) will land him back on Earth having travelled for 335 Earth years and a distance of 331 light years.<sup>[30]</sup> A full 40-year trip at 1 g will appear on Earth to last 58,000 years and cover a distance of 55,000 light years. A 40-year trip at 1.1 g will take 148,000 Earth years and cover about 140,000 light years. A one-way 28 year (14 years accelerating, 14 decelerating as measured with the cosmonaut's clock) trip at 1 g acceleration could reach 2,000,000 light-years to the Andromeda Galaxy.[31] This same time dilation is why a muon travelling close to c is observed to travel much further than c times its half-life (when at rest). [32]

### 6 Causality and prohibition of motion faster than light

See also: Causality (physics) and Tachyonic antitelephone In diagram 2 the interval AB is 'time-like'; i.e., there is a frame of reference in which events A and B occur at the same location in space, separated only by occurring at different times. If A precedes B in that frame, then A precedes B in all frames. It is hypothetically possible for matter (or information) to travel from A to B, so there can be a causal relationship (with A the cause and B the effect).

The interval AC in the diagram is 'space-like'; i.e., there is a frame of reference in which events A and C occur simultaneously, separated only in space. There are also frames in which A precedes C (as shown) and frames in which C precedes A. If it were possible for a cause-and-effect relationship to exist between events A and C, then para-

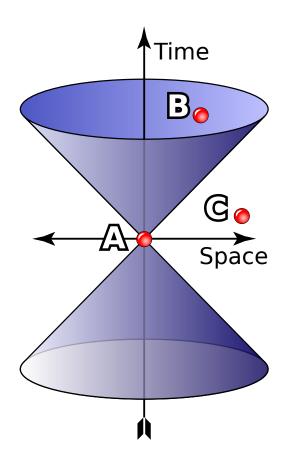


Diagram 2. Light cone

doxes of causality would result. For example, if A was the cause, and C the effect, then there would be frames of reference in which the effect preceded the cause. Although this in itself won't give rise to a paradox, one can show<sup>[33][34]</sup> that faster than light signals can be sent back into one's own past. A causal paradox can then be constructed by sending the signal if and only if no signal was received previously.

Therefore, if causality is to be preserved, one of the consequences of special relativity is that no information signal or material object can travel faster than light in vacuum. However, some "things" can still move faster than light. For example, the location where the beam of a search light hits the bottom of a cloud can move faster than light when the search light is turned rapidly.<sup>[35]</sup>

Even without considerations of causality, there are other strong reasons why faster-than-light travel is forbidden by special relativity. For example, if a constant force is applied to an object for a limitless amount of time, then integrating F = dp/dt gives a momentum that grows without bound, but this is simply because  $p = m\gamma v$  approaches infinity as v approaches c. To an observer who is not accelerating, it appears as though the object's inertia is increasing, so as to produce a smaller acceleration in response to the same force. This behavior is observed in particle accelerators, where each charged particle is accelerated by the electromagnetic force.

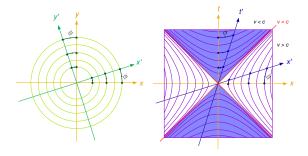
### 7 Geometry of spacetime

Main article: Minkowski space

# 7.1 Comparison between flat Euclidean space and Minkowski space

See also: line element

Special relativity uses a 'flat' 4-dimensional Minkowski



Orthogonality and rotation of coordinate systems compared between **left:** Euclidean space through circular angle  $\varphi$ , **right:** in Minkowski spacetime through hyperbolic angle  $\varphi$  (red lines labelled c denote the worldlines of a light signal, a vector is orthogonal to itself if it lies on this line). [36]

space – an example of a spacetime. Minkowski spacetime appears to be very similar to the standard 3-dimensional Euclidean space, but there is a crucial difference with respect to time.

In 3D space, the differential of distance (line element) *ds* is defined by

$$ds^2 = d\mathbf{x} \cdot d\mathbf{x} = dx_1^2 + dx_2^2 + dx_3^2$$

where  $d\mathbf{x} = (dx_1, dx_2, dx_3)$  are the differentials of the three spatial dimensions. In Minkowski geometry, there is an extra dimension with coordinate  $X^0$  derived from time, such that the distance differential fulfills

$$ds^2 = -dX_0^2 + dX_1^2 + dX_2^2 + dX_3^2,$$

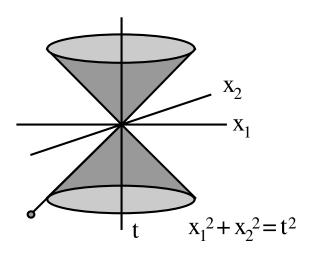
where  $d\mathbf{X} = (dX_0, dX_1, dX_2, dX_3)$  are the differentials of the four spacetime dimensions. This suggests a deep theoretical insight: special relativity is simply a rotational symmetry of our spacetime, analogous to the rotational symmetry of Euclidean space (see image right). [37] Just as Euclidean space uses a Euclidean metric, so spacetime uses a Minkowski metric. Basically, special relativity can be stated as the *invariance of any spacetime interval* (that is the 4D distance between any two events) when viewed from *any inertial reference frame*. All equations and effects of special relativity can be derived from this rotational symmetry (the Poincaré group) of Minkowski spacetime.

8 PHYSICS IN SPACETIME

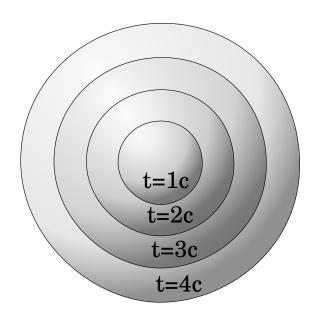
The actual form of ds above depends on the metric and on the choices for the  $X^0$  coordinate. To make the time coordinate look like the space coordinates, it can be treated as imaginary:  $X_0 = ict$  (this is called a Wick rotation). According to Misner, Thorne and Wheeler (1971, §2.3), ultimately the deeper understanding of both special and general relativity will come from the study of the Minkowski metric (described below) and to take  $X^0 = ct$ , rather than a "disguised" Euclidean metric using ict as the time coordinate.

Some authors use  $X^0 = t$ , with factors of c elsewhere to compensate; for instance, spatial coordinates are divided by c or factors of  $c^{\pm 2}$  are included in the metric tensor. [38] These numerous conventions can be superseded by using natural units where c = 1. Then space and time have equivalent units, and no factors of c appear anywhere.

#### 7.2 3D spacetime



Three-dimensional dual-cone.



Null spherical space.

If we reduce the spatial dimensions to 2, so that we can represent the physics in a 3D space

$$ds^2 = dx_1^2 + dx_2^2 - c^2 dt^2,$$

we see that the null geodesics lie along a dual-cone (see image right) defined by the equation;

$$ds^2 = 0 = dx_1^2 + dx_2^2 - c^2 dt^2$$

or simply

$$dx_1^2 + dx_2^2 = c^2 dt^2$$

which is the equation of a circle of radius c dt.

#### 7.3 4D spacetime

If we extend this to three spatial dimensions, the null geodesics are the 4-dimensional cone:

$$ds^2 = 0 = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2$$

SC

$$dx_1^2 + dx_2^2 + dx_3^2 = c^2 dt^2$$
.

This null dual-cone represents the "line of sight" of a point in space. That is, when we look at the stars and say "The light from that star which I am receiving is X years old", we are looking down this line of sight: a null geodesic. We are looking at an event a distance  $d=\sqrt{x_1^2+x_2^2+x_3^2}$  away and a time d/c in the past. For this reason the null dual cone is also known as the 'light cone'. (The point in the lower left of the picture above right represents the star, the origin represents the observer, and the line represents the null geodesic "line of sight".)

The cone in the -t region is the information that the point is 'receiving', while the cone in the +t section is the information that the point is 'sending'.

The geometry of Minkowski space can be depicted using Minkowski diagrams, which are useful also in understanding many of the thought-experiments in special relativity.

Note that, in 4d spacetime, the concept of the center of mass becomes more complicated, see center of mass (relativistic).

### 8 Physics in spacetime

## 8.1 Transformations of physical quantities between reference frames

Above, the Lorentz transformation for the time coordinate and three space coordinates illustrates that they are intertwined. This is true more generally: certain pairs of "timelike" and "spacelike" quantities naturally combine on equal footing under the same Lorentz transformation.

The Lorentz transformation in standard configuration above, i.e. for a boost in the x direction, can be recast into matrix form as follows:

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma ct - \gamma\beta x \\ \gamma x - \beta\gamma ct \\ y \\ z \end{pmatrix} \\ \psi^{\mu} = \frac{dX^{\mu}}{d\tau} = \gamma(v)(c, v_x, v_y, v_z) = \gamma(v)(c, \mathbf{v}).$$
 where the Lorentz factor is:

In Newtonian mechanics, quantities which have magnitude and direction are mathematically described as 3d vectors in Euclidean space, and in general they are parametrized by time. In special relativity, this notion is extended by adding the appropriate timelike quantity to a spacelike vector quantity, and we have 4d vectors, or "four vectors", in Minkowski spacetime. The components of vectors are written using tensor index notation, as this has numerous advantages. The notation makes it clear the equations are manifestly covariant under the Poincaré group, thus bypassing the tedious calculations to check this fact. In constructing such equations, we often find that equations previously thought to be unrelated are, in fact, closely connected being part of the same tensor equation. Recognizing other physical quantities as tensors simplifies their transformation laws. Throughout, upper indices (superscripts) are contravariant indices rather than exponents except when they indicate a square (this is should be clear from the context), and lower indices (subscripts) are covariant indices. For simplicity and consistency with the earlier equations, Cartesian coordinates will be used.

The simplest example of a four-vector is the position of an event in spacetime, which constitutes a timelike component ct and spacelike component  $\mathbf{x} = (x, y, z)$ , in a contravariant position four vector with components:

$$X^{\nu} = (X^0, X^1, X^2, X^3) = (ct, x, y, z) = (ct, \mathbf{x}).$$

where we define  $X^0 = ct$  so that the time coordinate has the same dimension of distance as the other spatial dimensions; so that space and time are treated equally. Now the transformation of the contravariant components of the position 4-vector can be compactly written as:

$$X^{\mu'} = \Lambda^{\mu'}{}_{\nu}X^{\nu}$$

where there is an implied summation on  $\nu$  from 0 to 3, and  $\Lambda^{\mu'}{}_{\nu}$  is a matrix.

More generally, all contravariant components of a four-vector  $T^{\nu}$  transform from one frame to another frame by a Lorentz transformation:

$$T^{\mu'} = \Lambda^{\mu'}_{\phantom{\mu'}\nu} T^{\nu}$$

Examples of other 4-vectors include the four-velocity  $U^{\mu}$ , defined as the derivative of the position 4-vector with respect to proper time:

$$\gamma(v) = \frac{1}{\sqrt{1 - (v/c)^2}}, \quad v^2 = v_x^2 + v_y^2 + v_z^2.$$

The relativistic energy  $E = \gamma(v)mc^2$  and relativistic momentum  $\mathbf{p} = \gamma(v)m\mathbf{v}$  of an object are respectively the timelike and spacelike components of a contravariant four momentum vector:

$$P^{\mu} = mU^{\mu} = m\gamma(v)(c, v_x, v_y, v_z) = (E/c, p_x, p_y, p_z) = (E/c, \mathbf{p}).$$

where m is the invariant mass.

The four-acceleration is the proper time derivative of 4-velocity:

$$A^{\mu} = \frac{dU^{\mu}}{d\tau} \, .$$

The transformation rules for *three*-dimensional velocities and accelerations are very awkward; even above in standard configuration the velocity equations are quite complicated owing to their non-linearity. On the other hand, the transformation of *four*-velocity and *four*-acceleration are simpler by means of the Lorentz transformation matrix.

The four-gradient of a scalar field  $\phi$  transforms covariantly rather than contravariantly:

$$\begin{pmatrix} \frac{1}{c} \frac{\partial \phi}{\partial t'} & \frac{\partial \phi}{\partial x'} & \frac{\partial \phi}{\partial y'} & \frac{\partial \phi}{\partial z'} \end{pmatrix} = \begin{pmatrix} \frac{1}{c} \frac{\partial \phi}{\partial t} & \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{pmatrix} \begin{pmatrix} \gamma & -\beta \gamma & 0 \\ -\beta \gamma & \gamma & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

that is:

$$(\partial_{\mu'}\phi) = \Lambda_{\mu'}{}^{\nu}(\partial_{\nu}\phi), \quad \partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}.$$

10 8 PHYSICS IN SPACETIME

only in Cartesian coordinates. It's the covariant derivative which transforms in manifest covariance, in Cartesian coordinates this happens to reduce to the partial derivatives, but not in other coordinates.

More generally, the *co*variant components of a 4-vector transform according to the *inverse* Lorentz transformation:

$$\Lambda_{\mu'}{}^{\nu}T^{\mu'}=T^{\nu}$$

where  ${\Lambda_{\mu'}}^{\nu}$  is the reciprocal matrix of  ${\Lambda^{\mu'}}_{\nu}$  .

The postulates of special relativity constrain the exact form the Lorentz transformation matrices take.

More generally, most physical quantities are best described as (components of) tensors. So to transform from one frame to another, we use the well-known tensor transformation law<sup>[42]</sup>

$$T_{\theta'\iota'\cdots\kappa'}^{\alpha'\beta'\cdots\zeta'} = \Lambda^{\alpha'}{}_{\mu}\Lambda^{\beta'}{}_{\nu}\cdots\Lambda^{\zeta'}{}_{\rho}\Lambda_{\theta'}{}^{\sigma}\Lambda_{\iota'}{}^{\upsilon}\cdots\Lambda_{\kappa'}{}^{\phi}T_{\sigma\upsilon\cdots\phi}^{\mu\nu\cdots\rho}$$

where  ${\Lambda_{\chi'}}^{\psi}$  is the reciprocal matrix of  ${\Lambda^{\chi'}}_{\psi}$  . All tensors transform by this rule.

An example of a four dimensional second order antisymmetric tensor is the relativistic angular momentum, which has six components: three are the classical angular momentum, and the other three are related to the boost of the center of mass of the system. The derivative of the relativistic angular momentum with respect to proper time is the relativistic torque, also second order antisymmetric tensor.

The electromagnetic field tensor is another second order antisymmetric tensor field, with six components: three for the electric field and another three for the magnetic field. There is also the stress—energy tensor for the electromagnetic field, namely the electromagnetic stress—energy tensor.

#### 8.2 Metric

The metric tensor allows one to define the inner product of two vectors, which in turn allows one to assign a magnitude to the vector. Given the four-dimensional nature of spacetime the Minkowski metric  $\eta$  has components (valid in any inertial reference frame) which can be arranged in a  $4 \times 4$  matrix:

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which is equal to its reciprocal,  $\eta^{\alpha\beta}$ , in those frames. Throughout we use the signs as above, different authors

use different conventions – see Minkowski metric alternative signs.

The Poincaré group is the most general group of transformations which preserves the Minkowski metric:

$$\eta_{\alpha\beta} = \eta_{\mu'\nu'} \Lambda^{\mu'}{}_{\alpha} \Lambda^{\nu'}{}_{\beta}$$

and this is the physical symmetry underlying special relativity.

The metric can be used for raising and lowering indices on vectors and tensors. Invariants can be constructed using the metric, the inner product of a 4-vector *T* with another 4-vector *S* is:

$$T^{\alpha}S_{\alpha}=T^{\alpha}\eta_{\alpha\beta}S^{\beta}=T_{\alpha}\eta^{\alpha\beta}S_{\beta}=\text{scalar invariant}$$

Invariant means that it takes the same value in all inertial frames, because it is a scalar (0 rank tensor), and so no  $\Lambda$  appears in its trivial transformation. The magnitude of the 4-vector T is the positive square root of the inner product with itself:

$$|\mathbf{T}| = \sqrt{T^{\alpha}T_{\alpha}}$$

One can extend this idea to tensors of higher order, for a second order tensor we can form the invariants:

$$T^{\alpha}{}_{\alpha}$$
,  $T^{\alpha}{}_{\beta}T^{\beta}{}_{\alpha}$ ,  $T^{\alpha}{}_{\beta}T^{\beta}{}_{\gamma}T^{\gamma}{}_{\alpha}$  = scalars invariant,

similarly for higher order tensors. Invariant expressions, particularly inner products of 4-vectors with themselves, provide equations that are useful for calculations, because one doesn't need to perform Lorentz transformations to determine the invariants.

#### 8.3 Relativistic kinematics and invariance

The coordinate differentials transform also contravariantly:

$$dX^{\mu'} = \Lambda^{\mu'}{}_{\nu} dX^{\nu}$$

so the squared length of the differential of the position four-vector  $dX^{\mu}$  constructed using

$$d\mathbf{X}^2 = dX^{\mu} dX_{\mu} = \eta_{\mu\nu} dX^{\mu} dX^{\nu} = -(cdt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

is an invariant. Notice that when the line element  $d\mathbf{X}^2$  is negative that  $\sqrt{-d\mathbf{X}^2}$  is the differential of proper time, while when  $d\mathbf{X}^2$  is positive,  $\sqrt{d\mathbf{X}^2}$  is differential of the proper distance.

The 4-velocity  $U^{\mu}$  has an invariant form:

$$\mathbf{U}^2 = \eta_{\nu\mu} U^{\nu} U^{\mu} = -c^2 \,,$$

which means all velocity four-vectors have a magnitude of c. This is an expression of the fact that there is no such thing as being at coordinate rest in relativity: at the least, you are always moving forward through time. Differentiating the above equation by  $\tau$  produces:

$$2\eta_{\mu\nu}A^{\mu}U^{\nu}=0.$$

So in special relativity, the acceleration four-vector and the velocity four-vector are orthogonal.

#### 8.4 Relativistic dynamics and invariance

The invariant magnitude of the momentum 4-vector generates the energy–momentum relation:

$$\mathbf{P}^2 = \eta^{\mu\nu} P_{\mu} P_{\nu} = -(E/c)^2 + p^2.$$

We can work out what this invariant is by first arguing that, since it is a scalar, it doesn't matter in which reference frame we calculate it, and then by transforming to a frame where the total momentum is zero.

$$\mathbf{P}^2 = -(E_{\text{rest}}/c)^2 = -(mc)^2$$
.

We see that the rest energy is an independent invariant. A rest energy can be calculated even for particles and systems in motion, by translating to a frame in which momentum is zero.

The rest energy is related to the mass according to the celebrated equation discussed above:

$$E_{\text{rest}} = mc^2$$
.

Note that the mass of systems measured in their center of momentum frame (where total momentum is zero) is given by the total energy of the system in this frame. It may not be equal to the sum of individual system masses measured in other frames.

To use Newton's third law of motion, both forces must be defined as the rate of change of momentum with respect to the same time coordinate. That is, it requires the 3D force defined above. Unfortunately, there is no tensor in 4D which contains the components of the 3D force vector among its components.

If a particle is not traveling at c, one can transform the 3D force from the particle's co-moving reference frame

into the observer's reference frame. This yields a 4-vector called the four-force. It is the rate of change of the above energy momentum four-vector with respect to proper time. The covariant version of the four-force is:

$$F_{\nu} = \frac{dP_{\nu}}{d\tau} = mA_{\nu}$$

In the rest frame of the object, the time component of the four force is zero unless the "invariant mass" of the object is changing (this requires a non-closed system in which energy/mass is being directly added or removed from the object) in which case it is the negative of that rate of change of mass, times c. In general, though, the components of the four force are not equal to the components of the three-force, because the three force is defined by the rate of change of momentum with respect to coordinate time, i.e. dp/dt while the four force is defined by the rate of change of momentum with respect to proper time, i.e.  $dp/d\tau$ .

In a continuous medium, the 3D *density of force* combines with the *density of power* to form a covariant 4-vector. The spatial part is the result of dividing the force on a small cell (in 3-space) by the volume of that cell. The time component is -1/c times the power transferred to that cell divided by the volume of the cell. This will be used below in the section on electromagnetism.

# 9 Relativity and unifying electromagnetism

Main articles: Classical electromagnetism and special relativity and Covariant formulation of classical electromagnetism

Theoretical investigation in classical electromagnetism led to the discovery of wave propagation. Equations generalizing the electromagnetic effects found that finite propagation speed of the **E** and **B** fields required certain behaviors on charged particles. The general study of moving charges forms the Liénard–Wiechert potential, which is a step towards special relativity.

The Lorentz transformation of the electric field of a moving charge into a non-moving observer's reference frame results in the appearance of a mathematical term commonly called the magnetic field. Conversely, the *magnetic* field generated by a moving charge disappears and becomes a purely *electrostatic* field in a comoving frame of reference. Maxwell's equations are thus simply an empirical fit to special relativistic effects in a classical model of the Universe. As electric and magnetic fields are reference frame dependent and thus intertwined, one speaks of *electromagnetic* fields. Special relativity provides the transformation rules for how an electromagnetic field in one inertial frame appears in another inertial frame.

Maxwell's equations in the 3D form are already consistent with the physical content of special relativity, although they are easier to manipulate in a manifestly covariant form, i.e. in the language of tensor calculus. [43] See main links for more detail.

#### 10 Status

Main articles: Tests of special relativity and Criticism of relativity theory

Special relativity in its Minkowski spacetime is accurate only when the absolute value of the gravitational potential is much less than  $c^2$  in the region of interest. [44] In a strong gravitational field, one must use general relativity. General relativity becomes special relativity at the limit of weak field. At very small scales, such as at the Planck length and below, quantum effects must be taken into consideration resulting in quantum gravity. However, at macroscopic scales and in the absence of strong gravitational fields, special relativity is experimentally tested to extremely high degree of accuracy  $(10^{-20})^{[45]}$  and thus accepted by the physics community. Experimental results which appear to contradict it are not reproducible and are thus widely believed to be due to experimental errors.

Special relativity is mathematically self-consistent, and it is an organic part of all modern physical theories, most notably quantum field theory, string theory, and general relativity (in the limiting case of negligible gravitational fields).

Newtonian mechanics mathematically follows from special relativity at small velocities (compared to the speed of light) – thus Newtonian mechanics can be considered as a special relativity of slow moving bodies. See classical mechanics for a more detailed discussion.

Several experiments predating Einstein's 1905 paper are now interpreted as evidence for relativity. Of these it is known Einstein was aware of the Fizeau experiment before 1905, [46] and historians have concluded that Einstein was at least aware of the Michelson–Morley experiment as early as 1899 despite claims he made in his later years that it played no role in his development of the theory. [20]

- The Fizeau experiment (1851, repeated by Michelson and Morley in 1886) measured the speed of light in moving media, with results that are consistent with relativistic addition of colinear velocities.
- The famous Michelson–Morley experiment (1881, 1887) gave further support to the postulate that detecting an absolute reference velocity was not achievable. It should be stated here that, contrary to many alternative claims, it said little about the invariance of the speed of light with respect to the source and observer's velocity, as both source and

- observer were travelling together at the same velocity at all times.
- The Trouton–Noble experiment (1903) showed that the torque on a capacitor is independent of position and inertial reference frame.
- The Experiments of Rayleigh and Brace (1902, 1904) showed that length contraction doesn't lead to birefringence for a co-moving observer, in accordance with the relativity principle.

Particle accelerators routinely accelerate and measure the properties of particles moving at near the speed of light, where their behavior is completely consistent with relativity theory and inconsistent with the earlier Newtonian mechanics. These machines would simply not work if they were not engineered according to relativistic principles. In addition, a considerable number of modern experiments have been conducted to test special relativity. Some examples:

- Tests of relativistic energy and momentum testing the limiting speed of particles
- Ives–Stilwell experiment testing relativistic Doppler effect and time dilation
- Time dilation of moving particles relativistic effects on a fast-moving particle's half-life
- Kennedy–Thorndike experiment time dilation in accordance with Lorentz transformations
- Hughes–Drever experiment testing isotropy of space and mass
- Modern searches for Lorentz violation various modern tests
- Experiments to test emission theory demonstrated that the speed of light is independent of the speed of the emitter.
- Experiments to test the aether drag hypothesis no "aether flow obstruction".

# 11 Theories of relativity and quantum mechanics

Special relativity can be combined with quantum mechanics to form relativistic quantum mechanics. It is an unsolved problem in physics how *general* relativity and quantum mechanics can be unified; quantum gravity and a "theory of everything", which require such a unification, are active and ongoing areas in theoretical research.

The early Bohr–Sommerfeld atomic model explained the fine structure of alkali metal atoms using both special relativity and the preliminary knowledge on quantum mechanics of the time.<sup>[47]</sup>

In 1928, Paul Dirac constructed an influential relativistic wave equation, now known as the Dirac equation in his honour, [48] that is fully compatible both with special relativity and with the final version of quantum theory existing after 1926. This equation explained not only the intrinsic angular momentum of the electrons called *spin*, it also led to the prediction of the antiparticle of the electron (the positron), [48][49] and fine structure could only be fully explained with special relativity. It was the first foundation of *relativistic quantum mechanics*. In non-relativistic quantum mechanics, spin is phenomenological and cannot be explained.

On the other hand, the existence of antiparticles leads to the conclusion that relativistic quantum mechanics is not enough for a more accurate and complete theory of particle interactions. Instead, a theory of particles interpreted as quantized fields, called *quantum field theory*, becomes necessary; in which particles can be created and destroyed throughout space and time.

#### 12 See also

**People**: Hendrik Lorentz | Henri Poincaré | Albert Einstein | Max Planck | Hermann Minkowski | Max von Laue | Arnold Sommerfeld | Max Born | Gustav Herglotz | Richard C. Tolman

Relativity: Theory of relativity | History of special relativity | Principle of relativity | General relativity | Frame of reference | Inertial frame of reference | Lorentz transformations | Bondi k-calculus | Einstein synchronisation | Rietdijk—Putnam argument | Special relativity (alternative formulations) | Criticism of relativity theory | Relativity priority dispute

Physics: Newtonian Mechanics | spacetime | speed of light | simultaneity | center of mass (relativistic) | physical cosmology | Doppler effect | relativistic Euler equations | Aether drag hypothesis | Lorentz ether theory | Moving magnet and conductor problem | Shape waves | Relativistic heat conduction | Relativistic disk | Thomas precession | Born rigidity | Born coordinates

Mathematics: Derivations of the Lorentz transformations | Minkowski space | four-vector | world line | light cone | Lorentz group | Poincaré group | geometry | tensors | split-complex number | Relativity in the APS formalism

**Philosophy**: actualism | conventionalism | formalism

**Paradoxes**: Twin paradox | Ehrenfest paradox | Ladder paradox | Bell's spaceship paradox | Velocity composition paradox

#### 13 References

- [1] Albert Einstein (1905) "Zur Elektrodynamik bewegter Körper", Annalen der Physik 17: 891; English translation On the Electrodynamics of Moving Bodies by George Barker Jeffery and Wilfrid Perrett (1923); Another English translation On the Electrodynamics of Moving Bodies by Megh Nad Saha (1920).
- [2] Tom Roberts and Siegmar Schleif (October 2007). "What is the experimental basis of Special Relativity?". *Usenet Physics FAQ*. Retrieved 2008-09-17.
- [3] Albert Einstein (2001). Relativity: The Special and the General Theory (Reprint of 1920 translation by Robert W. Lawson ed.). Routledge. p. 48. ISBN 0-415-25384-5.
- [4] Richard Phillips Feynman (1998). Six Not-so-easy Pieces: Einstein's relativity, symmetry, and space-time (Reprint of 1995 ed.). Basic Books. p. 68. ISBN 0-201-32842-9.
- [5] Sean Carroll, Lecture Notes on General Relativity, ch. 1, "Special relativity and flat spacetime," http://ned.ipac. caltech.edu/level5/March01/Carroll3/Carroll1.html
- [6] Wald, General Relativity, p. 60: "...the special theory of relativity asserts that spacetime is the manifold  $\mathbb{R}^4$  with a flat metric of Lorentz signature defined on it. Conversely, the entire content of special relativity ... is contained in this statement ..."
- [7] Rindler, W., 1969, Essential Relativity: Special, General, and Cosmological
- [8] Edwin F. Taylor and John Archibald Wheeler (1992).
   Spacetime Physics: Introduction to Special Relativity. W.
   H. Freeman. ISBN 0-7167-2327-1.
- [9] Wolfgang Rindler (1977). Essential Relativity. Birkhäuser. p. §1,11 p. 7. ISBN 3-540-07970-X.
- [10] Einstein, Autobiographical Notes, 1949.
- [11] Einstein, "Fundamental Ideas and Methods of the Theory of Relativity", 1920
- [12] For a survey of such derivations, see Lucas and Hodgson, Spacetime and Electromagnetism, 1990
- [13] Einstein, A., Lorentz, H. A., Minkowski, H., & Weyl, H. (1952). The Principle of Relativity: a collection of original memoirs on the special and general theory of relativity. Courier Dover Publications. p. 111. ISBN 0-486-60081-5.
- [14] Einstein, On the Relativity Principle and the Conclusions Drawn from It, 1907; "The Principle of Relativity and Its Consequences in Modern Physics", 1910; "The Theory of Relativity", 1911; Manuscript on the Special Theory of Relativity, 1912; Theory of Relativity, 1913; Einstein, Relativity, the Special and General Theory, 1916; The Principle Ideas of the Theory of Relativity, 1916; What Is The Theory of Relativity?, 1919; The Principle of Relativity (Princeton Lectures), 1921; Physics and Reality, 1936; The Theory of Relativity, 1949.
- [15] Das, A. (1993) The Special Theory of Relativity, A Mathematical Exposition, Springer, ISBN 0-387-94042-1.

14 13 REFERENCES

- [16] Schutz, J. (1997) Independent Axioms for Minkowski Spacetime, Addison Wesley Longman Limited, ISBN 0-582-31760-6.
- [17] Yaakov Friedman (2004). Physical Applications of Homogeneous Balls. Progress in Mathematical Physics 40. pp. 1–21. ISBN 0-8176-3339-1.
- [18] David Morin (2007) Introduction to Classical Mechanics, Cambridge University Press, Cambridge, chapter 11, Appendix I, ISBN 1-139-46837-5.
- [19] Michael Polanyi (1974) *Personal Knowledge: Towards a Post-Critical Philosophy*, ISBN 0-226-67288-3, footnote page 10–11: Einstein reports, via Dr N Balzas in response to Polanyi's query, that "The Michelson–Morley experiment had no role in the foundation of the theory." and "..the theory of relativity was not founded to explain its outcome at all."
- [20] Jeroen van Dongen (2009). "On the role of the Michelson–Morley experiment: Einstein in Chicago" (PDF). Eprint arXiv:0908.1545 0908: 1545. arXiv:0908.1545. Bibcode:2009arXiv:0908.1545V.
- [21] Staley, Richard (2009), "Albert Michelson, the Velocity of Light, and the Ether Drift", Einstein's generation. The origins of the relativity revolution, Chicago: University of Chicago Press, ISBN 0-226-77057-5
- [22] Robert Resnick (1968). *Introduction to special relativity*. Wiley. pp. 62–63.
- [23] Daniel Kleppner and David Kolenkow (1973). An Introduction to Mechanics. pp. 468–70. ISBN 0-07-035048-5.
- [24] Does the inertia of a body depend upon its energy content? A. Einstein, *Annalen der Physik*. **18**:639, 1905 (English translation by W. Perrett and G.B. Jeffery)
- [25] Max Jammer (1997). Concepts of Mass in Classical and Modern Physics. Courier Dover Publications. pp. 177– 178. ISBN 0-486-29998-8.
- [26] John J. Stachel (2002). Einstein from B to Z. Springer. p. 221. ISBN 0-8176-4143-2.
- [27] On the Inertia of Energy Required by the Relativity Principle, A. Einstein, Annalen der Physik 23 (1907): 371–384
- [28] In a letter to Carl Seelig in 1955, Einstein wrote "I had already previously found that Maxwell's theory did not account for the micro-structure of radiation and could therefore have no general validity.", Einstein letter to Carl Seelig, 1955.
- [29] Baglio, Julien (26 May 2007). "Acceleration in special relativity: What is the meaning of "uniformly accelerated movement"?" (PDF). Physics Department, ENS Cachan. Retrieved 22 January 2016.
- [30] Philip Gibbs and Don Koks. "The Relativistic Rocket". Retrieved 30 August 2012.
- [31] Philip Gibbs and Don Koks. "The Relativistic Rocket". Retrieved 13 October 2013.

- [32] The special theory of relativity shows that time and space are affected by motion. Library.thinkquest.org. Retrieved on 2013-04-24.
- [33] R. C. Tolman, *The theory of the Relativity of Motion*, (Berkeley 1917), p. 54
- [34] G. A. Benford, D. L. Book, and W. A. Newcomb (1970). "The Tachyonic Antitelephone". *Physical Review D* **2** (2): 263. Bibcode:1970PhRvD...2..263B. doi:10.1103/PhysRevD.2.263.
- [35] Wesley C. Salmon (2006). Four Decades of Scientific Explanation. University of Pittsburgh. p. 107. ISBN 0-8229-5926-7., Section 3.7 page 107
- [36] J.A. Wheeler, C. Misner, K.S. Thorne (1973). Gravitation. W.H. Freeman & Co. p. 58. ISBN 0-7167-0344-0.
- [37] J.R. Forshaw, A.G. Smith (2009). Dynamics and Relativity. Wiley. p. 247. ISBN 978-0-470-01460-8.
- [38] R. Penrose (2007). *The Road to Reality*. Vintage books. ISBN 0-679-77631-1.
- [39] Jean-Bernard Zuber & Claude Itzykson, Quantum Field Theory, pg 5, ISBN 0-07-032071-3
- [40] Charles W. Misner, Kip S. Thorne & John A. Wheeler, Gravitation, pg 51, ISBN 0-7167-0344-0
- [41] George Sterman, An Introduction to Quantum Field Theory, pg 4, ISBN 0-521-31132-2
- [42] Sean M. Carroll (2004). Spacetime and Geometry: An Introduction to General Relativity. Addison Wesley. p. 22. ISBN 0-8053-8732-3.
- [43] E. J. Post (1962). Formal Structure of Electromagnetics: General Covariance and Electromagnetics. Dover Publications Inc. ISBN 0-486-65427-3.
- [44] Øyvind Grøn and Sigbjørn Hervik (2007). Einstein's general theory of relativity: with modern applications in cosmology. Springer. p. 195. ISBN 0-387-69199-5. Extract of page 195 (with units where c=1)
- [45] The number of works is vast, see as example: Sidney Coleman, Sheldon L. Glashow (1997). "Cosmic Ray and Neutrino Tests of Special Relativity". *Phys. Lett.* **B405** (3–4): 249–252. arXiv:hep-ph/9703240. Bibcode:1997PhLB..405..249C. doi:10.1016/S0370-2693(97)00638-2. An overview can be found on this page
- [46] John D. Norton, John D. (2004). "Einstein's Investigations of Galilean Covariant Electrodynamics prior to 1905". Archive for History of Exact Sciences 59: 45–105. Bibcode:2004AHES...59...45N. doi:10.1007/s00407-004-0085-6.
- [47] R. Resnick, R. Eisberg (1985). Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles (2nd ed.). John Wiley & Sons. pp. 114–116. ISBN 978-0-471-87373-0.

- [48] P.A.M. Dirac (1930). "A Theory of Electrons and Protons". *Proceedings of the Royal Society* **A126** (801): 360. Bibcode:1930RSPSA.126..360D. doi:10.1098/rspa.1930.0013. JSTOR 95359.
- [49] C.D. Anderson (1933). "The Positive Electron". *Phys. Rev.* **43** (6): 491–494. Bibcode:1933PhRv...43..491A. doi:10.1103/PhysRev.43.491.

#### 13.1 Textbooks

- Einstein, Albert (1920). Relativity: The Special and General Theory.
- Einstein, Albert (1996). *The Meaning of Relativity*. Fine Communications. ISBN 1-56731-136-9
- Logunov, Anatoly A. (2005) Henri Poincaré and the Relativity Theory (transl. from Russian by G. Pontocorvo and V. O. Soleviev, edited by V. A. Petrov) Nauka, Moscow.
- Charles Misner, Kip Thorne, and John Archibald Wheeler (1971) *Gravitation*. W. H. Freeman & Co. ISBN 0-7167-0334-3
- Post, E.J., 1997 (1962) Formal Structure of Electromagnetics: General Covariance and Electromagnetics. Dover Publications.
- Wolfgang Rindler (1991). Introduction to Special Relativity (2nd ed.), Oxford University Press. ISBN 978-0-19-853952-0; ISBN 0-19-853952-5
- Harvey R. Brown (2005). Physical relativity: spacetime structure from a dynamical perspective, Oxford University Press, ISBN 0-19-927583-1; ISBN 978-0-19-927583-0
- Qadir, Asghar (1989). *Relativity: An Introduction to the Special Theory*. Singapore: World Scientific Publications. p. 128. ISBN 9971-5-0612-2.
- Silberstein, Ludwik (1914) The Theory of Relativity.
- Lawrence Sklar (1977). Space, Time and Spacetime.
   University of California Press. ISBN 0-520-03174-1.
- Lawrence Sklar (1992). Philosophy of Physics. Westview Press. ISBN 0-8133-0625-6.
- Taylor, Edwin, and John Archibald Wheeler (1992) Spacetime Physics (2nd ed.). W.H. Freeman & Co. ISBN 0-7167-2327-1
- Tipler, Paul, and Llewellyn, Ralph (2002). *Modern Physics* (4th ed.). W. H. Freeman & Co. ISBN 0-7167-4345-0

#### 13.2 Journal articles

- Alvager, T.; Farley, F. J. M.; Kjellman, J.; Wallin, L.; et al. (1964). "Test of the Second Postulate of Special Relativity in the GeV region". *Physics Letters* 12 (3): 260. Bibcode:1964PhL...12..260A. doi:10.1016/0031-9163(64)91095-9.
- Darrigol, Olivier (2004). "The Mystery of the Poincaré–Einstein Connection". *Isis* 95 (4): 614– 26. doi:10.1086/430652. PMID 16011297.
- Wolf, Peter; Petit, Gerard (1997). "Satellite test of Special Relativity using the Global Positioning System". *Physical Review A* 56 (6): 4405–09. Bibcode:1997PhRvA..56.4405W. doi:10.1103/PhysRevA.56.4405.
- Special Relativity Scholarpedia
- Special relativity: Kinematics Wolfgang Rindler, Scholarpedia, 6(2):8520. doi: 10.4249/scholarpedia.8520

### 14 External links

#### 14.1 Original works

- Zur Elektrodynamik bewegter Körper Einstein's original work in German, Annalen der Physik, Bern 1905
- On the Electrodynamics of Moving Bodies English
  Translation as published in the 1923 book The Principle of Relativity.

# 14.2 Special relativity for a general audience (no mathematical knowledge required)

- Einstein Light An award-winning, non-technical introduction (film clips and demonstrations) supported by dozens of pages of further explanations and animations, at levels with or without mathematics.
- Einstein Online Introduction to relativity theory, from the Max Planck Institute for Gravitational Physics.
- Audio: Cain/Gay (2006) Astronomy Cast. Einstein's Theory of Special Relativity

# 14.3 Special relativity explained (using simple or more advanced mathematics)

• Greg Egan's Foundations.

16 14 EXTERNAL LINKS

- The Hogg Notes on Special Relativity A good introduction to special relativity at the undergraduate level, using calculus.
- Relativity Calculator: Special Relativity An algebraic and integral calculus derivation for  $E = mc^2$ .
- MathPages Reflections on Relativity A complete online book on relativity with an extensive bibliography.
- Relativity An introduction to special relativity at the undergraduate level, without calculus.

•

- Relativity: the Special and General Theory at Project Gutenberg, by Albert Einstein
- Special Relativity Lecture Notes is a standard introduction to special relativity containing illustrative explanations based on drawings and spacetime diagrams from Virginia Polytechnic Institute and State University.
- Understanding Special Relativity The theory of special relativity in an easily understandable way.
- An Introduction to the Special Theory of Relativity (1964) by Robert Katz, "an introduction ... that is accessible to any student who has had an introduction to general physics and some slight acquaintance with the calculus" (130 pp; pdf format).
- Lecture Notes on Special Relativity by J D Cresser Department of Physics Macquarie University.
- SpecialRelativity.net An overview with visualizations and minimal mathematics.

#### 14.4 Visualization

- Raytracing Special Relativity Software visualizing several scenarios under the influence of special relativity.
- Real Time Relativity The Australian National University. Relativistic visual effects experienced through an interactive program.
- Spacetime travel A variety of visualizations of relativistic effects, from relativistic motion to black holes.
- Through Einstein's Eyes The Australian National University. Relativistic visual effects explained with movies and images.
- Warp Special Relativity Simulator A computer program to show the effects of traveling close to the speed of light.

- Animation clip on YouTube visualizing the Lorentz transformation.
- Original interactive FLASH Animations from John de Pillis illustrating Lorentz and Galilean frames, Train and Tunnel Paradox, the Twin Paradox, Wave Propagation, Clock Synchronization, etc.
- Relativistic Optics at the ANU
- lightspeed An OpenGL-based program developed to illustrate the effects of special relativity on the appearance of moving objects.
- Animation showing the stars near Earth, as seen from a spacecraft accelerating rapidly to light speed.

### 15 Text and image sources, contributors, and licenses

#### 15.1 Text

• Special relativity Source: https://en.wikipedia.org/wiki/Special\_relativity?oldid=708416854 Contributors: AxelBoldt, RjLesch, The Cunctator, Derek Ross, Mav, Bryan Derksen, The Anome, Tarquin, Ap, -- April, XJaM, Christian List, William Avery, Roadrunner, DrBob, Heron, Karl Palmen, Bth, Ram-Man, Stevertigo, Edward, Bdesham, Nealmcb, Patrick, Chas zzz brown, Boud, Michael Hardy, Tim Starling, Alan Peakall, Cole Kitchen, Ixfd64, Chinju, Zulu~enwiki, Alfio, Dgrant, CesarB, Looxix~enwiki, Ellywa, Ahoerstemeier, Cyp, Stevenj, CatherineMunro, Kevin Baas, Salsa Shark, Whkoh, Wooster, Agtx, Reddi, Jitse Niesen, Kbk, Wik, Jeepien, Lfwlfw, Patrick0Moran, Morwen, Taxman, Bevo, Sokane, Banno, PuzzletChung, Craig Stuntz, Astronautics~enwiki, Gregors, Fredrik, Altenmann, Lowellian, Mayooranathan, Gandalf61, Academic Challenger, Sdw25, PxT, DHN, Robinh, Fuelbottle, Anthony, Cyrius, Carnildo, Davedx, Alan Liefting, Giftlite, Christopher Parham, Patrik~enwiki, Wolfkeeper, Netoholic, Fastfission, Fropuff, Xerxes314, Muke, Alex.atkins, Wwoods, Everyking, Bkonrad, Michael Devore, Alvestrand, DÅ, ugosz, Leonard Vertighel, Chowbok, Utcursch, Andycjp, Alexf, Knutux, Antandrus, Sbyers11, MisfitToys, Ablewisuk, MadIce, Kesac, Spoirier~enwiki, Icairns, Sam Hocevar, Avihu, Fermion, Zondor, Bluemask, Mike Rosoft, Chris Howard, D6, Smokris, Sysy, Ultratomio, Klaas van Aarsen, Discospinster, Mageaere, ElTyrant, Rich Farmbrough, ThomasK, Masudr, Piacobi, Kooo, Xezbeth, Francois-Dominique~enwiki, Paul August, Gonzalo Diethelm, Dmr2, Mwm126, Bender235, Kjoonlee, Blogjack, RJHall, Livajo, El C, Rgdboer, Lankiveil, Laurascudder, Joaopais, Bobo192, Nigelj, Army1987, Marco Polo, Wood Thrush, Robotje, Teorth, GTubio, 19Q79oL78KiL0QTFHgyc, Novakyu, Rje, Jonathunder, Geschichte, Passw0rd, Jumbuck, Red Winged Duck, Danski 14, Hackwrench, Keenan Pepper, Jengeldk, Ricky 81682, Slim Virgin, PAR, BernardH, Hu, Ksnow, King TT, Psmither, DonQuixote, Eddie Dealtry, Count Iblis, CloudNine, DAnderson, DV8 2XL, HenryLi, KTC, Loxley~enwiki, Siafu, Hoziron, Boothy443, Igny, Etacar11, Sabejias, StradivariusTV, TheNightFly, Neomundus, Robert K S, Mpatel, Jok2000, Shahbaznihal, El Suizo, Joke 137, Jonathan 48, Christopher Thomas, Palica, Pfalstad, Theo F, Gerbrant, LRC, Marudubshinki, Rnt 20, Graham 87, Magister Mathematicae, Ilya, Qwertyus, RxS, Melesse, Ketiltrout, Sjakkalle, Rjwilmsi, Seidenstud, Саша Стефановић, WingZero, Mkuehn10, R.O.C, Thechamelon, R.e.b., Ems57fcva, Yamamoto Ichiro, Wragge, FlaBot, Gringo300, Old Moonraker, Arnero, Musical Linguist, Latka, Alfred Centauri, Nimur, Jrtayloriv, Enon, Fresheneesz, Tysto, Srleffler, David H Braun (1964), Silivrenion, Chobot, DVdm, VolatileChemical, Bgwhite, YurikBot, Wavelength, Ec-, Kafziel, Hillman, Brandmeister (old), Hyad, Loom91, JabberWok, Stephenb, Manop, Archelon, Gaius Cornelius, Chaos, NawlinWiki, Msikma, ErkDemon, Welsh, Trovatore, Długosz, Proyster, Tailpig, Schlafly, Krea, SCZenz, Mingshey~enwiki, Muu-karhu, Fr~enwiki, SFC9394, Elkman, Wknight94, Yisraelasper, Perspicacious, Light current, Enormousdude, Licensedlunacy, Zzuuzz, Lt-wiki-bot, Errabee, Izabela Furjan, Sambc, Willtron, Ilmari Karonen, Profero, Infinity0, Sbyrnes321, Sardanaphalus, SmackBot, RDBury, Slashme, KnowledgeOfSelf, WilyD, Nickst, Eskimbot, Harald88, Gilliam, Ohnoitsjamie, Dauto, Bluebot, Keegan, Quinsareth, MalafayaBot, Silly rabbit, Complexica, RayAYang, Whispering, Geometer, Sbharris, Rkinch, Scwlong, Can't sleep, clown will eat me, Danielkueh, Ioscius, Rrburke, Xyzzyplugh, E4mmacro, Downwards, Khukri, Nakon, Bubby the Tour G, Lpgeffen, Jon Awbrey, DMacks, Murf42, Acdx, Andrei Stroe, Mearnhardtfan, Ohconfucius, Nmnogueira, Lambiam, Nathanael Bar-Aur L., Cronholm144, SilkTork, Adj08, Loadmaster, Dr Greg, Techsmith, Kpengboy, SandyGeorgia, Dr.K., Politepunk, Quaeler, Belltower, Ssiruuk25, Joseph Solis in Australia, Andreas Rejbrand, MOBle, GDallimore, Nfutvol, CapitalR, Gornly, Yskyflyer, Tawkerbot2, JRSpriggs, DKqwerty, Falconus, Robinhw, JForget, TristanJ, Sakurambo, CmdrObot, Diegueins, Linus M., Rusliko, Minor Crank, Arnavion, WeggeBot, Myasuda, Icek~enwiki, Cydebot, Abebasal, UncleBubba, Gogo Dodo, Xxanthippe, Agne27, Edgerck, Francis Pym, Michael C Price, Christian75, DumbBOT, DBaba, ErrantX, Omicronpersei8, SummonerMarc, Epbr123, LeBofSportif, Markus Pössel, Pmkost, Martin Hogbin, N5iln, Headbomb, Jojan, Yzmo, X201, Tellyaddict, Davidhorman, Elhector, Iviney, Thljcl, D.H, Big Bird, Sean William, Austin Maxwell, AntiVandalBot, Gioto, Der alte Hexenmeister, Elmoosecapitan, Magencalc, Gnixon, Xaethos-enwiki, Fashionslide, Tim Shuba, Knotwork, JAnDbot, Husond, Em3ryguy, CosineKitty, Moshier, Hut 8.5, Autotheist, Absolution., Pervect, Magioladitis, Bongwarrior, VoABot II, Faizhaider, TxAlien, Stijn Vermeeren, Cyktsui, JJ Harrison, LorenzoB, StuFifeScotland, User A1, DerHexer, EightBall1989, Edward321, Biggy P, MartinBot, Ken Seto, Vox Rationis, E.Shubee, Jschilling, N4nojohn, Vanwhistler, J.delanoy, Trizow, R. Baley, James R, 3halfinchfloppy, OttoMäkelä, Hodja Nasreddin, Lantonov, M C Y 1008, BobEnyart, Silas S. Brown, Iimd1931, WaiteDavid137, M-le-mot-dit, SouthParkFan, JohnAnthonyCarey, Pope Bonny Eberndu, NewEnglandYankee, SJP, Tgooding, Touch Of Light, I gave you a twenty, I know the pizza didn't cost that much, Fylwind, Shadow Android, Jake roman, Chris O'Riordan, Ken g6, Moroder~enwiki, Championdante, Wildsurmise, Pdcook, Bgtgwazi, VolkovBot, Jeff G., JohnBlackburne, Optokinetics, Fences and windows, Philip Trueman, TXiKiBoT, Coder Dan, Red Act, Nxavar, Anonymous Dissident, Qxz, Einstein, NOT!, DennyColt, Corvus cornix, Fizzackerly, Oren neu dag, Wjkiely, Fbs. 13, Seb az86556, Geometry guy, Wenli, RobertFritzius, Andy Dingley, Synthebot, Antixt, Sylviaelse, Webrider, Woojamon, Gth759k, C0N6R355, SaltyBoatr, Dnarby, SieBot, Timb66, Nubiatech, Hertz1888, Likebox, RadicalOne, Arjen Dijksman, Udirock, Rafimoor, Arthana, Fedosin, Coldcreation, H1nkles, TFCforever, ShajiA, ImageRemovalBot, Martarius, ClueBot, Dypteran, NickCT, IceUnshattered, Plastikspork, Wwheaton, R000t, RYNORT, Flyingcar73242, Glibik, Eric.brasseur, Agge1000, Anaholic, ChandlerMap-Bot, Oxnard27, RAmesbury, Paulcmnt, Ajoykt, Denveron, PixelBot, Tamaratrouts, Brews ohare, NuclearWarfare, Hans Adler, Hasteur, DS1000, Friedlibend und tapfer, Roger491127, Aitias, John C. Huang, Dkress14, Steve D. Gage, DumZiBoT, Antti29, TimothyRias, RQG, Helixweb, BarretB, XLinkBot, Namreh ekim, Johanley, Rror, WikHead, Cleatus 69, Truthnlove, Mitrg, MTessier, Addbot, Eric Drexler, Some jerk on the Internet, Jojhutton, Smdowney, Haruth, Booger Eating Moron, Fluffernutter, Cst17, EconoPhysicist, Delaszk, Favonian, LinkFA-Bot, Bob K31416, Patton123, Lightbot, ScAvenger, Gameseeker, Blablablob, Luckas-bot, Yobot, Ptbotgourou, LGB, RHB100, KamikazeBot, Szajci, Synchronism, AnomieBOT, Jim1138, 9258fahsflkh917fas, Piano non troppo, Classicalmatter, Materialscientist, The High Fin Sperm Whale, Citation bot, ArthurBot, LilHelpa, Xqbot, Wavgfkl, Tasudrty, Amareto2, Capricorn42, Nasnema, Nanog, Jzhuo, Gap9551, NOrbeck, Almabot, GrouchoBot, Efield, RibotBOT, Waleswatcher, Victamonn, Charvest, Rayclipper, The Wiki ghost, Schekinov Alexey Victorovich, A. di M., Nym6433, Cooguy77, CES1596, FrescoBot, Baz.77.243.99.32, Paine Ellsworth, Ironboy11, Lipsquid, Tzviscarr, Sławomir Biały, Rymmen, Citation bot 1, Jimfarley, Greenrev, Pinethicket, Tsester, Tom.Reding, Zanzapod, Lithium cyanide, Number67c, Tcnuk, Riccardo.fabris, IVAN3MAN, Corinne68, Meier99, Kfchurchill, TobeBot, Trappist the monk, Callanecc, Cardinality, Iphegenia, Earthandmoon, Sandman888, Andrea105, Mean as custard, RjwilmsiBot, Alph Bot, Ripchip Bot, WildBot, Emaus-Bot, John of Reading, Troubled asset, WikitanvirBot, Docjudith, Jazzalex, Heracles31, Benschopp8, RenamedUser01302013, Slightsmile, Netheril96, K6ka, Coffeefilter, Hhhippo, Empty Buffer, Albertolanzoni71, Kharoutinian, Juan Fco. Araya, Quondum, EWikist, Iiar, Ashell2, Bsmith671, Maschen, Donner60, Bill william compton, Rjowsey, ChuispastonBot, RockMagnetist, NTox, Vadher lalit, Alexandrevennes, Petrb, ClueBot NG, Bloodjam, TehGrauniad, SusikMkr, Cntras, David.bertalan, Braincricket, Richukuttan, Bopomofo, Helpful Pixie Bot, Witherpshins, Anofein, Bibcode Bot, BG19bot, Hewhoamareismyself, Gresavage, Datechanger, Bonginkosi zwane, Afbrat151, F=q(E+v^B), Bobo123456, Harizotoh9, Tremere2, Klilidiplomus, Tabish rafiq, Jffjgfjgfgf, Stigmatella aurantiaca, Khazar2, Ewjohnsonjohnson, JYBot, Dexbot, Twhitguy14, CuriousMind01, TwoTwoHello, 1Todd1, OneToddWon, Shivamshaiv, Mrvd42, Jamesx12345, Shivamshaivpatel, Andyhowlett, Shivam scientist, Gmxian89, NeapleBerlina, Jwratner1, DavRosen, Frinthruit, Anrnusna, Unifiedfields, Mezafo, Macjames4444, Mahusha, Abhishek.m.patel89, Monkbot, JhonASF, Gjbayes, TE5ITA, Zachman727, Neeraj Bhakta, Jayache88, ChamithN, The Average Wikipedian, Bondy11u, Corsairio, Tetra quark, Spyridon Vossos, 2TonyTony, Grammarian3.14159265359, KasparBot, Kafishabbir, Are you freaking kidding me, DamnLetMeLogIn, Trambak J Chall, Harshumarathe, JGS952 and Anonymous: 755

#### **15.2 Images**

- File:Commons-logo.svg Source: https://upload.wikimedia.org/wikipedia/en/4/4a/Commons-logo.svg License: CC-BY-SA-3.0 Contributors: ? Original artist: ?
- File:Einstein\_patentoffice.jpg Source: https://upload.wikimedia.org/wikipedia/commons/a/a0/Einstein\_patentoffice.jpg License: Public domain Contributors: Transferred from en.wikipedia; transferred to Commons by User:Guerillero using CommonsHelper.

  Original artist: Lucien Chavan [#cite\_note-author-1 [1]] (1868 1942), a friend of Einstein's when he was living in Berne.
- File:Light\_cone.svg Source: https://upload.wikimedia.org/wikipedia/commons/2/27/Light\_cone.svg License: Public domain Contributors: Own work Original artist: Sakurambo
- File:Orthogonality\_and\_rotation.svg Source: https://upload.wikimedia.org/wikipedia/commons/5/5e/Orthogonality\_and\_rotation.svg
   License: CC0 Contributors: Own work Original artist: Maschen
- File:Relativity\_of\_Simultaneity.svg Source: https://upload.wikimedia.org/wikipedia/commons/b/b1/Relativity\_of\_Simultaneity.svg
   License: CC BY-SA 3.0 Contributors: Derivative work of en:File:Relativity\_of\_simultaneity\_(color).png Original artist: User:Army1987 created the original PNG file; Acdx converted it to SVG.
- File:Spacetime\_curvature.png Source: https://upload.wikimedia.org/wikipedia/commons/2/22/Spacetime\_curvature.png License: CC-BY-SA-3.0 Contributors: ? Original artist: ?

Original artist:

User:Yzmo

- File:Sr3.svg Source: https://upload.wikimedia.org/wikipedia/commons/7/78/Sr3.svg License: CC-BY-SA-3.0 Contributors:
- Sr3.jpg Original artist: Sr3.jpg: user:Kevin Baas
- File:Stylised\_Lithium\_Atom.svg Source: https://upload.wikimedia.org/wikipedia/commons/e/e1/Stylised\_Lithium\_Atom.svg License: CC-BY-SA-3.0 Contributors: based off of Image:Stylised Lithium Atom.png by Halfdan. Original artist: SVG by Indolences. Recoloring and ironing out some glitches done by Rainer Klute.
- File:Wikibooks-logo-en-noslogan.svg Source: https://upload.wikimedia.org/wikipedia/commons/d/df/Wikibooks-logo-en-noslogan.svg License: CC BY-SA 3.0 Contributors: Own work Original artist: User:Bastique, User:Ramac et al.
- File:Wikiquote-logo.svg Source: https://upload.wikimedia.org/wikipedia/commons/f/fa/Wikiquote-logo.svg License: Public domain Contributors: ? Original artist: ?
- File:Wikisource-logo.svg Source: https://upload.wikimedia.org/wikipedia/commons/4/4c/Wikisource-logo.svg License: CC BY-SA 3.0
  Contributors: Rei-artur Original artist: Nicholas Moreau
- File:Wikiversity-logo.svg Source: https://upload.wikimedia.org/wikipedia/commons/9/91/Wikiversity-logo.svg License: CC BY-SA 3.0 Contributors: Snorky (optimized and cleaned up by verdy\_p) Original artist: Snorky (optimized and cleaned up by verdy\_p)
- File:Wiktionary-logo-en.svg Source: https://upload.wikimedia.org/wikipedia/commons/f/f8/Wiktionary-logo-en.svg License: Public domain Contributors: Vector version of Image:Wiktionary-logo-en.png. Original artist: Vectorized by Fvasconcellos (talk · contribs), based on original logo tossed together by Brion Vibber
- File:World\_line.svg Source: https://upload.wikimedia.org/wikipedia/commons/1/16/World\_line.svg License: CC-BY-SA-3.0 Contributors: Transferred from en.wikipedia.

Original artist: SVG version: K. Aainsqatsi at en.wikipedia

#### 15.3 Content license

• Creative Commons Attribution-Share Alike 3.0