Bézier Triangles

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Triangular patches

- How do we build triangular patches instead of quads?

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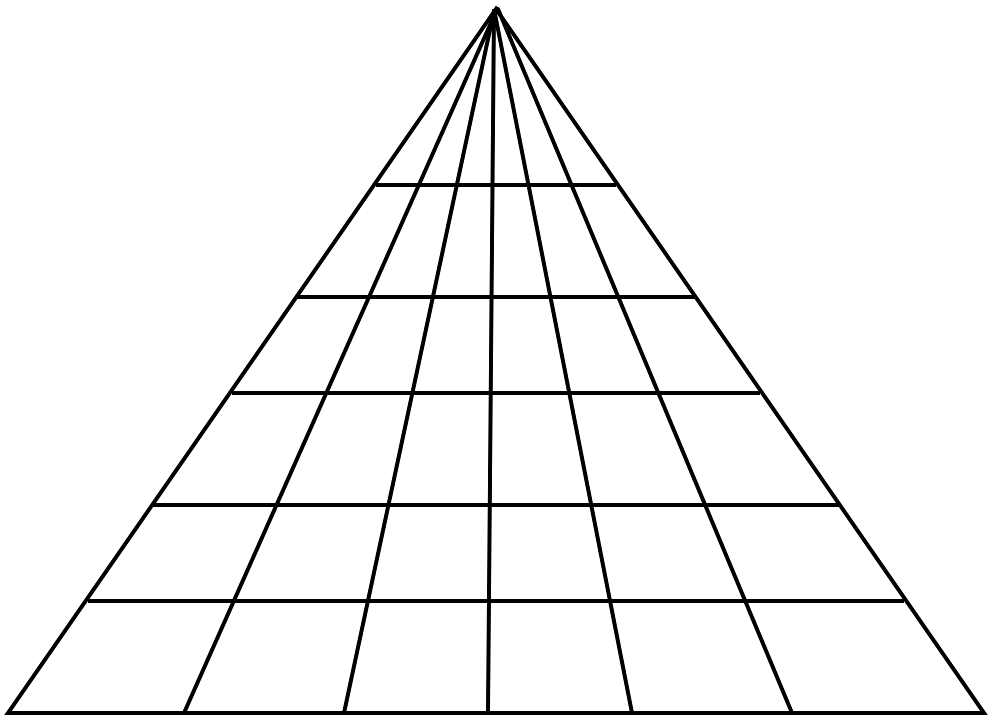

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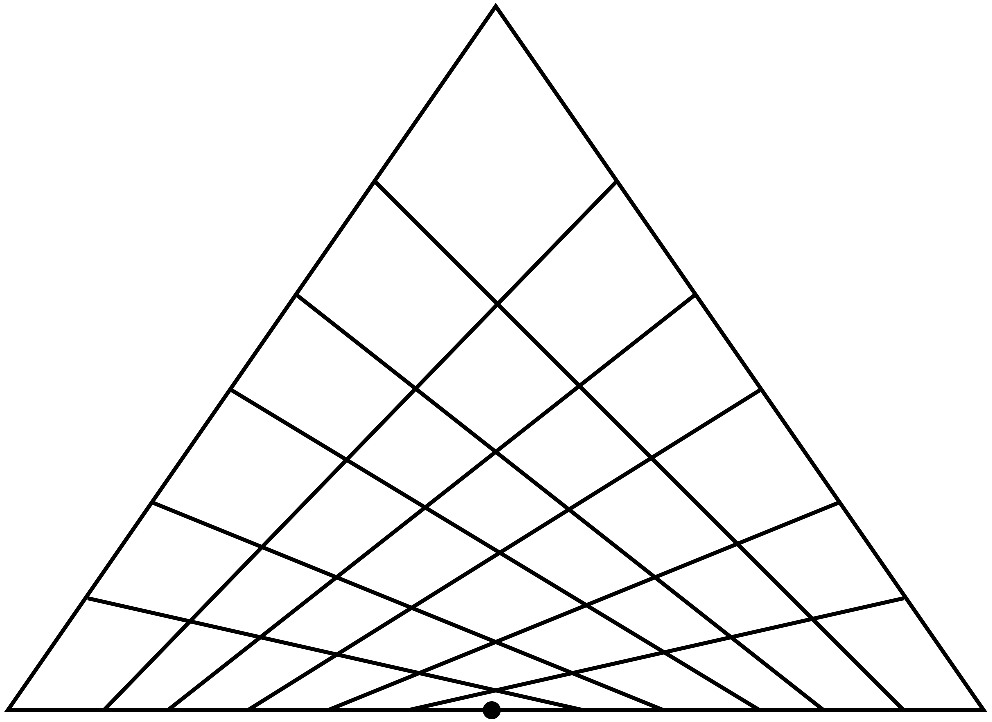
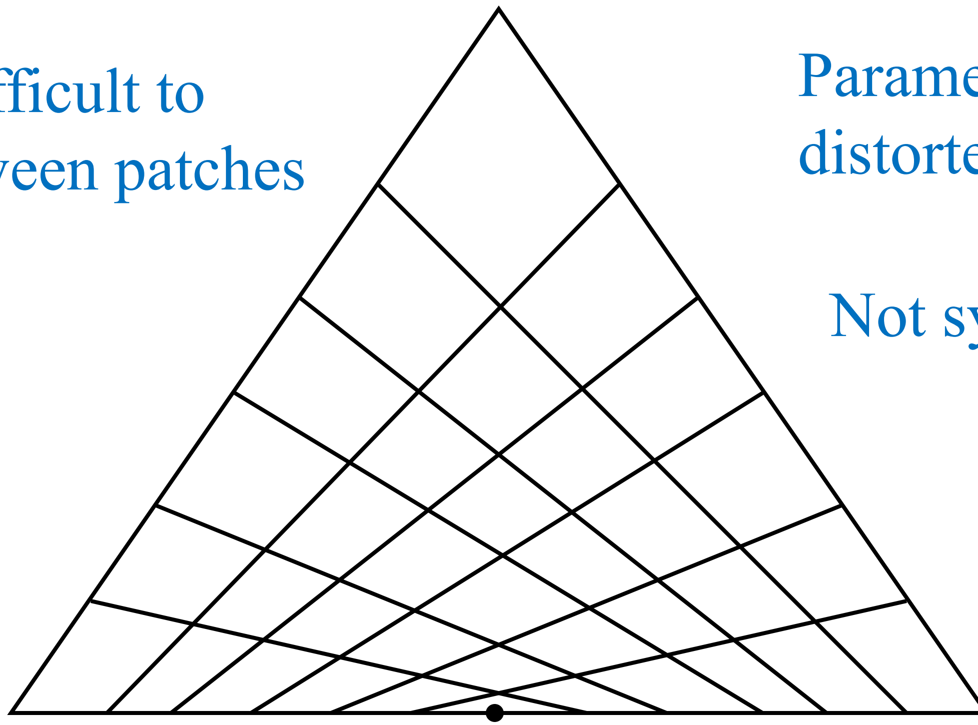


Triangular patches

- How do we build triangular patches instead of quads?

Continuity difficult to maintain between patches

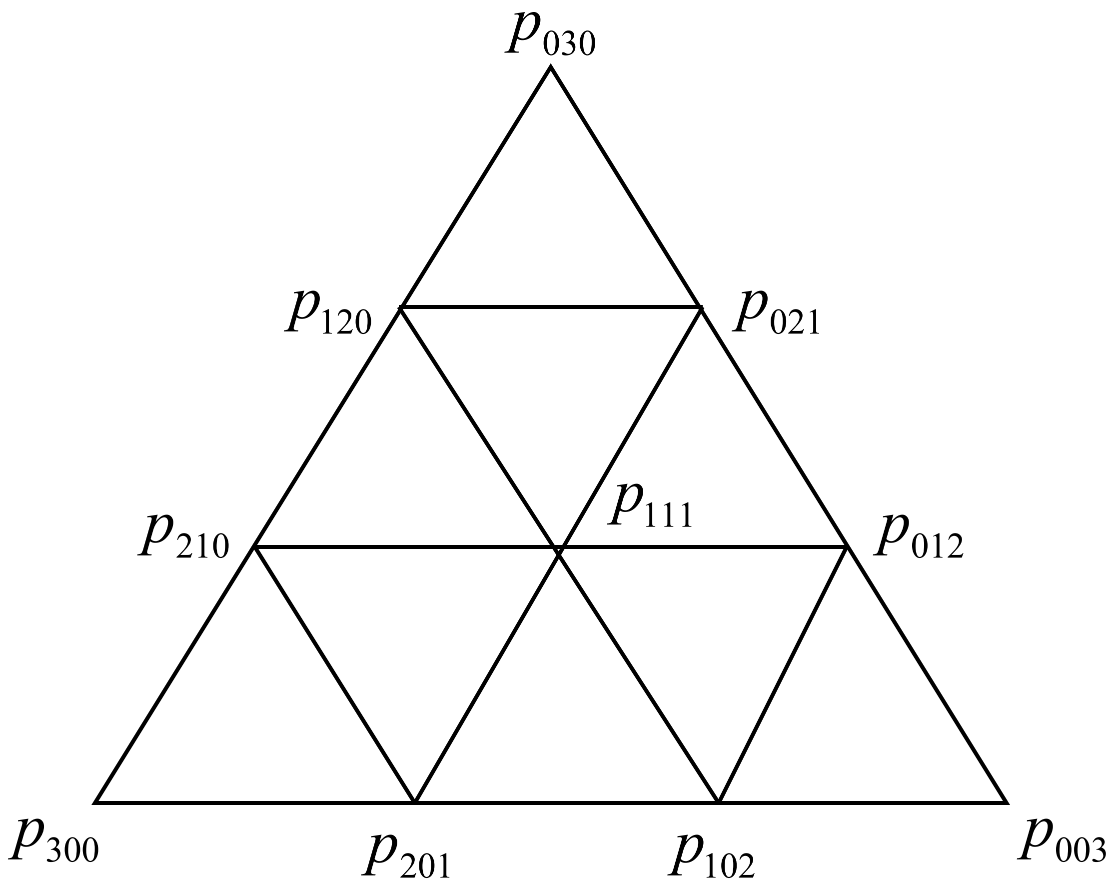


Parameterization very distorted

Not symmetric

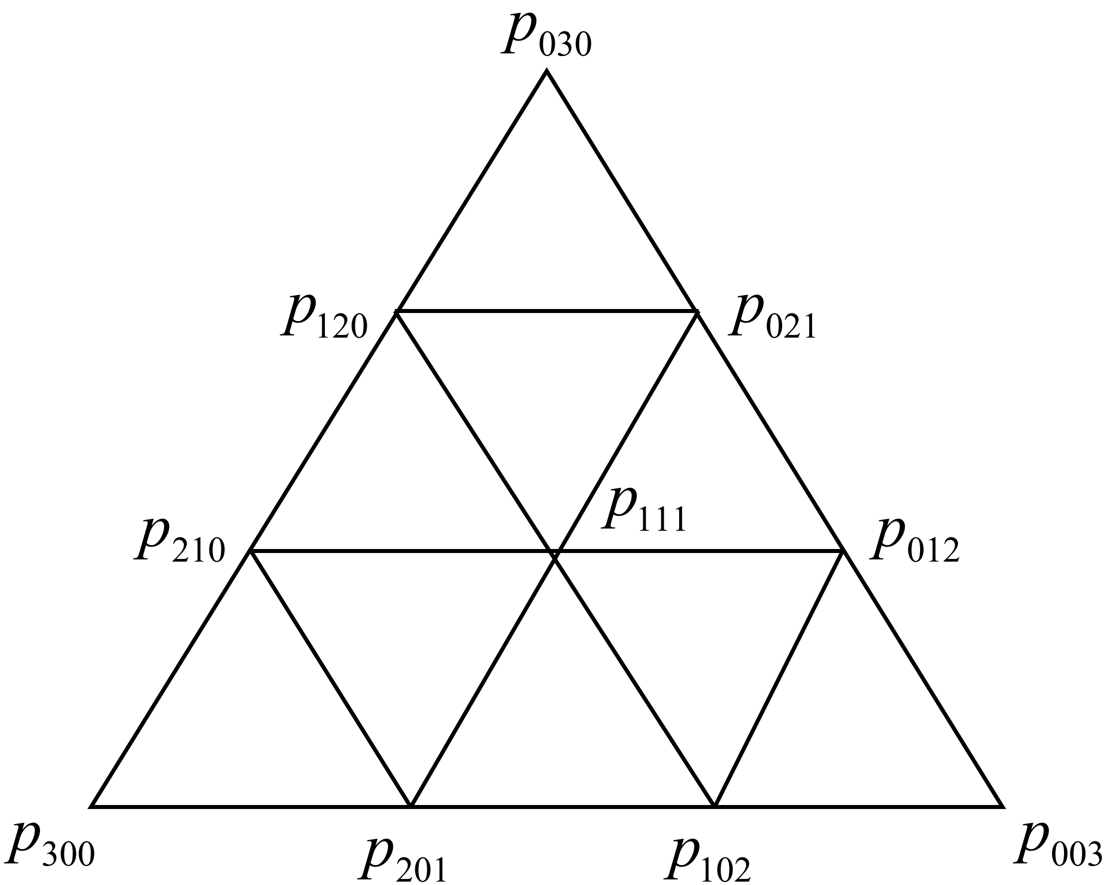
Bézier triangles

- Control points p_{ijk} defined in triangular array



The de Casteljau algorithm for Bézier triangles

- Evaluate at (s,t,u) where $s+t+u=1$



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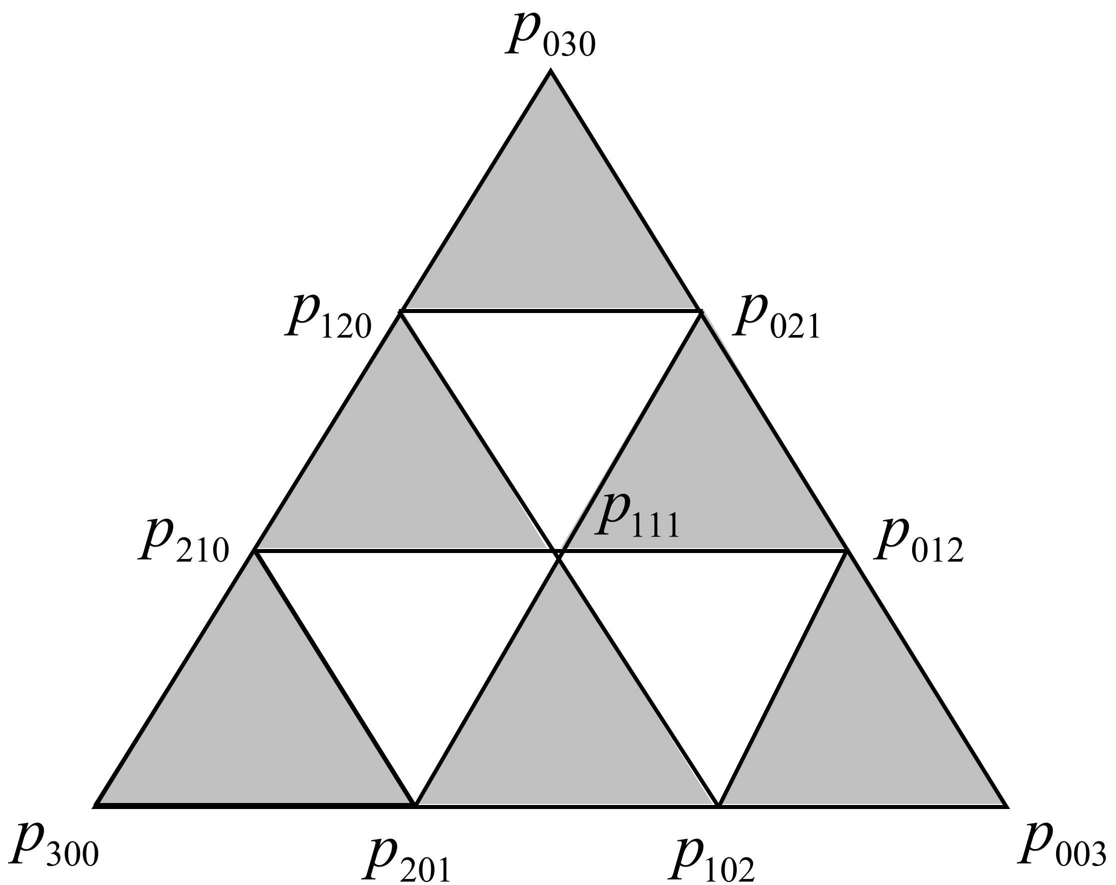

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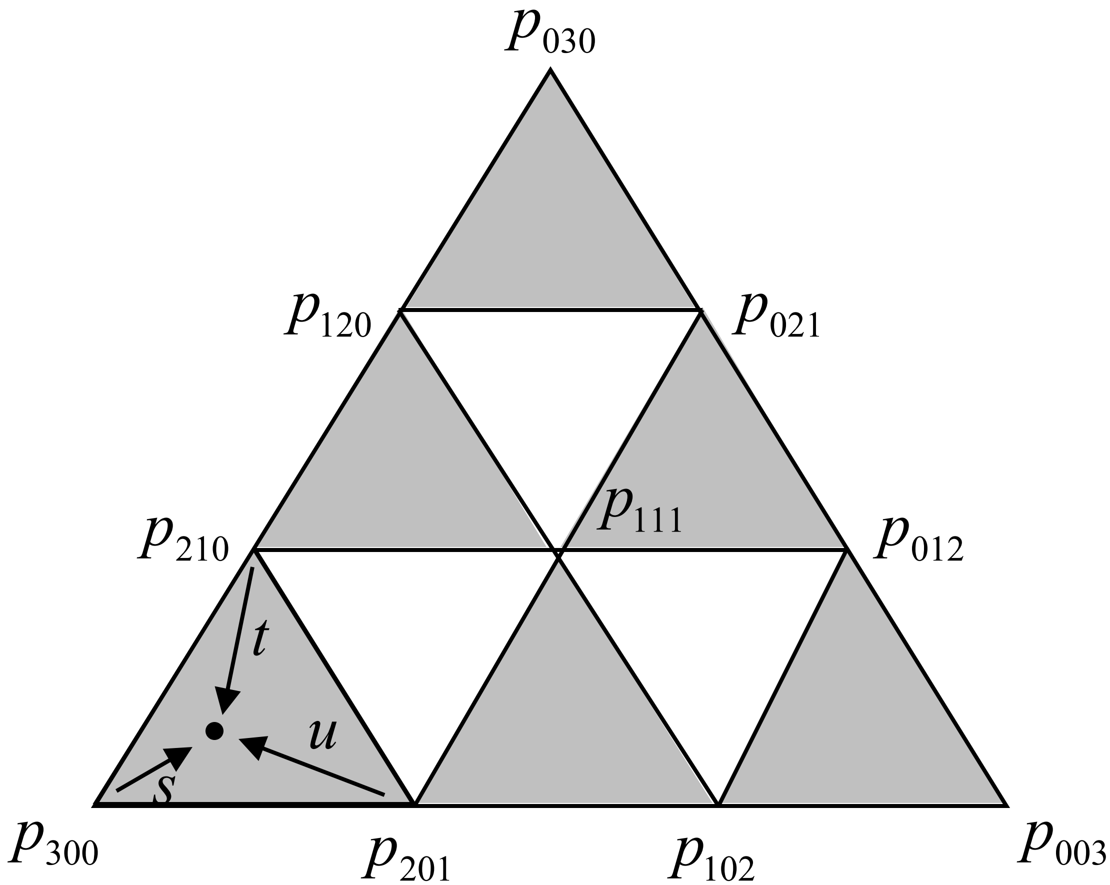

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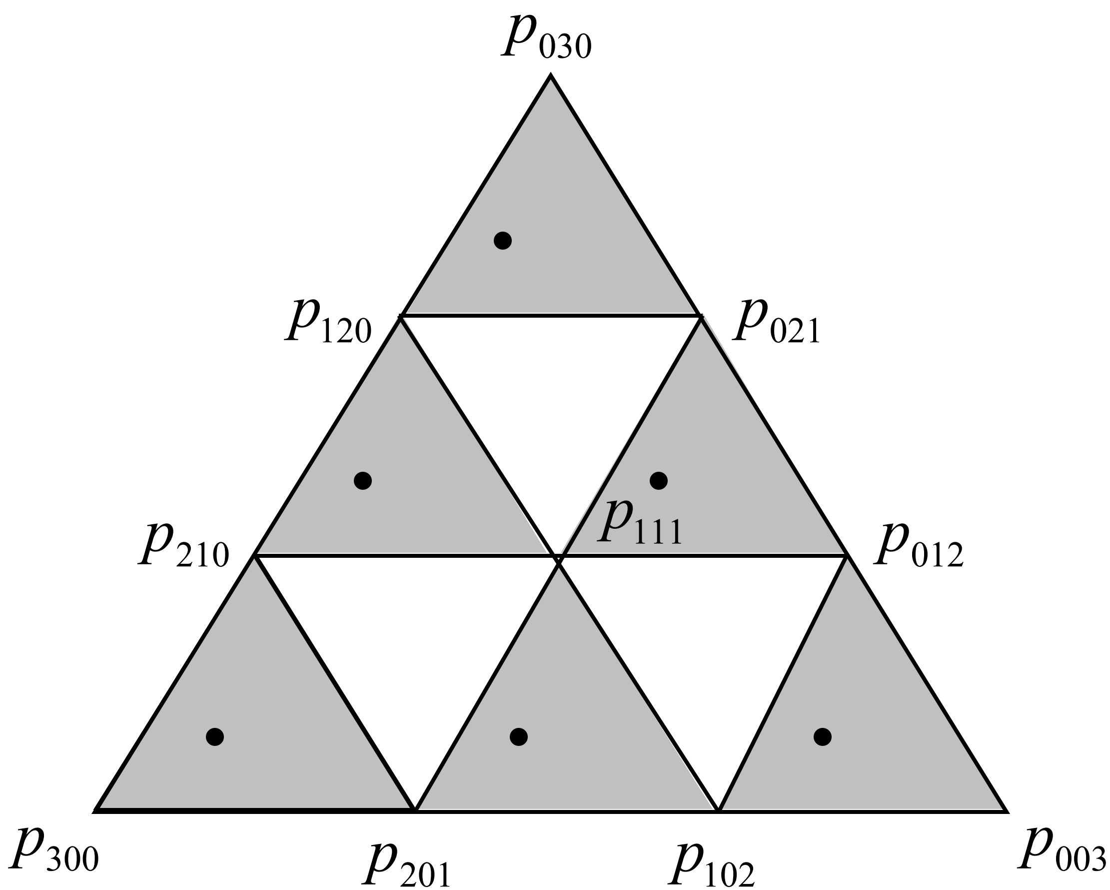

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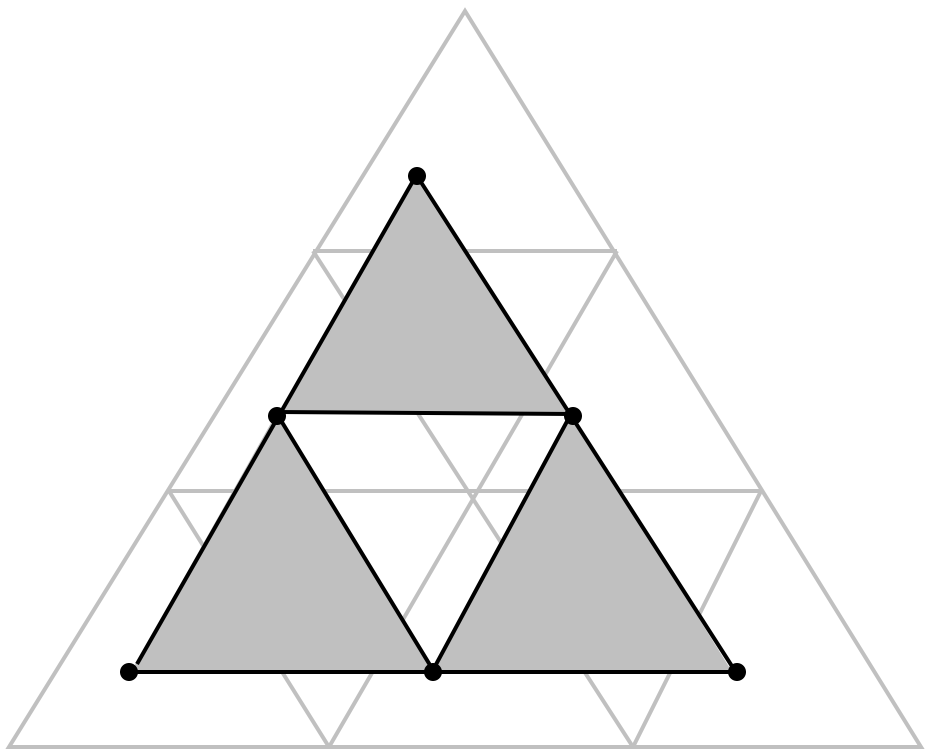

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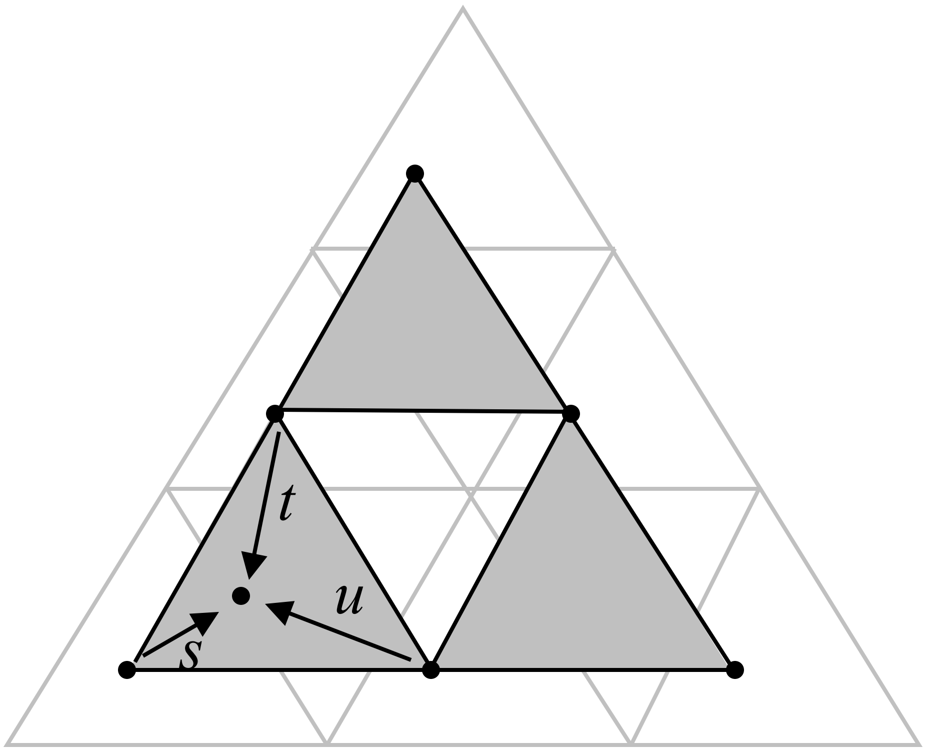

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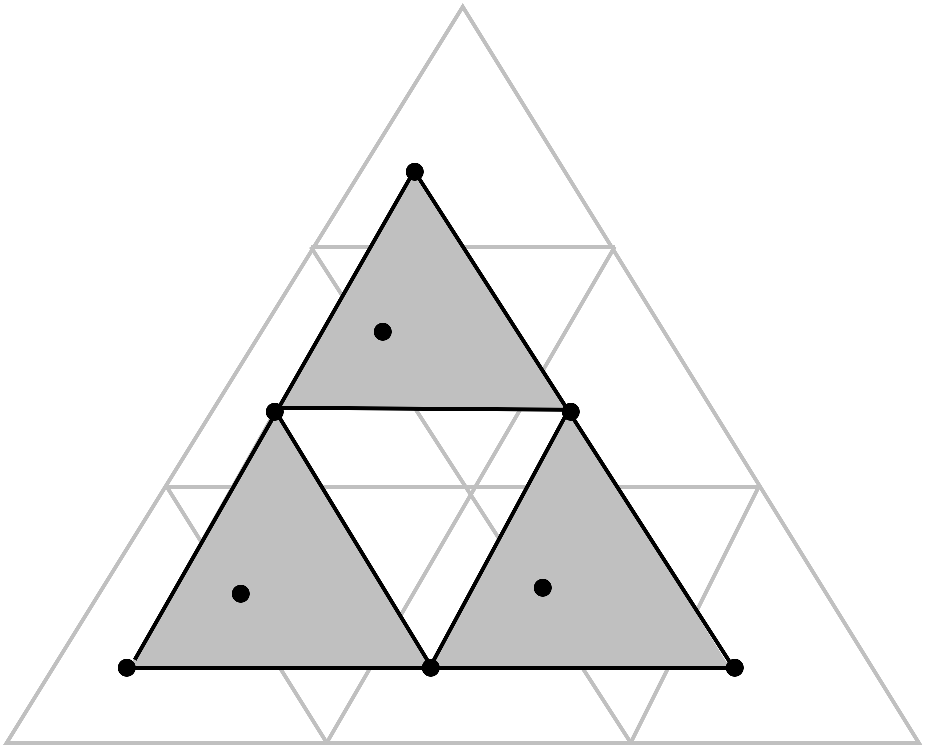

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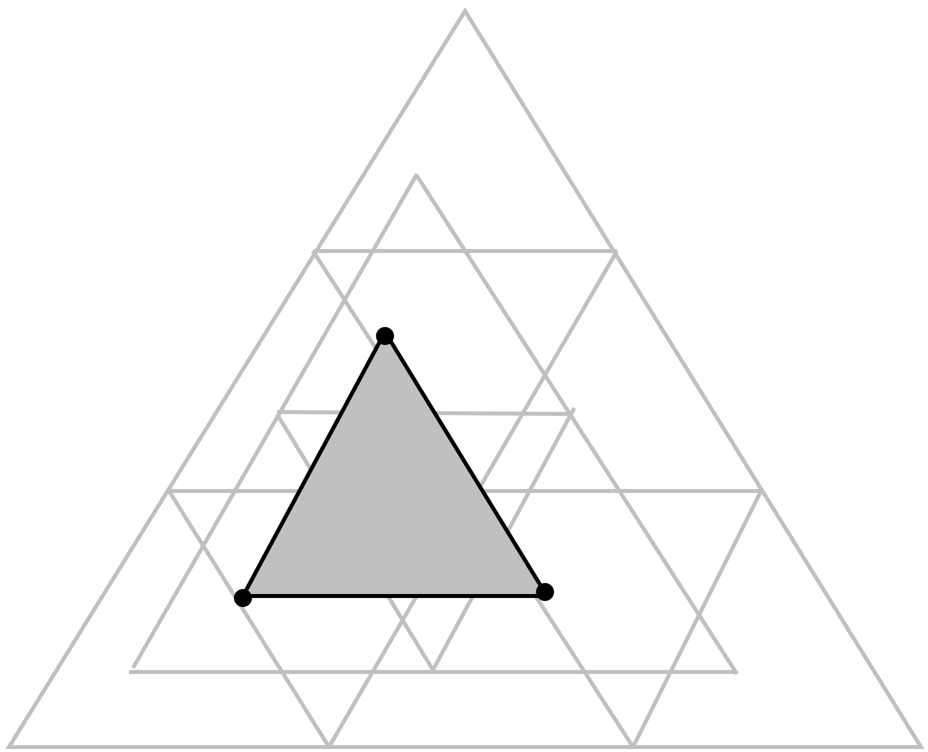

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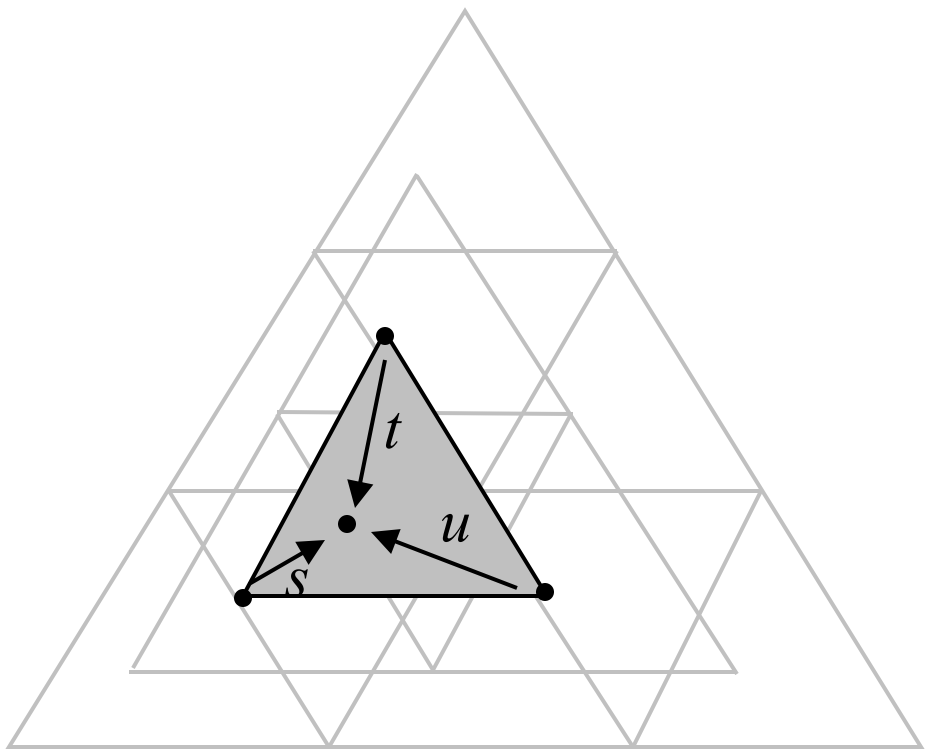
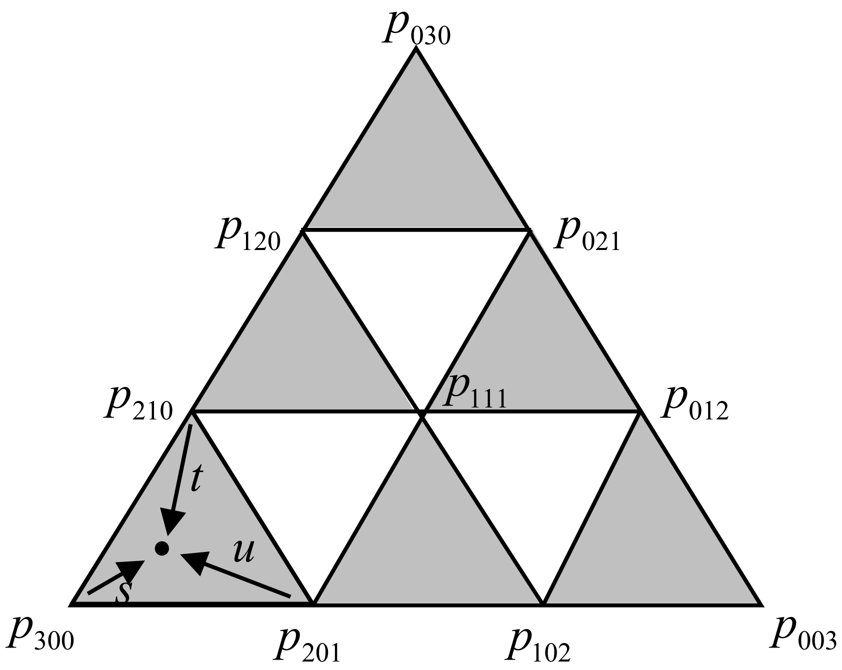


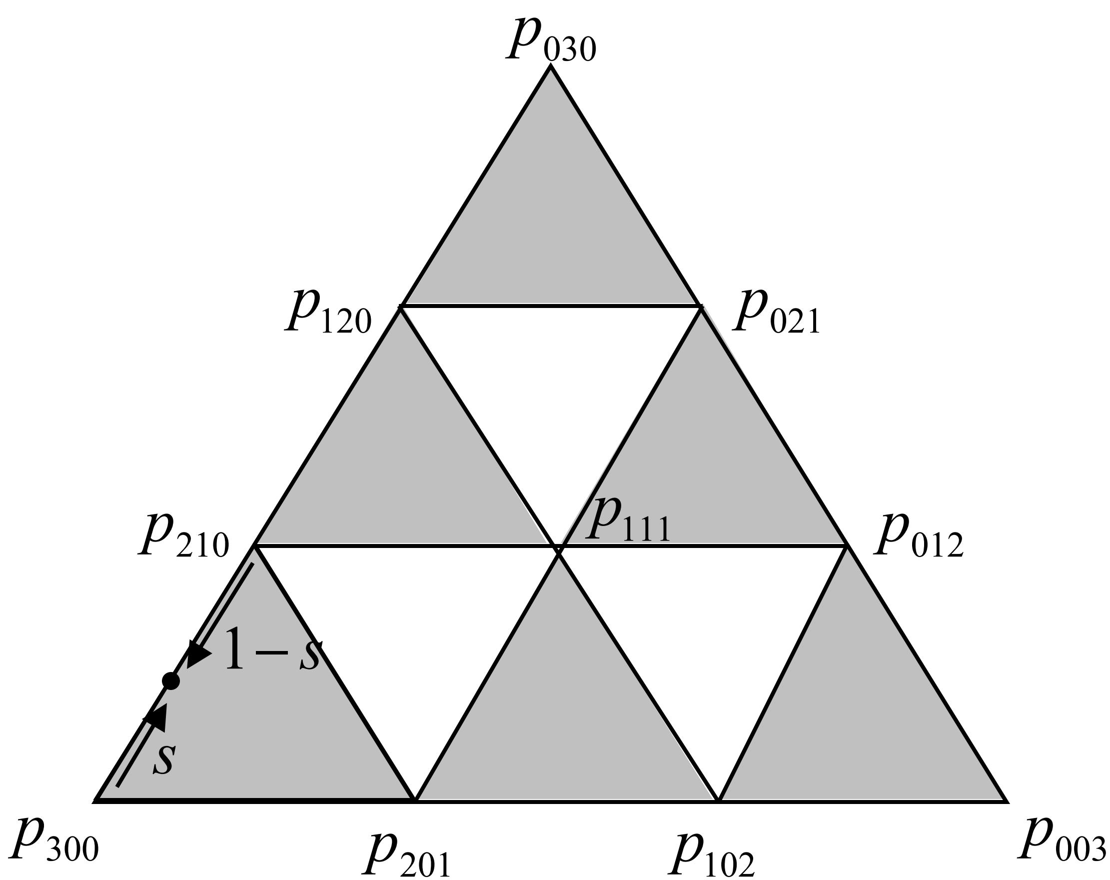
Properties of Bézier triangles

- Convex hull



Properties of Bézier triangles

- Convex hull
- Boundaries are Bézier curves



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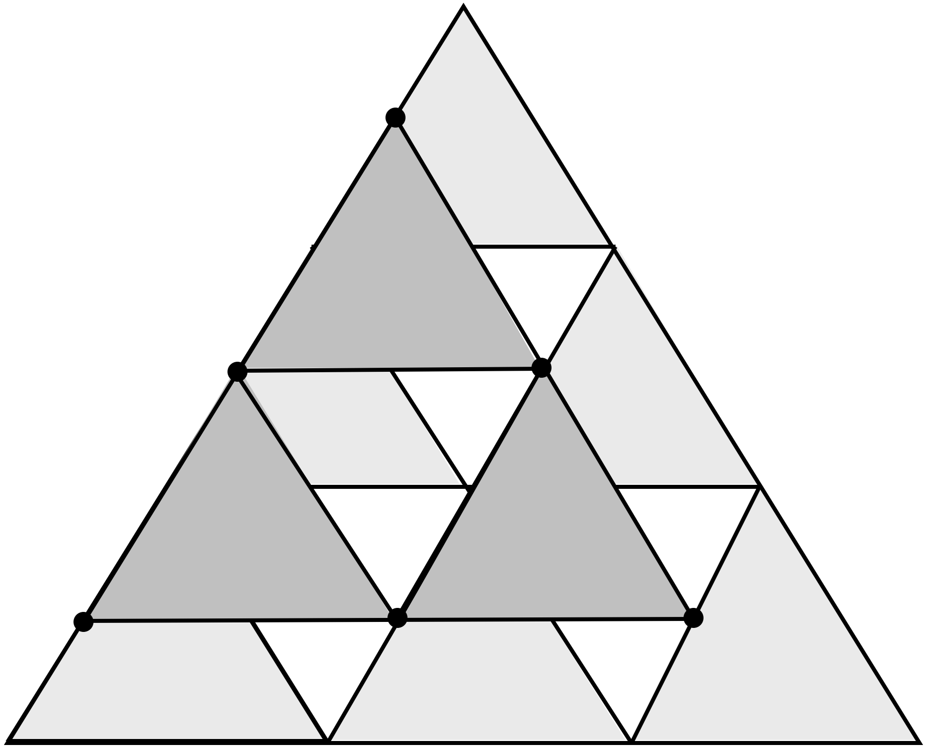

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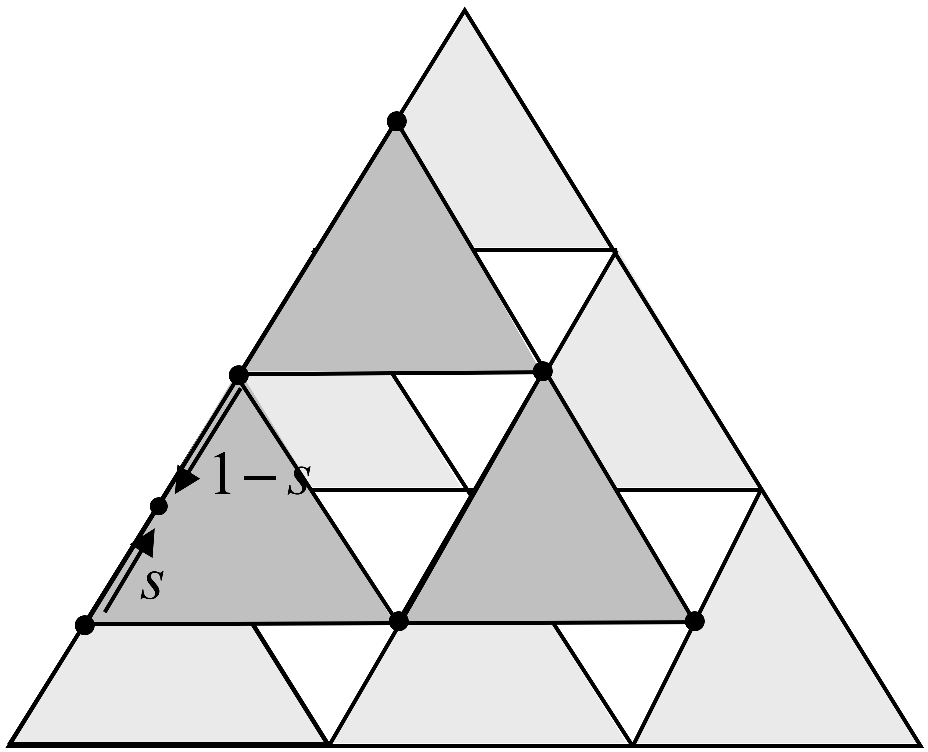

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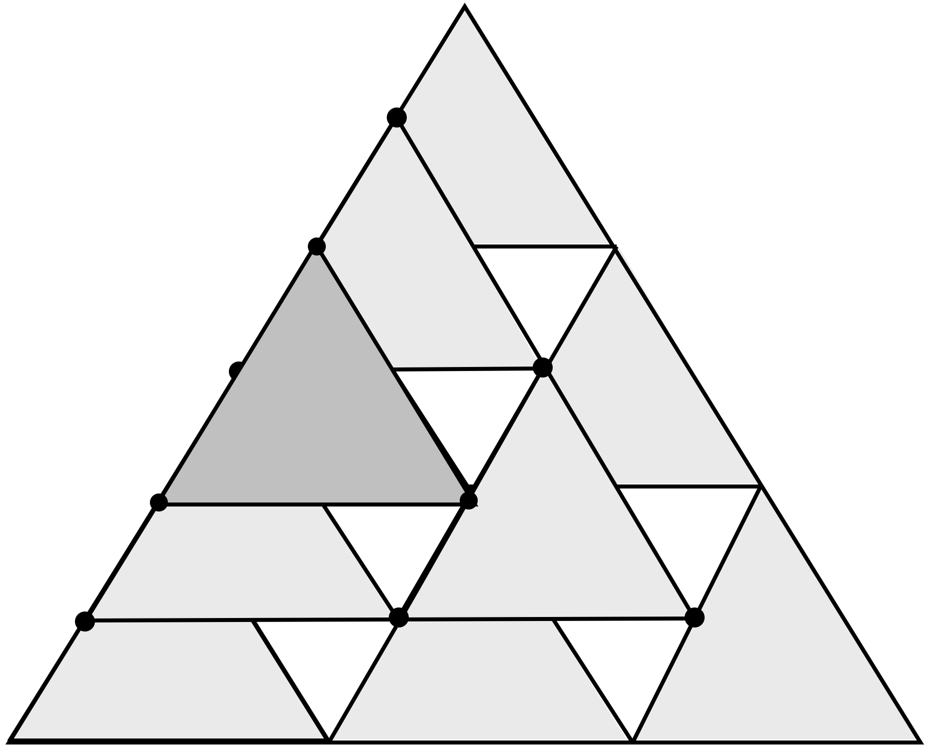

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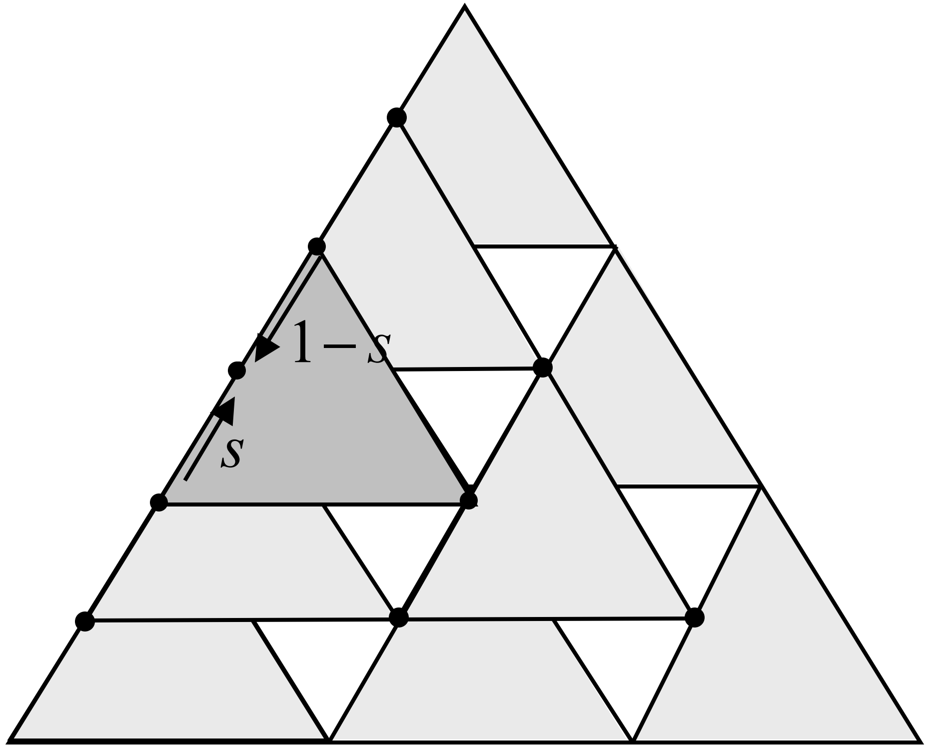
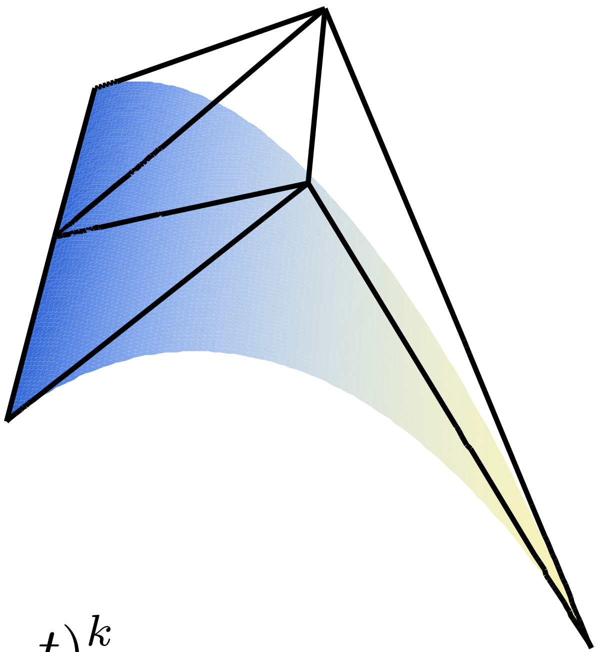


Properties of Bézier triangles

- Convex hull
- Boundaries are Bézier curves
- Explicit polynomial form

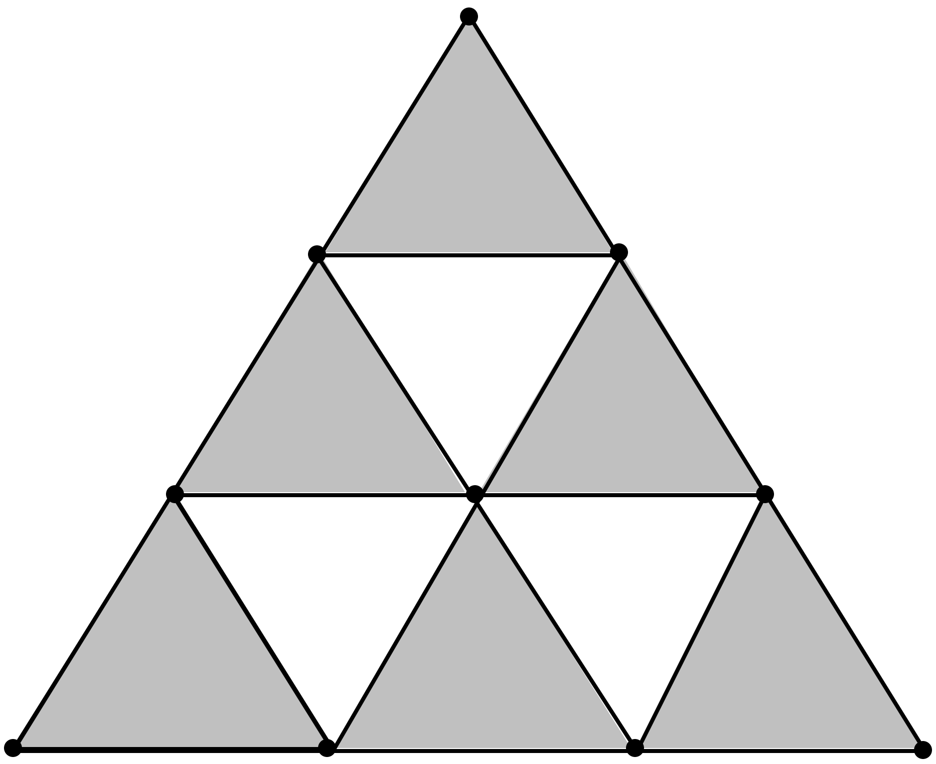
$$b^n(s,t) = \sum_{i+j+k=n} p_{i,j,k} B_{i,j,k}(s,t)$$



Bernstein polynomials:

$$B_{i,j,k}(s,t) := \frac{n!}{i!j!k!} s^i t^j (1-s-t)^k$$

Subdividing Bézier triangles



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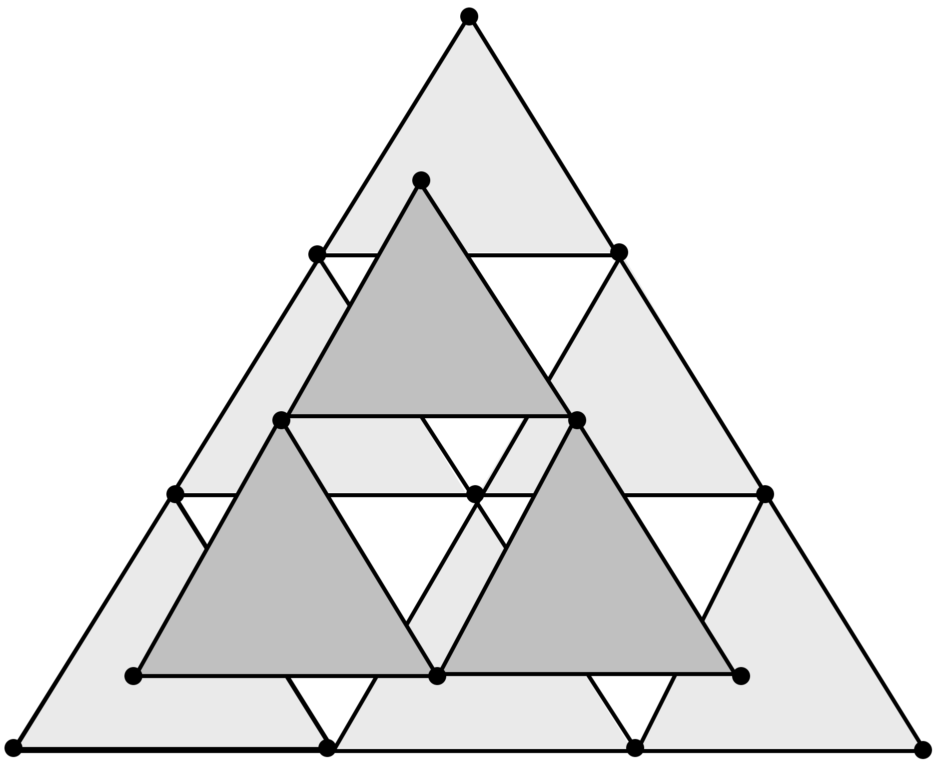

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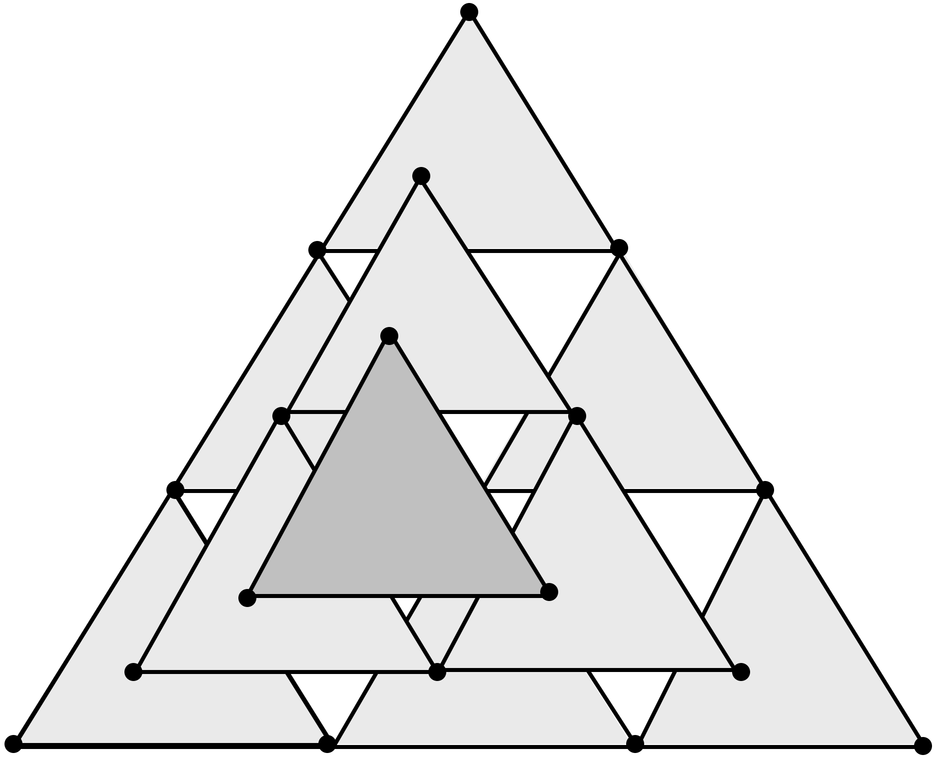

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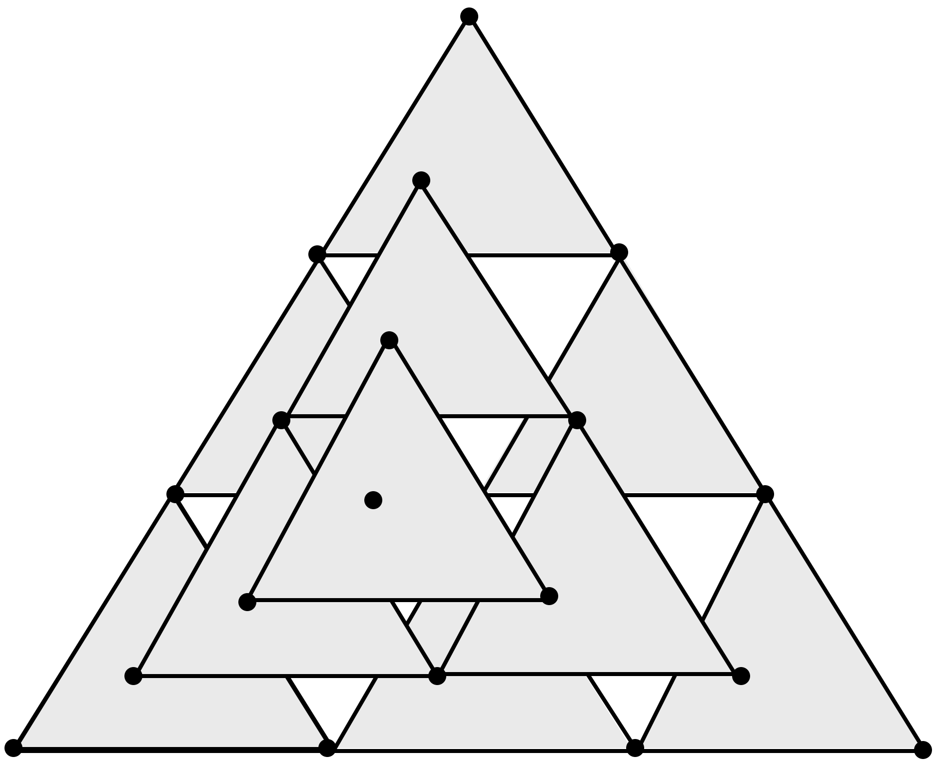

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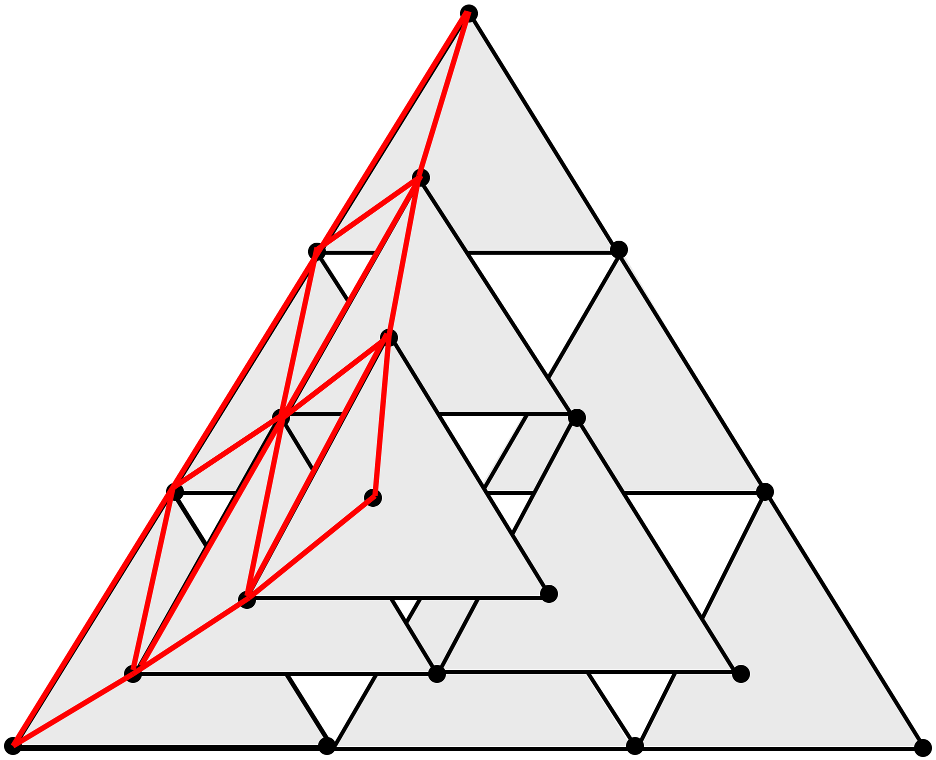

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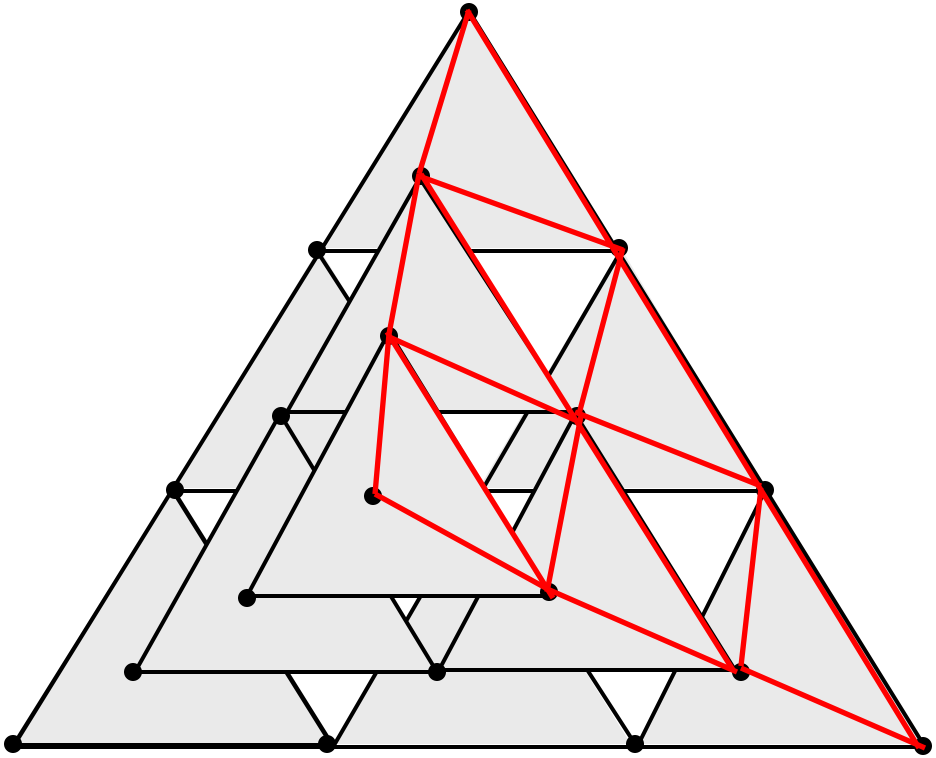

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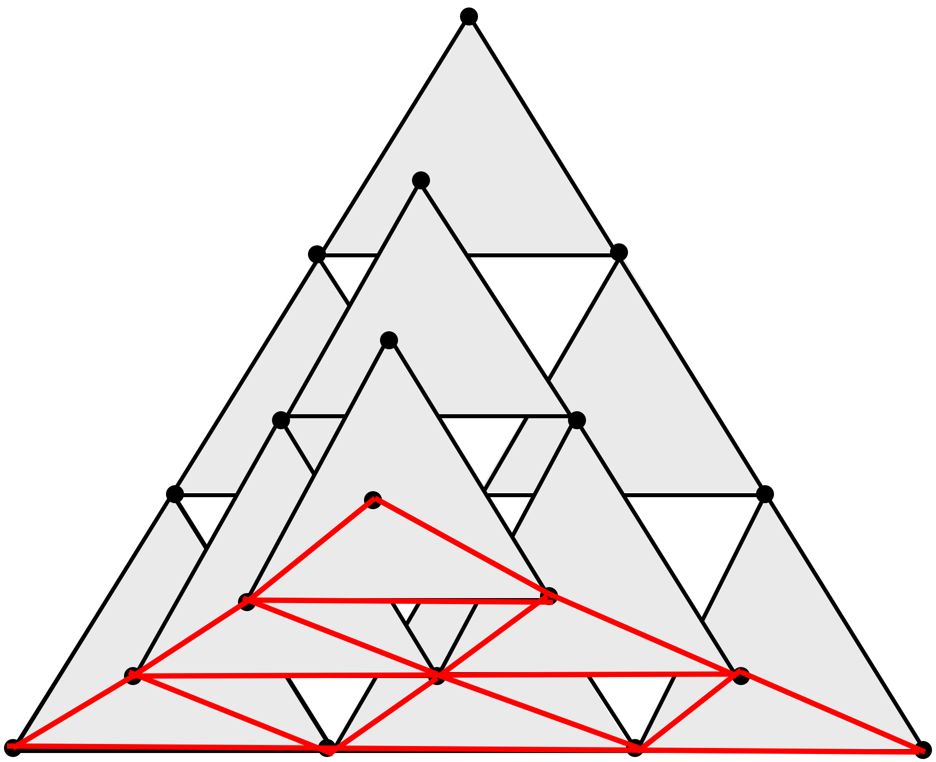
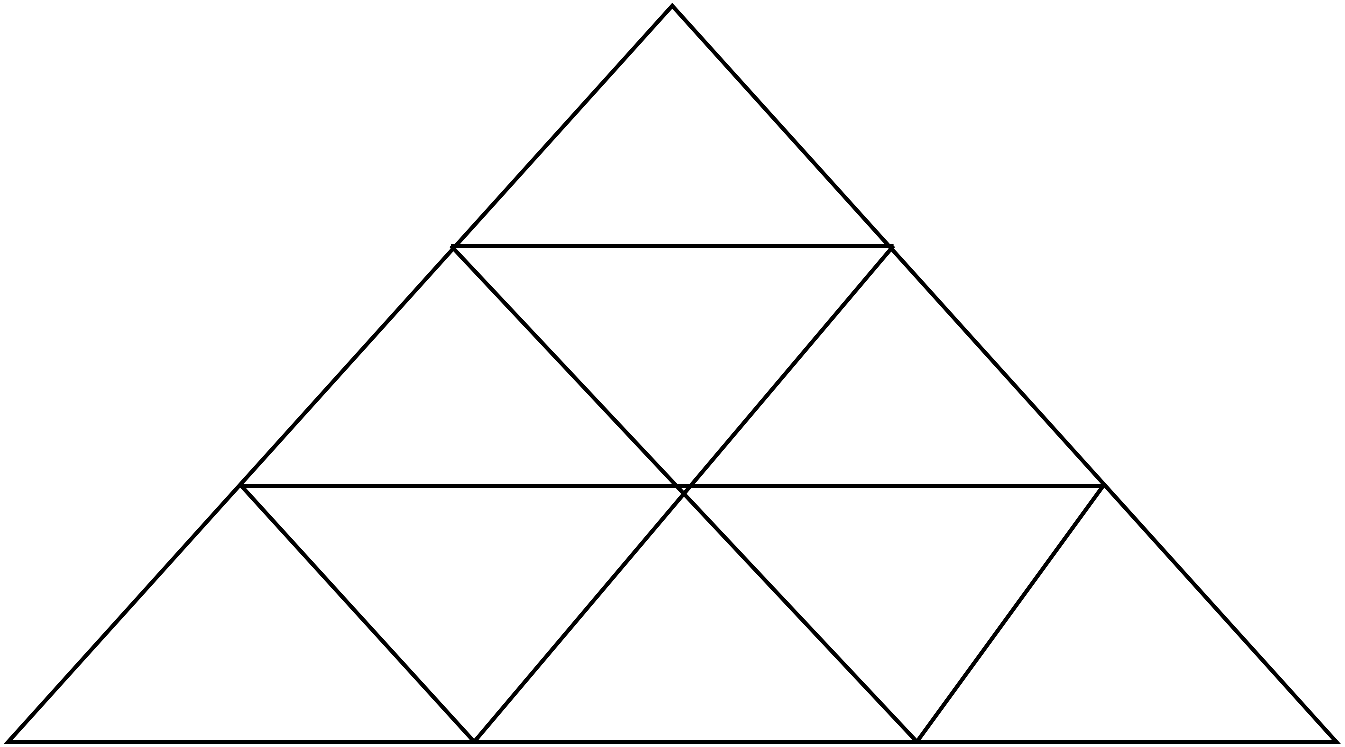


Subdividing Bézier triangles

- Split along longest edge



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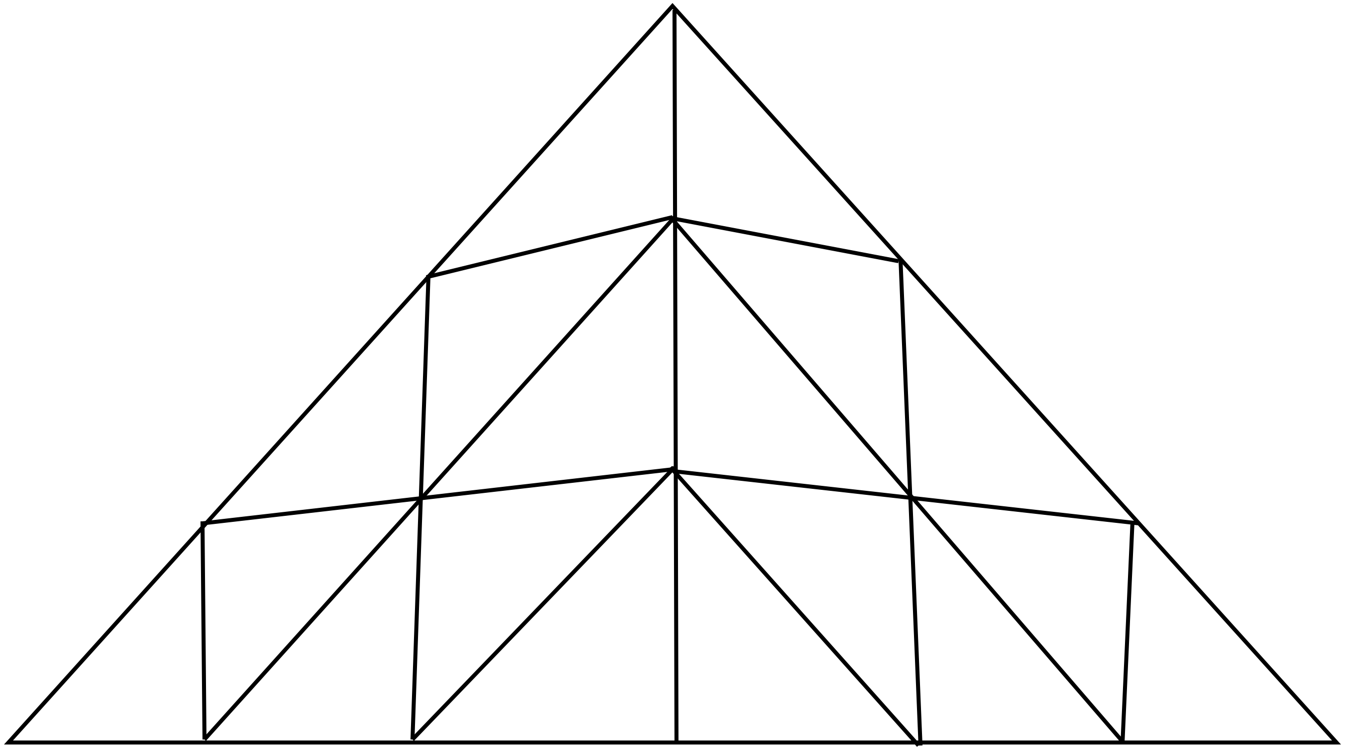
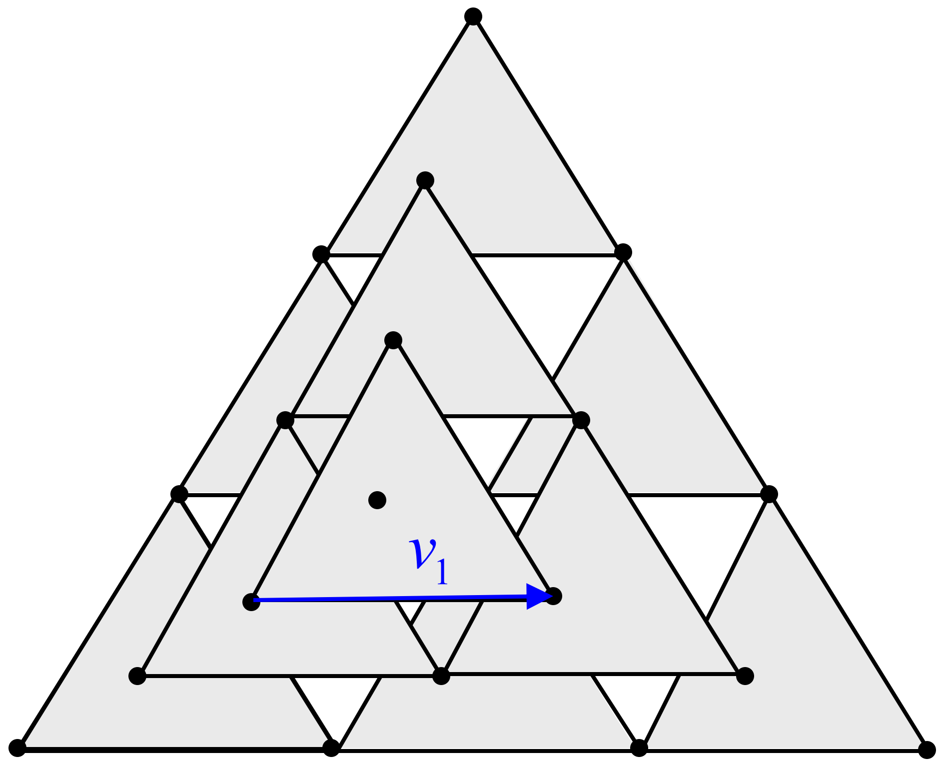


Derivatives of Bézier Triangles



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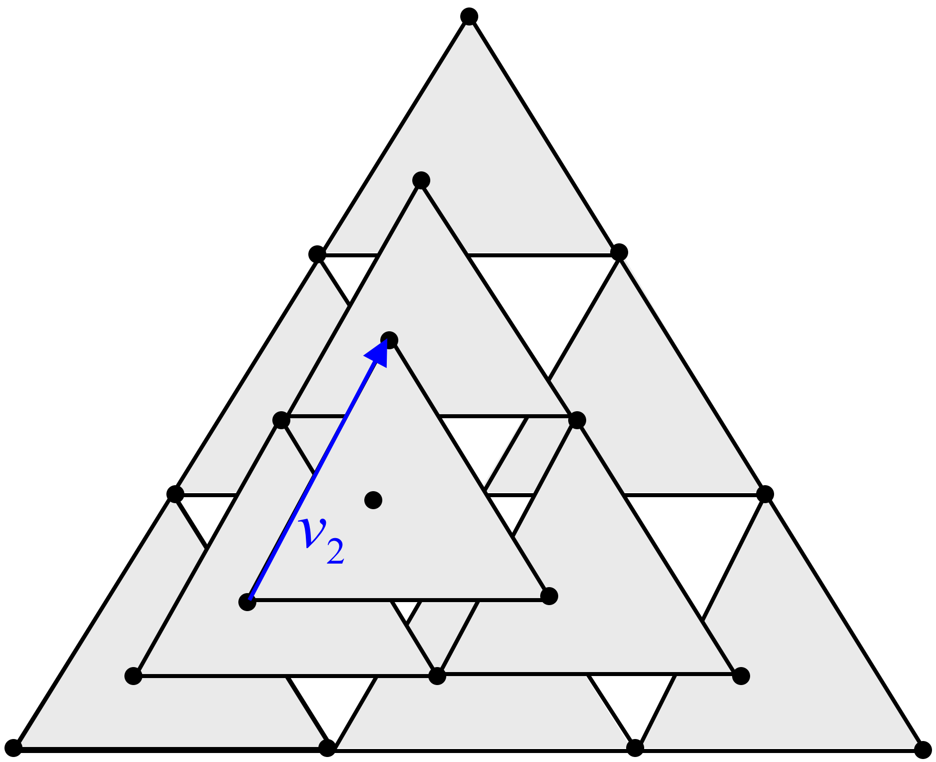

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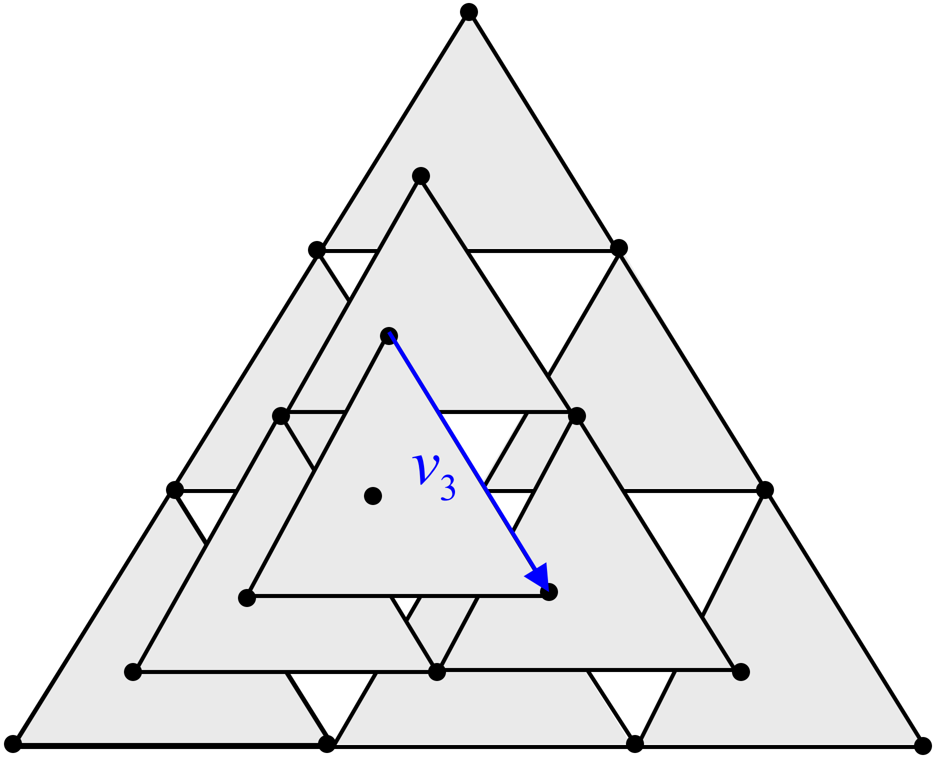
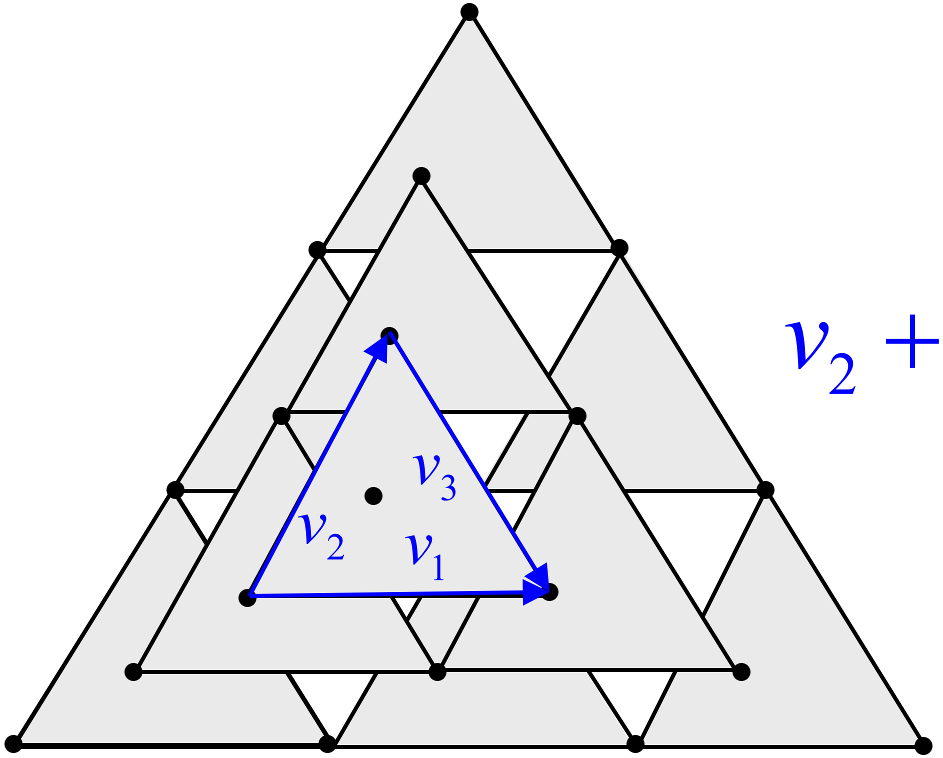


Derivatives of Bézier Triangles

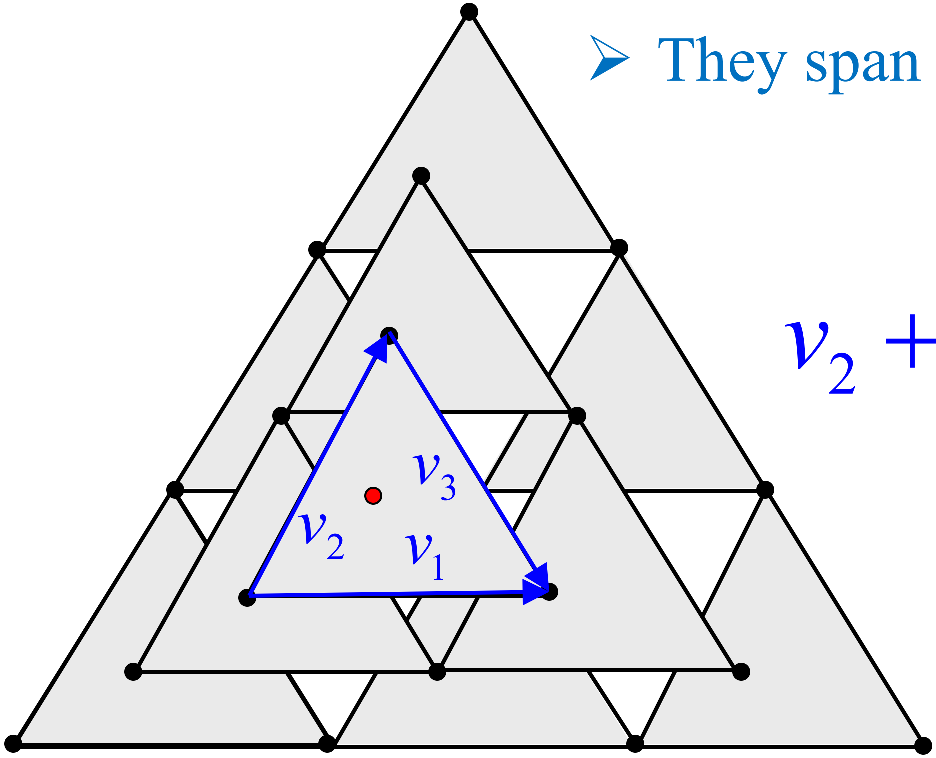
- Really only 2 directions for derivatives



$$v_2 + v_3 = v_1$$

Derivatives of Bézier Triangles

- Really only 2 directions for derivatives
- They span the tangent plane



$$v_2 + v_3 = v_1$$

Many properties are similar to the rectangular case

- Degree elevation:

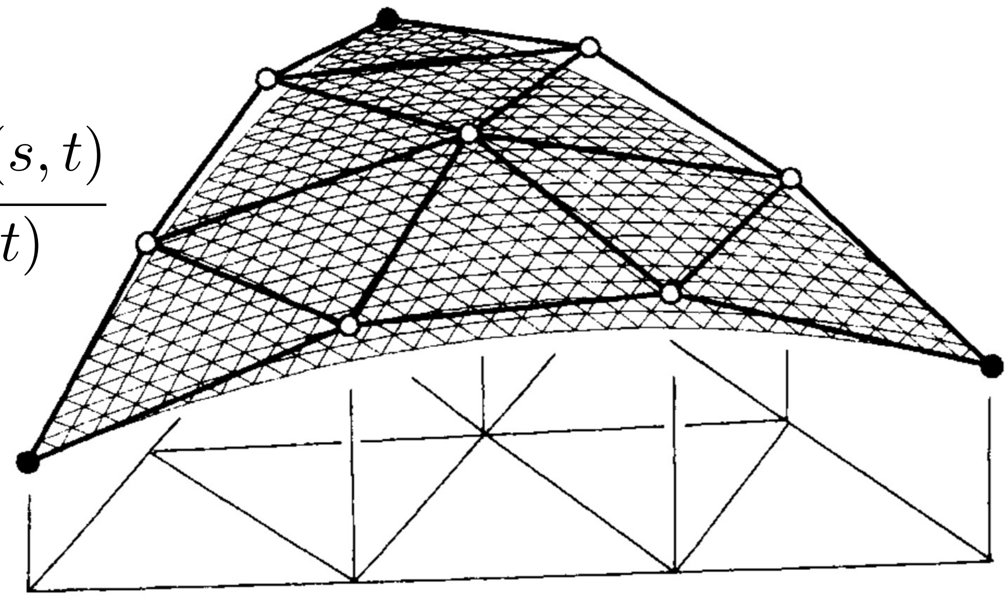
$$\sum_{i+j+k=n} p_{i,j,k} B_{i,j,k}(s,t) = \sum_{i+j+k=n+1} p_{i,j,k}^{(1)} B_{i,j,k}(s,t)$$

$$p_{i,j,k}^{(1)} = \frac{1}{n+1}(ip_{i-1,j,k} + jp_{i,j-1,k} + kp_{i,j,k-1})$$

- Rational Bézier triangles:

$$\frac{\sum_{i+j+k=n} w_{i,j,k} p_{i,j,k} B_{i,j,k}(s,t)}{\sum_{i+j+k=n} w_{i,j,k} B_{i,j,k}(s,t)}$$

- Control points of a graph:



Continuity between Bézier triangles

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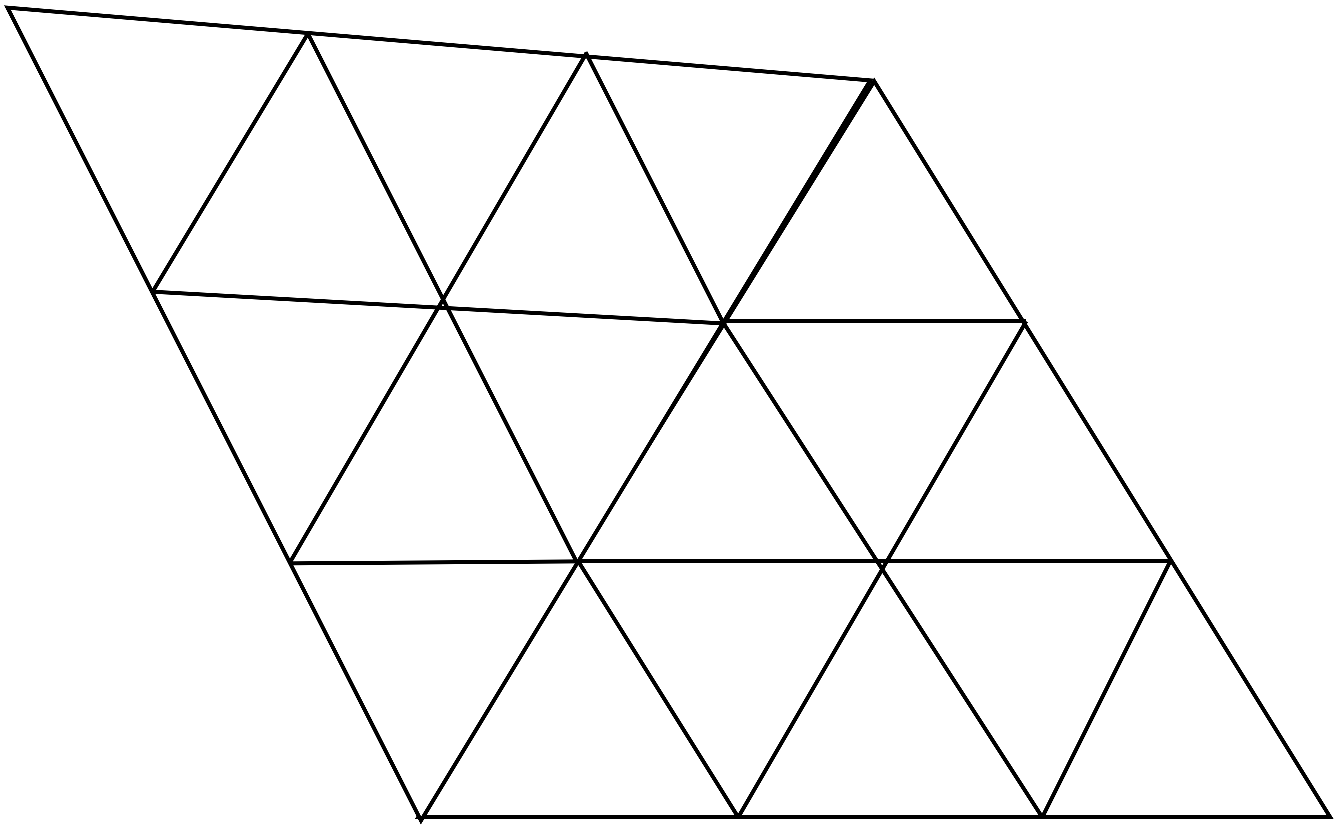

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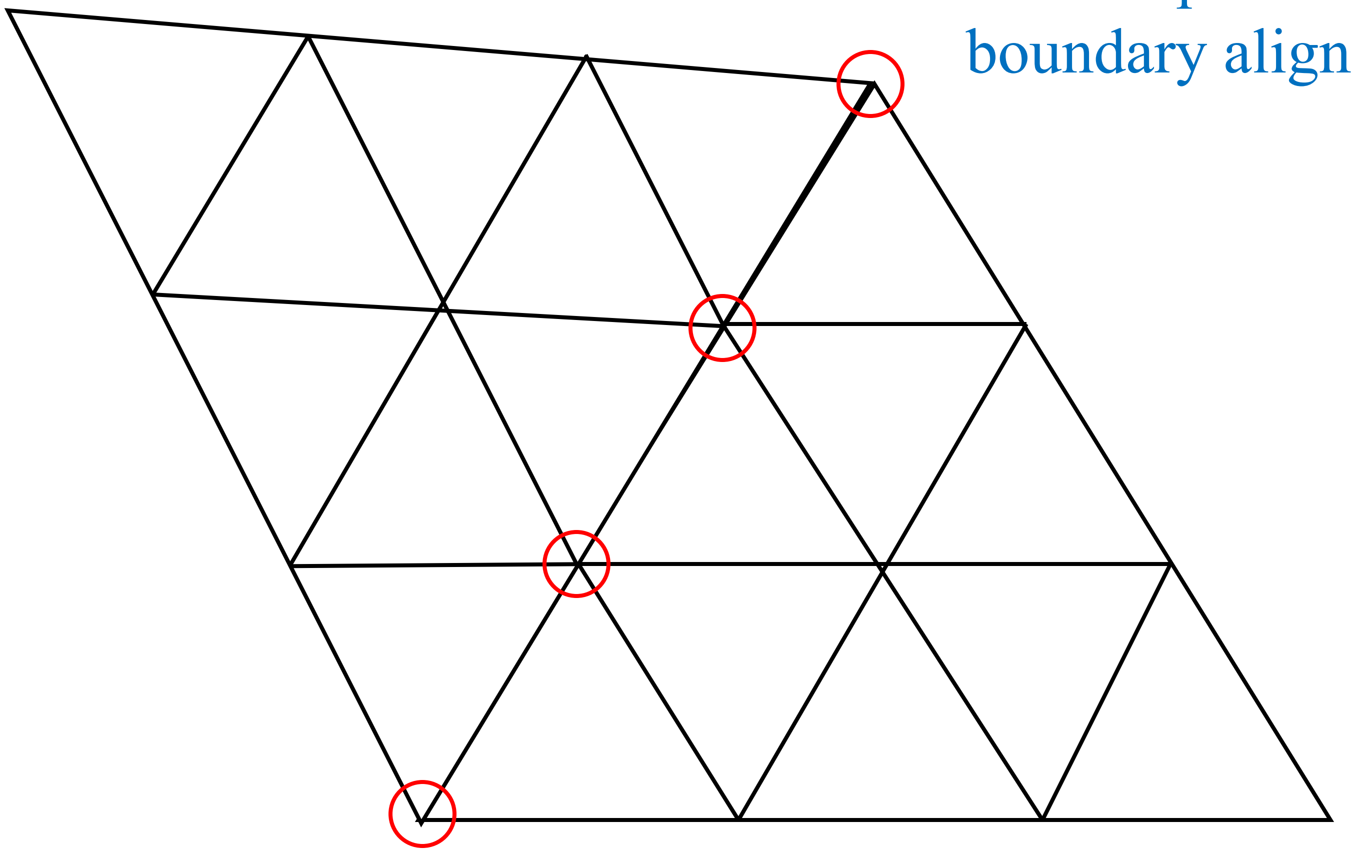
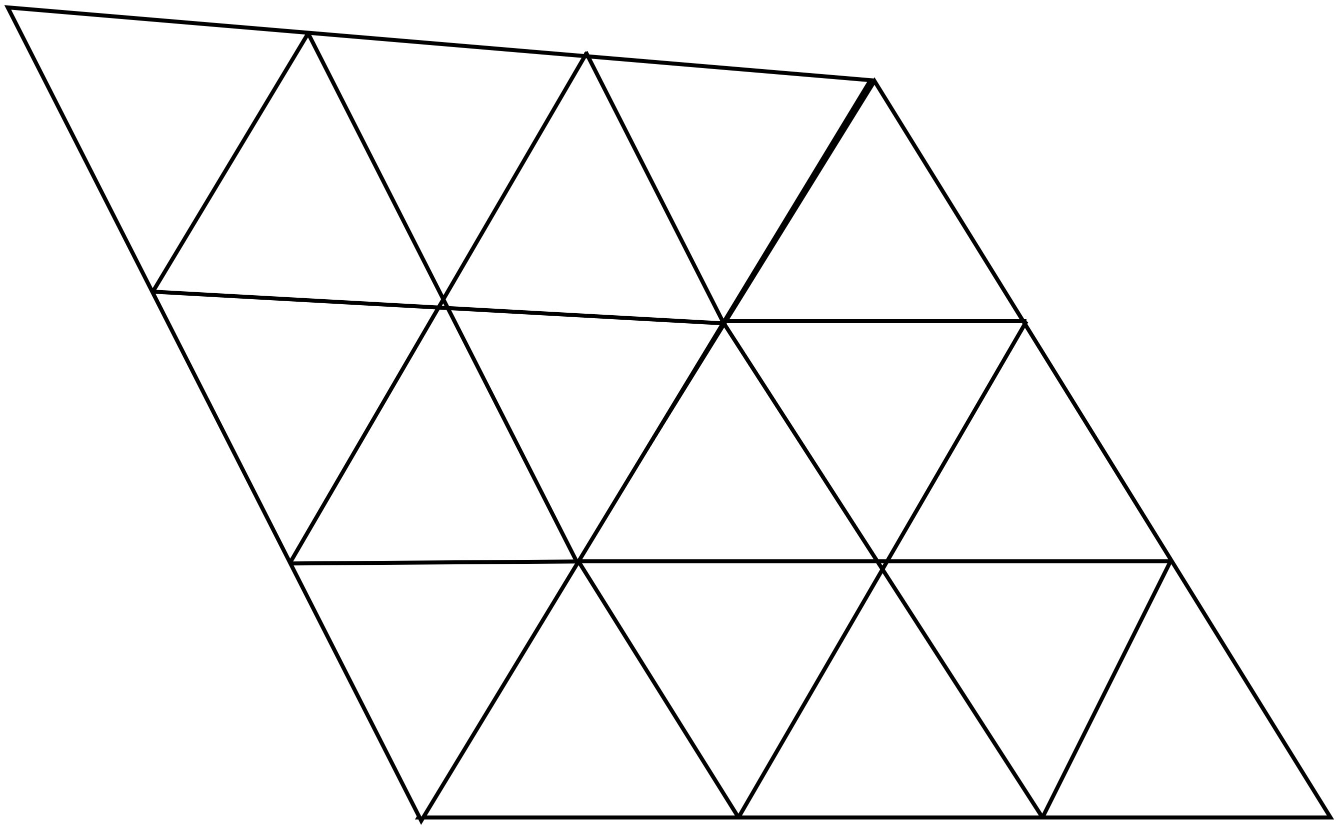


Control points on boundary align for C^0

Continuity between Bézier triangles

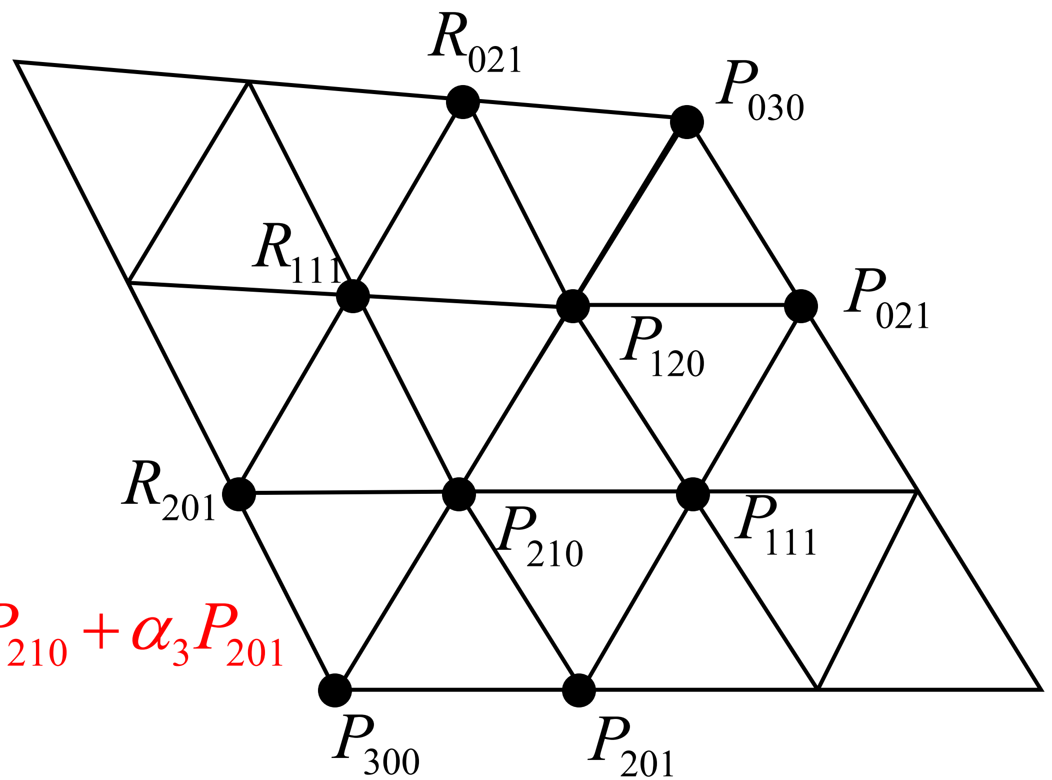
- How do we determine continuity conditions between Bézier triangles?

What about C^1?



Continuity between Bézier triangles

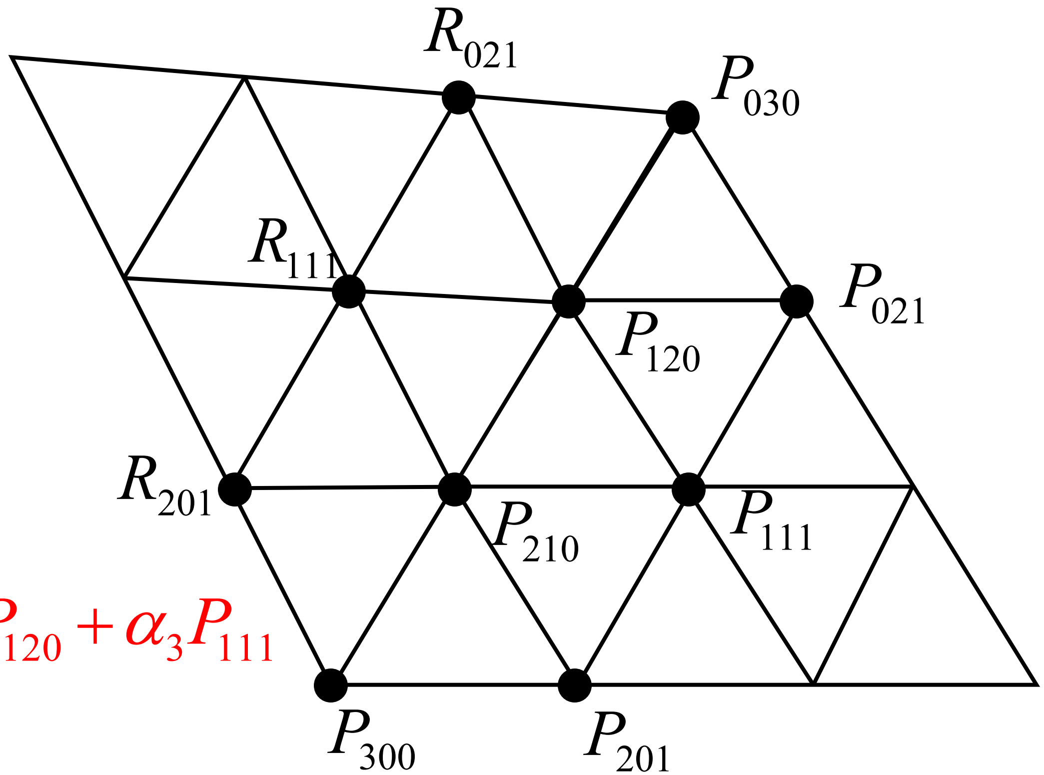
- C^1 continuity



$$R_{201} = \alpha_1 P_{300} + \alpha_2 P_{210} + \alpha_3 P_{201}$$

Continuity between Bézier triangles

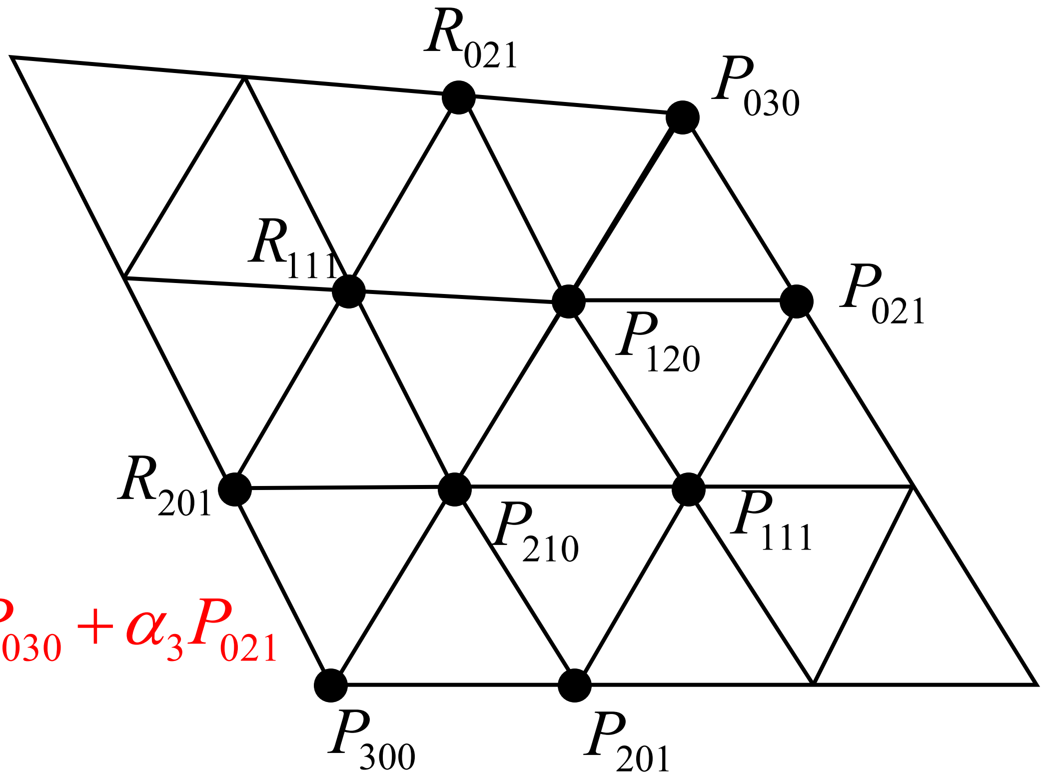
- C^1 continuity



$$R_{111} = \alpha_1 P_{210} + \alpha_2 P_{120} + \alpha_3 P_{111}$$

Continuity between Bézier triangles

- C^1 continuity



$$R_{021} = \alpha_1 P_{120} + \alpha_2 P_{030} + \alpha_3 P_{021}$$

Exercice

On considère une surface triangulaire de Bézier de degré 2

$$P(s,t) = \sum_{i+j+k=2} b_{i,j,k} B_{i,j,k}(s,t)$$

définie par le réseau de points de contrôle :

$$b_{2,0,0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, b_{0,2,0} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, b_{0,0,2} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix},$$

$$b_{1,1,0} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, b_{0,1,1} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, b_{1,0,1} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

1. Esquisser une ébauche de cette surface et de son réseau de contrôle.
2. Quelle est la nature des courbes au bord de cette surface ?
3. Calculer le point de cette surface correspondant aux paramètres $(s,t) = (1/3, 1/3)$ à l'aide de l'algorithme de De Casteljau. Expliquer.