Gravity is the 'classicalized' entanglement?

Zhi-Hang Liu

Is it possible that gravity can be understood as the 'classicalized' entanglement? We try to initialize such a work in this manuscript. The theory scheme is quantum in its origin, but with classicality mechanism inherent in it. We propose to start from the completely quantum regime and figure out the classicality mechanism leading to the current universe. In the context of holographic (random) tensor network, we argue that the emergent space is completely quantum, while time and locality emerging from quantum fluctuation measurements are signatures of classicality. We conjecture that, through quantum fluctuation measurements and subsequent quantum field excitations, space (entanglement), energy, particles, and time are intertwined and form the space-time manifold eventually, which is described by general relativity. One of the results is that the inherent holographic quantum error correction code (QECC) mechanism leads to the inflationary universe. Fine-tuning the QECC can probably lead to the observed accelerating inflation. From the perspective of quantum vacuum fluctuation engines, we propose an entanglement-energy conservation law.

I. THE THEORY SCHEME

Imagining in the early universe, the whole universe is totally (or maximally) entangled, the inseparable one and only (mathematically, the trivial topology). The universe is the one and only observer and observable. In the spirit of the correspondence between CFT vacuum and (Anti-)dS space, we conjecture that there is no bulk or boundary degrees of freedom (uncontracted legs) in this period. In this early universe, the concept of system-environment does not exist and there is no notion of time and observer locality. When we talk about locality, there should be at least two observers (observables). This early universe is a completely quantum vacuum and there are only several fundamental entities: entanglement structure, energy, information and quantum fluctuations. In the context of holographic tensor network models, such as holographic MERA tensor network, exact holographic mapping, and HaPPY code [1–9], it is showed that vacuum space geometry emerges from the entanglement structure of the quantum vacuum, i.e., the origin of space geometry is completely quantum, leading to the Reeh-Schlieder theorem [10, 11].

However, classicality exists in the current universe. Where does the classicality come from? We argue that classicality is inherent to the quantum vacuum because of quantum fluctuation (we can not strictly distinguish between quantum and classical), and time and locality are signatures of classicality.

First of all, we need to establish the relationship between energy and entanglement, which we show naturally leads to the emergent time and local observers (observables) through spontaneous quantum fluctuation measurements. For quantum many-body systems with maximally entangled ground states, there is a minimum separable energy for the set of separable states, which is strictly greater than the ground state energy [12], i.e., less entanglement corresponds to a higher energy level and the probability of finding a particle to be in excited states indicates the existence of entanglement. This principle is utilised in the design of many-body quantum vacuum-powered engines, an engine cycle whereby work is extracted from the quantum vacuum via local measurements [13]. How do such measurement-powered machines exist naturally in our universe ? The core question is the natural existence of local measurement, and indeed, quantum fluctuation serves this role. It was shown that a quantum mean field could be decohered by its own quantum fluctuations, an example of how a quantum 'system' (local observable) emerges and is viewed as effectively open and decoheres through its own dynamics [14]. We refer to this transition of the totally entangled universe to separable systems as the (open) system-environment mechanism (mathematically, the advent of non-trivial topology), which is closely related to the nature of time as emergent for a system by separating it from its environment (the Page-Wootters relational time approach) [15]. In particular, starting from the invariance principle for the global, entangled state $|\Psi\rangle$

$$\exp\left[i\lambda(\hat{H}-E)\right]|\Psi\rangle = |\Psi\rangle, \tag{1}$$

the crucial role of the entanglement in $|\Psi\rangle$ with respect to the states of system and environment for non-trivial system dynamics is revealed. Otherwise, system and environment fulfill separately a "global" invariance principle, leaving the emergent time relation undetermined [16]. In brief, time and locality emerge when separability comes into existence, indicating changes of entanglement structure. Quantum fluctuation measurements of quantum vacuum fields lead to the initial quantum field excitations, which interact with each other and affect the underling entanglement structure in their special ways. Based on the classicality mechanism discussed above, space (entanglement), energy, particles, and time are intertwined and form the space-time manifold eventually, which is described by general relativity. In this sense, we conjecture that gravity is the 'classicalized' entanglement. It appears that we need to reformulate quantum field theory with a focus on entanglement structure dynamics.

How do we ensure locality between observers? The entanglement structure nature of the universe is captured by the bulk/boundary duality [17–19], such as (Anti-)dS/CFT correspondence, which we assume to be exact. The bulk/boundary duality ensures that boundary is able to access the bulk information instantaneously, contradicting speed of light and locality every observer feels. This contradiction is solved by the QECC inherently incorporated into holographic tensor networks [7, 8]. For a holographic tensor network, we argue that the spontaneous quantum fluctuation measurements introduce uncontracted legs to the bulk, which may break the contracted boundary legs and leads to boundary degree of freedom, and then the bulk/boundary duality begins. This bulk/boundary duality can be modeled by exact holographic mapping, but we still lack a mechanism to ensure locality between observers, which is why holographic QECC, such as the HaPPY code, is needed.

In particular, the ongoing spontaneous quantum fluctuation measurements and subsequent quantum field excitations and interactions continuously introduce uncontracted legs to bulk and boundary. Consequently, to ensure the effectiveness of the QECC mechanism, the universe is inflating (self-growing) to allowing for more boundary degree of freedom. The self-growing mechanism of holographic tensor network is still a mystery and needs further study. In other words, we argue that holographic QECC mechanism leads to the inflationary universe. The universe exists as the most ambitious quantum error correction code and fine-tuning the QECC can probably lead to the observed accelerating inflation.

Van Raamsdonk [20] showed that the geodesic distance between two regions of spacetime can increase if we disentangling the associated degrees of freedom, which is consistent with the QECC inflation mechanism, since quantum fluctuation measurements and quantum field excitations correspond to entanglement decrease. We thus conclude that in the QECC inflation mechanism, the geodesic distance in the bulk is increasing and the boundary is inflating. If there is no quantum entanglement between two degrees of freedom, the geodesic distance between the regions associated with them should go to infinity as argued by Van Raamsdonk. This distance infinity should lead to worries and question, which can be addressed primarily by investigating the quantum corrections to holographic entanglement entropy [21]. In the classical regime, the mutual information $I(A, B) = S(A) + S(B) - S(A \cup B)$ between two disjoint regions, A and B, is zero, by invoking the RT formula [22]. The bulk entanglement term of the quantum correction to the RT formula leads to nonzero $I(A, B)$ and thus we can have non-vanishing correlators, based on the general bound for correlators,

$$I(A, B) \geq \frac{\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle^2}{2|\mathcal{O}_A|^2 |\mathcal{O}_B|^2}. \tag{2}$$

Consequently, the geodesic distance between A and B will not go to infinity after we taking quantum corrections into consideration.

Currently, physicists assume there is no quantum entanglement between any two classical observables [23, 24]. Thus the geodesic distance between the observables should be infinity if we follow the argument of Van Raamsdonk, which is obviously wrong in the classical world we perceive. The problem lies in the existence of states with vanishing entanglement between its subsystems. Accepting that space geometry emerges from entanglement structure, we would argue that the whole space is connected. When we talk about disconnected space, we are actually talking about speed of light and locality, which are showed to be not exactly true by the holographic QECC mechanism. No space regions are actually disconnected leads to the statement: there is no two degrees of freedom which are entirely disentangled and we question the existence of product states $|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle$ in nature. To some extent, the vacuum entanglement and the Reeh-Schlieder theorem support this statement. Entanglement plays such a fundamental role in the universe, how come it seems to disappear in the classical world? We conjecture that, 'entanglement' never vanishes for any two degrees of freedom and entanglement manifests as gravity in the classical world through the classicality mechanism, which explains why gravity is universal in the classical world, just as entanglement is in the quantum world.

The model we propose above can help us to imagine the appearance of the early universe, its evolution, and perhaps the original universe. (Anti-)dS/CFT correspondence [25, 26] are connected to cosmological theory and observations in an explicitly way. We wonder about the origin of quantum fluctuations and the uncertainty principle (energy-time uncertainty relation). A promising candidate to study this problem is the holographic random tensor network [9, 27, 28].

II. IMPLICATIONS AND OPEN QUESTIONS

Through the triangle relation: geometry—(related by the RT formula [21, 29, 30])—entanglement—(related by CFT)—energy-momentum, the area of the extremal surface is related to the energy-momentum distribution at the boundary, which turns out to be equivalent to the linearized Einstein equation [31–34]. This derivation of the linearized Einstein equation works for small perturbations around the vacuum. We expect this derivation to be the supporting evidence for our theory scheme by reinterpretation. During the evolution of the universe, the underling vacuum entanglement structure changes and exists eternally . We argue that emergent gravity can still preserve some quantumness through an unknown mechanism. Consequently, gravity can still induce entanglement between two massive particles [35, 36].

The inflationary universe inspires us to construct self-growing tensor networks driven by their own randomness, which might be the origin of uncertainty principle. The energy-time uncertainty relation plays a fundamental role in our theory scheme, and we wonder if a derivation of it is possible. We expect the derivation of various generalized (information or gravitational perspective) uncertainty relations [37–39].

Inspired by the theory of quantum vacuum fluctuation engine, we might be able to establish an entanglement-energy conservation law. Virtual particles in the quantum fluctuation can be seen as the spontaneous tendency of classicalization. The energy threshold for virtual particles becoming real particles could be related to the entanglement-energy conservation law. Landauer's principle governs the minimal energy that must be consumed to erase a bit of information. Energy consumption corresponds to entanglement increase in our theory scheme. We expect an entanglement-energy-information conservation law.

Cosmological constant problem. It was shown that when the quantum to classical transition is properly treated, with due consideration of the relation of decoherence, noise, fluctuation, and dissipation, the amplitude of density contrast predicted falls in the acceptable range without requiring a fine-tuning of the coupling constant [14]. Recently, by treating the divergent vacuum energy density of quantum fluctuation seriously without trying renormalization, the fine-tune problem is circumvented, claiming that there is no necessity to introduce dark energy [40, 41]. It has been demonstrated that the observed phenomena that are currently attributed to dark matter are the consequence of the emergent nature of gravity and are caused by an elastic response due to the volume law contribution to the entanglement entropy in our universe [26].

III. EFFORTS TO RECONCILE QUANTUM MECHANICS AND GRAVITY

Reconciling quantum mechanics with gravity has long posed a challenge for physicists. Recent developments in quantum information theory shed new light to this fundamental problem. In particular, quantum entanglement comes to play a fundamental role in understanding spacetime geometry. Physicists try to provide a more microscopic and complete understanding of how spacetime geometry and gravity emerge from quantum information characteristics of many-body states [33, 42–46]. To be more specific, we explore quantum information structure of spacetime with various kinds of generalized entanglement entropy.

The study of entanglement entropy (von Neumann entropy) $S_A = -\mathrm{Tr}\,\rho_A \log \rho_A$ (ρ_A being the partial trace of total state.) in quantum field theory (QFT) and spacetime geometry has already significantly changed our view of gravity and spacetime, especially in the context of gauge-gravity correspondence. This process begins with the area law of black hole entropy $S = \frac{\mathrm{Area}(A)}{4G_N}$, which leads to the formulation of holographic principle and AdS/CFT correspondence [17–19]. With the conjecture of Ryu-Takayanagi (RT) formula $S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$ (γ_A being the minimal surface) [29, 47, 48], which states different entanglement structures correspond to different bulk geometries, quantum entanglement and entanglement entropy come to play a fundamental role in understanding spacetime geometry. Based on RT formula and its generalizations [21, 30], Maldacena et al suggest that the spacetime geometry is constructed from quantum entanglement [20, 49].

The Reeh-Schlieder theorem [50–52] in QFT indicates every Reeh-Schlieder state contains entanglement between any two spatially separated regions, highlighting the deeply non-local and entangled nature of QFT. Even the vacuum state contains rich and subtle structure, offering insights into the entanglement inherent in quantum spacetime. For two spacelike separated subsystems A and B, extrapolation of the Reeh-Schlieder theorem conjectures that any pair of modes, one in A and one in B, are entangled in the vacuum. A recent work by Agullo et al concludes that this extrapolation is not true [10]. With smeared field operators, they define a "pixel" of the field theory, which captures a simplified (smeared) version of the field and the related Cauchy hypersurface is divided into disjoint pixels correspondingly. In this way, Agullo et al argues that entanglement in Minkowski spacetime is significantly less ubiquitous than normally thought and one needs to carefully select the support of modes for entanglement to show up.

IV. RESEARCH QUESTION, METHODOLOGY AND IMPLICATION

We are not persuaded by this argument. The pixel process in [10, 53] is essentially the idea of holographic principle, but in an imprudent manner. In particular, we are worried that dividing the space into "disjoint" pixels would break the entanglement structure of the vacuum, which of course leads to the conclusion that entanglement between finite-dimensional subsystems in the vacuum is not common at all. It is reasonable to argue that two spatially separated regions get entangled through a series of intermediary entanglement connections, i.e., entanglement swapping. To study the extrapolation of the Reeh-Schlieder theorem, we think it is more appropriate to work with "compatible pixels", in the spirit of compatible charts in manifold. We aim to establish a model in which any compact spacetime is covered by finite number of compatible pixels. This model should be consistent with the conjecture that entanglement knits spacetime together [20, 49, 54].

Moreover, if spacetime is knitted by non-local quantum entanglement, how does locality emerge? Within our model, locality and integration property of spacetime shall be studied. Our model proposal is far from concrete. As the first step, we need a reasonable definition of compatible pixels, which may be derived from appropriate smeared field operators. An implication of this model is the magnitude of cutoff ℓ and the nature of Planck-scale physics [55]. When we study entanglement entropy in this model, the magnitude of ℓ might be derived from the size of compatible pixels. The appropriate size of pixels may differ for different energy scales. To make this model tractable, initially, we shall work in the regime of Gaussian state [10, 56].

Measuring degrees of freedom in QFT defined in a time-dependent spacetime background is an especially important open problem. Entanglement entropy can serve as a measure of the effective operative degrees of freedom in a given region of the background geometry [57]. Aiming at a better understanding of the time-dependence of entanglement entropy in generic QFT (interacting QFT or QFT in time-dependent background spacetime), covariant generalization of entanglement entropy is needed. Following the RT formula, Hubeny et al [30] propose a covariant generalization of the holographic entanglement entropy by generalizing the Euclidean minimal surface γ_A to an extremal surface. For Gaussian scalar field theory, Sorkin proposed a manifestly covariant formulation of entanglement entropy using quantum Peierls brackets and by replacing spacetime with a causal set [55]. For interacting QFT in gravitational settings, extending Sorkin's entropy to non-Gaussian theories is established using perturbation theory [58, 59], suggesting avenues for further extensions to generic interacting theories.

Studying spacetime entanglement entropy in interacting QFTs is an important research question. Since there are relatively few techniques to calculate the entanglement entropy in interacting QFTs, the geometric perspective provided by the minimal surface construction and its covariant generalization has many advantages, especially for QFTs in dimensions $d > 2$. With the help of gauge-gravity correspondence, we can understand time-dependent entanglement entropy in an interacting QFT better. Evolution of entanglement entropy in strongly coupled QFTs has been studied via holographic description in terms of a Vaidya geometry [60]. For two CFTs interacting via massless interactions, evolution of Rényi entanglement entropy is computed from the viewpoint of AdS/CFT correspondence [61], leaving the massive interactions case as an open problem. There is still a long way to go before we arrive at a satisfactory theory for interacting QFTs.

With many significant progresses been made, entanglement entropy has its limitations. Given a specific state of the boundary QFT, we shall ask how much information is encoded in the entanglement entropy, i.e., how much metric information can be extracted from the entanglement entropy data. In particular, for a system in a given state admitting a gravitational dual, if we know the entanglement entropy for all subsystems of the boundary, can we decode the full geometry of the gravitational dual corresponding to that state. A natural question is whether there is some more information to be gained from studying other entanglement entropies, such as Rényi entanglement entropy and pseudo-entropy, and the answer is yes. It is well understood that a full understanding of a quantum state requires Rényi entropies $S_n = \frac{1}{1-n}\ln \mathrm{Tr}(\rho^n)$ and the RT formula has been generalized to Rényi entropies using bulk codimension-2 cosmic brane [62, 63].

$$n^2\partial_n\left(\frac{n-1}{n}S_n\right) = \frac{\mathrm{Area}(\text{Cosmic Brane}_n)}{4G_N}. \tag{3}$$

The subscript n on the cosmic brane denotes that its brane tension as a function of n is given by $T_n = \frac{n-1}{4nG_N}$.

Rényi entropies introduced as a one-parameter generalization of the von Neumann entropy, contain richer physical information about the entanglement structure of a quantum state and have been extensively studied in free field theories [64], in two-dimensional CFTs or higher [65–67], and in the context of gauge/gravity correspondence [68, 69]. In particular, Rényi entropy $S_{1/2}$ (the max-entropy) gives the entanglement negativity [70]. One of our research questions is exploring the role of Rényi entropies at different indices, such as smooth min-entropy $S_{\min} \equiv S_\infty$ and collision entropy S_2 [71]. The physical meaning of Rényi entropies at different order n can be investigated through the brane tension T_n. For example, in the $n \to 1$ limit, the cosmic brane becomes tensionless and no longer backreacts on the bulk geometry and the Rényi entropy area law Eq (**??**) reduces to the RT formula. In generic spacetime geometry, we need to consider backreactions between subregions, and thus decoding the full geometry of the gravitational dual needs the study of Rényi entropies.

Pseudo-(Rényi) entropy [73], as a generalization of entanglement entropy via postselection, is a complex-valued measure of information, which can be viewed as the von Neumann (Rényi) entropy of a reduced transition matrix and provide a new class of order parameters in quantum many-body systems. In the AdS/CFT correspondence, pseudo-entropy is dual to areas of minimal area surfaces in time-dependent Euclidean spaces, which serves as the geometric computation of it. The imaginary part of the pseudo-entropy describes an emergent time which generalizes the notion of an emergent space from quantum entanglement. Pseudo-entropy then leads to the definition of timelike entanglement entropy [74], which in the boundary theory can be viewed as a Wick rotation that changes a spacelike boundary subregion to a timelike one. Since the investigation of pseudo-entropy have been constrained to pure quantum states in free QFTs, the first step of our work is investigating pseudo-entropy with Gaussian states [43, 56, 75–77], paving the way for future study in interacting QFTs.

Essentially, the pseudo-entropy theory is a non-Hermitian theory [78–80], as the reduced transition matrix it based on is non-Hermitian and it works in the context of postselection. Therefore, it is closely related to the theory of open quantum systems [81–83]. Last but not least, we look forward that under the interplay of pseudo-entropy, non-Hermitian physics and the geometry of quantum many-body systems, this project can flourish and yield significant results.

Conclusion: Quantum information has come to play a fundamental role in emergent gravity, but the desired theory is still far away. There seems to be missing a crucial piece and a general theory scheme, which we argue to be the entanglement-energy relation and the inherent classicality mechanism, respectively. Further work is needed to concretize and substantiate this theory scheme.

[1] Guifre Vidal. Entanglement renormalization. *Physical review letters*, 99(22):220405, 2007.

[2] Vidal and G. A class of quantum many-body states that can be efficiently simulated. *Physical Review Letters*, 101 (11):6037–6040, 2008.

[3] Brian Swingle. Entanglement renormalization and holography. *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, 86(6):065007, 2012.

[4] Bartlomiej Czech, Lampros Lamprou, Samuel McCandlish, and James Sully. Integral geometry and holography. *Journal of High Energy Physics*, 2015(10):1–41, 2015.

[5] Bartlomiej Czech, Lampros Lamprou, Samuel McCandlish, and James Sully. Tensor networks from kinematic space. *Journal of High Energy Physics*, 2016(7):1–38, 2016.

[6] Xiao Liang Qi. Exact holographic mapping and emergent space-time geometry. *Physics*, 2013.

[7] Ahmed Almheiri, Xi Dong, and Daniel Harlow. Bulk locality and quantum error correction in ads/cft. *Journal of High Energy Physics*, 2015(4):1–34, 2015.

[8] Fernando Pastawski, Beni Yoshida, Daniel Harlow, and John Preskill. Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence. *Journal of High Energy Physics*, 2015(6):1–55, 2015.

[9] Patrick Hayden, Sepehr Nezami, Xiao-Liang Qi, Nathaniel Thomas, Michael Walter, and Zhao Yang. Holographic duality from random tensor networks. *Journal of High Energy Physics*, 2016(11):1–56, 2016.

[10] Ivan Agullo, Béatrice Bonga, Patricia Ribes-Metidieri, Dimitrios Kranas, and Sergi Nadal-Gisbert. How ubiquitous is entanglement in quantum field theory? *Physical Review D*, 108(8):085005, 2023.

[11] Ivan Agullo, Beatrice Bonga, Eduardo Martin-Martinez, Sergi Nadal-Gisbert, T Rick Perche, Jose Polo-Gomez, Patricia Ribes-Metidieri, and Bruno de SL Torres. The multimode nature of spacetime entanglement in qft. *arXiv preprint arXiv:2409.16368*, 2024.

[12] Mark R. Dowling, Andrew C. Doherty, and Stephen D. Bartlett. Energy as an entanglement witness for quantum many-body systems. *Physical Review A*, 70(6):102–102, 2004.

[13] Étienne Jussiau, Léa Bresque, Alexia Auffèves, Kater W Murch, and Andrew N Jordan. Many-body quantum vacuum fluctuation engines. *Physical Review Research*, 5(3): 033122, 2023.

[14] E Calzetta and BL Hu. Quantum fluctuations, decoherence of the mean field, and structure formation in the early universe. *Physical Review D*, 52(12):6770, 1995.

[15] Don N Page and William K Wootters. Evolution without evolution: Dynamics described by stationary observables. *Physical Review D*, 27(12):2885, 1983.

[16] Sebastian Gemsheim and Jan M Rost. Emergence of time from quantum interaction with the environment. *Physical Review Letters*, 131(14):140202, 2023.

[17] Gerard't Hooft. Dimensional reduction in quantum gravity. *arXiv preprint gr-qc/9310026*, 1993.

[18] Leonard Susskind. The world as a hologram. *Journal of Mathematical Physics*, 36(11):6377–6396, 1995.

[19] Juan Maldacena. The large-n limit of superconformal field theories and supergravity. *International journal of theoretical physics*, 38(4):1113–1133, 1999.

[20] Mark Van Raamsdonk. Building up space–time with quantum entanglement. *International Journal of Modern Physics D*, 19(14):2429–2435, 2010.

[21] Thomas Faulkner, Aitor Lewkowycz, and Juan Maldacena. Quantum corrections to holographic entanglement entropy. *Journal of High Energy Physics*, 2013(11):1–18, 2013.

[22] Matthew Headrick. Entanglement renyi entropies in holographic theories. *Physical Review D*, 82(12), 2013.

[23] Ebrahim Karimi and Robert W Boyd. Classical entanglement? *Science*, 350(6265):1172–1173, 2015.

[24] Dilip Paneru, Eliahu Cohen, Robert Fickler, Robert W Boyd, and Ebrahim Karimi. Entanglement: quantum or classical? *Reports on Progress in Physics*, 83(6):064001, 2020.

[25] Strominger and Andrew. The ds/cft correspondence. *Journal of High Energy Physics*, 2001(10):034–034, 2001.

[26] Erik P Verlinde. Emergent gravity and the dark universe. *SciPost Physics*, 2(3):016, 2017.

[27] Muxin Han and Shilin Huang. Discrete gravity on random tensor network and holographic rényi entropy. *Journal of High Energy Physics*, 2017(11):1–25, 2017.

[28] Jeevan Chandra and Thomas Hartman. Toward random tensor networks and holographic codes in cft. *Journal of High Energy Physics*, 2023(5):1–42, 2023.

[29] Shinsei Ryu and Tadashi Takayanagi. Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence. *Physical review letters*, 96(18):181602, 2006.

[30] Veronika E Hubeny, Mukund Rangamani, and Tadashi Takayanagi. A covariant holographic entanglement entropy proposal. *Journal of High Energy Physics*, 2007 (07):062, 2007.

[31] David D Blanco, Horacio Casini, Ling-Yan Hung, and Robert C Myers. Relative entropy and holography. *Journal of High Energy Physics*, 2013(8):1–65, 2013.

[32] Thomas Faulkner, Monica Guica, Thomas Hartman,

Robert C Myers, and Mark Van Raamsdonk. Gravitation from entanglement in holographic cfts. *Journal of High Energy Physics*, 2014(3):1–41, 2014.
[33] Xiao-Liang Qi. Does gravity come from quantum information? *Nature Physics*, 14(10):984–987, 2018.
[34] Nima Lashkari, Michael B McDermott, and Mark Van Raamsdonk. Gravitational dynamics from entanglement “thermodynamics”. *Journal of High Energy Physics*, 2014(4):1–16, 2014.
[35] Chiara Marletto and Vlatko Vedral. Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity. *Physical review letters*, 119(24):240402, 2017.
[36] Sougato Bose, Anupam Mazumdar, Gavin W Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew A Geraci, Peter F Barker, MS Kim, and Gerard Milburn. Spin entanglement witness for quantum gravity. *Physical review letters*, 119(24):240401, 2017.
[37] Ahmed Farag Ali, Saurya Das, and Elias C Vagenas. Discreteness of space from the generalized uncertainty principle. *Physics Letters B*, 678(5):497–499, 2009.
[38] Jang Young Bang and Micheal S Berger. Quantum mechanics and the generalized uncertainty principle. *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, 74(12):125012, 2006.
[39] Mario Berta, Matthias Christandl, Roger Colbeck, Joseph M Renes, and Renato Renner. The uncertainty principle in the presence of quantum memory. *Nature Physics*, 6(9):659–662, 2010.
[40] Qingdi Wang, Zhen Zhu, and William G. Unruh. How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the universe. *Phys.rev.d*, 95(10):103504, 2017.
[41] Qingdi Wang. Reformulation of the cosmological constant problem. *Physical Review Letters*, 125(5), 2020.
[42] Luigi Amico, Rosario Fazio, Andreas Osterloh, and Vlatko Vedral. Entanglement in many-body systems. *Reviews of modern physics*, 80(2):517–576, 2008.
[43] Erik Aurell, Lucas Hackl, Pawe Horodecki, Robert H Jonsson, and Mario Kieburg. Random pure gaussian states and hawking radiation. *Physical Review Letters*, 133(6): 060202, 2024.
[44] Rajibul Islam, Ruichao Ma, Philipp M Preiss, M Eric Tai, Alexander Lukin, Matthew Rispoli, and Markus Greiner. Measuring entanglement entropy in a quantum many-body system. *Nature*, 528(7580):77–83, 2015.
[45] Dmitry A Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn. Colloquium: Many-body localization, thermalization, and entanglement. *Reviews of Modern Physics*, 91(2):021001, 2019.
[46] Jens Eisert, Mathis Friesdorf, and Christian Gogolin. Quantum many-body systems out of equilibrium. *Nature Physics*, 11(2):124–130, 2015.
[47] Shinsei Ryu and Tadashi Takayanagi. Aspects of holographic entanglement entropy. *Journal of High Energy Physics*, 2006(08):045, 2006.
[48] Dmitri V Fursaev. Proof of the holographic formula for entanglement entropy. *Journal of High Energy Physics*, 2006(09):018, 2006.
[49] Juan Maldacena and Leonard Susskind. Cool horizons for entangled black holes. *Fortschritte der Physik*, 61(9): 781–811, 2013.
[50] Helmut Reeh and Siegfried Schlieder. Bemerkungen zur unitäräquivalenz von lorentzinvarianten feldern. *Il Nuovo Cimento (1955-1965)*, 22:1051–1068, 1961.
[51] Ian A Morrison. Boundary-to-bulk maps for ads causal wedges and the reeh-schlieder property in holography. *Journal of High Energy Physics*, 2014(5):1–29, 2014.
[52] Alexander Strohmaier, Rainer Verch, and Manfred Wollenberg. Microlocal analysis of quantum fields on curved space–times: Analytic wave front sets and reeh–schlieder theorems. *Journal of Mathematical Physics*, 43(11):5514–5530, 2002.
[53] Bruno de SL Torres, Kelly Wurtz, Jose Polo-Gomez, and Eduardo Martin-Martinez. Entanglement structure of quantum fields through local probes. *Journal of High Energy Physics*, 2023(5):1–38, 2023.
[54] Xi Dong, Eva Silverstein, and Gonzalo Torroba. De sitter holography and entanglement entropy. *Journal of High Energy Physics*, 2018(7):1–24, 2018.
[55] Rafael D Sorkin. Expressing entropy globally in terms of (4d) field-correlations. In *Journal of Physics: Conference Series*, volume 484, page 012004. IOP Publishing, 2014.
[56] Christian Weedbrook, Stefano Pirandola, Raul Garcia-Patron, Nicolas J Cerf, Timothy C Ralph, Jeffrey H Shapiro, and Seth Lloyd. Gaussian quantum information. *Reviews of Modern Physics*, 84(2):621–669, 2012.
[57] Pasquale Calabrese and John Cardy. Entanglement entropy and quantum field theory: a non-technical introduction. *International Journal of Quantum Information*, 4(03):429–438, 2006.
[58] Yangang Chen, Lucas Hackl, Ravi Kunjwal, Heidar Moradi, Yasaman K Yazdi, and Miguel Zilhão. Towards spacetime entanglement entropy for interacting theories. *Journal of High Energy Physics*, 2020(11):1–32, 2020.
[59] José J Fernández-Melgarejo and Javier Molina-Vilaplana. Entanglement entropy: non-gaussian states and strong coupling. *Journal of High Energy Physics*, 2021(2):1–21, 2021.
[60] Javier Abajo-Arrastia, Joao Aparicio, and Esperanza López. Holographic evolution of entanglement entropy. *Journal of High Energy Physics*, 2010(11):1–27, 2010.
[61] Ali Mollabashi, Noburo Shiba, and Tadashi Takayanagi. Entanglement between two interacting cfts and generalized holographic entanglement entropy. *Journal of High Energy Physics*, 2014(4):1–36, 2014.
[62] Xi Dong. The gravity dual of rényi entropy. *Nature communications*, 7(1):12472, 2016.
[63] Xi Dong, Aitor Lewkowycz, and Mukund Rangamani. Deriving covariant holographic entanglement. *Journal of High Energy Physics*, 2016(11):1–39, 2016.
[64] Igor R Klebanov, Silviu S Pufu, Subir Sachdev, and Benjamin R Safdi. Rényi entropies for free field theories. *Journal of High Energy Physics*, 2012(4):1–28, 2012.
[65] Shouvik Datta and Justin R David. Rényi entropies of free bosons on the torus and holography. *Journal of High Energy Physics*, 2014(4):1–52, 2014.
[66] Jeongseog Lee, Lauren McGough, and Benjamin R Safdi. Rényi entropy and geometry. *Physical Review D*, 89(12): 125016, 2014.
[67] Aitor Lewkowycz and Eric Perlmutter. Universality in the geometric dependence of renyi entropy. *Journal of High Energy Physics*, 2015(1):1–36, 2015.
[68] Dmitri V Fursaev. Entanglement rényi entropies in conformal field theories and holography. *Journal of High Energy Physics*, 2012(5):1–24, 2012.
[69] Damián A Galante and Robert C Myers. Holographic renyi entropies at finite coupling. *Journal of High Energy*

Physics, 2013(8):1–28, 2013.

[70] Guifré Vidal and Reinhard F Werner. Computable measure of entanglement. *Physical Review A*, 65(3):032314, 2002.

[71] Renato Renner and Stefan Wolf. Smooth rényi entropy and applications. In *International Symposium onInformation Theory, 2004. ISIT 2004. Proceedings.*, page 233. IEEE, 2004.

[72] Christoph Holzhey, Finn Larsen, and Frank Wilczek. Geometric and renormalized entropy in conformal field theory. *Nuclear physics b*, 424(3):443–467, 1994.

[73] Yoshifumi Nakata, Tadashi Takayanagi, Yusuke Taki, Kotaro Tamaoka, and Zixia Wei. New holographic generalization of entanglement entropy. *Physical Review D*, 103(2):026005, 2021.

[74] Jonathan Harper, Ali Mollabashi, Tadashi Takayanagi, Yusuke Taki, et al. Timelike entanglement entropy. *Journal of High Energy Physics*, 2023(5):1–62, 2023.

[75] Lucas Hackl and Eugenio Bianchi. Bosonic and fermionic gaussian states from kähler structures. *SciPost Physics Core*, 4(3):025, 2021.

[76] Bennet Windt, Alexander Jahn, Jens Eisert, and Lucas Hackl. Local optimization on pure gaussian state manifolds. *SciPost Physics*, 10(3):066, 2021.

[77] Lucas Fabian Hackl. *Aspects of Gaussian states entanglement, squeezing and complexity.* The Pennsylvania State University, 2018.

[78] Ali Mostafazadeh. Quantum brachistochrone problem and the geometry of the state space in pseudo-hermitian quantum mechanics. *Physical review letters*, 99(13): 130502, 2007.

[79] Wu-zhong Guo, Song He, and Yu-Xuan Zhang. Constructible reality condition of pseudo entropy via pseudo-hermiticity. *Journal of High Energy Physics*, 2023(5): 1–31, 2023.

[80] Wu-zhong Guo and Yao-zong Jiang. Pseudo entropy and pseudo-hermiticity in quantum field theories. *Journal of High Energy Physics*, 2024(5):1–34, 2024.

[81] Lucas Hackl, Tommaso Guaita, Tao Shi, Jutho Haegeman, Eugene Demler, and J Ignacio Cirac. Geometry of variational methods: dynamics of closed quantum systems. *SciPost Physics*, 9(4):048, 2020.

[82] Leonardo Banchi, Paolo Giorda, and Paolo Zanardi. Quantum information-geometry of dissipative quantum phase transitions. *Physical Review E*, 89(2):022102, 2014.

[83] Janusz Grabowski, Marek Kuś, and Giuseppe Marmo. Geometry of quantum systems: density states and entanglement. *Journal of Physics A: Mathematical and General*, 38(47):10217, 2005.