

# MATLAB

# Fundamentals

Heidelberg-EMBL  
12-15/09/2016

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(EMBL Centre for Biological Modelling)

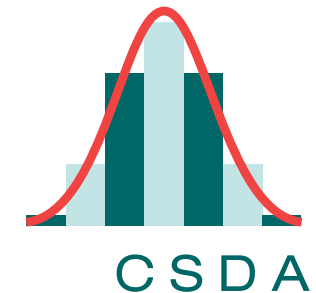
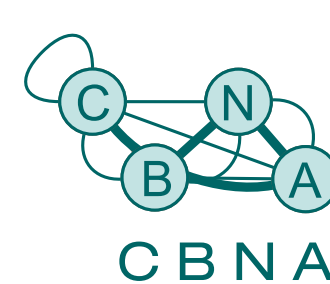
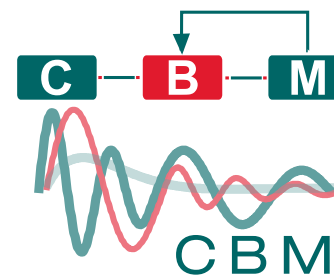
## Bio-IT



Bio-IT

## The EMBL Computational Centres

Modelling Networks Statistics



EMBL-HD's community for scientific computer users

- \* Courses and workshops
- \* Resources
- \* Networking and social events
- \* Web portal: <https://bio-it.embl.de>

Interdisciplinary research and support for **ALL** of EMBL.

- \* Consulting: advice or hands-on help
- \* Collaboration
- \* Training
- \* Online resources



# Introductions

Introduce your neighbour!

Find out:

1. Their name.
2. Something they think they are good at/enjoy doing.
3. If they were a component of a cell, what would they be and why.
4. How they think MATLAB will/has help their work.

# Aim of the course

1. MATLAB language
2. Basics of programming
3. Import and visually analyse data
4. More practice:
  - Build, simulate and explore mathematical models of biological systems
  - Image processing.

# Logistics

- Morning open session: 10 am -12 noon
- Afternoon practical sessions: 1 m - 5/6 pm
  - is ok if you need to leave!
- Informal course, so ask questions throughout.
  - Color coded help: Green for everything is ok, Pink for need help
- Break at 3:30
- Any questions?

# Introductions and expectation

What is your name?

What do you do?

Why do you want to learn MATLAB?

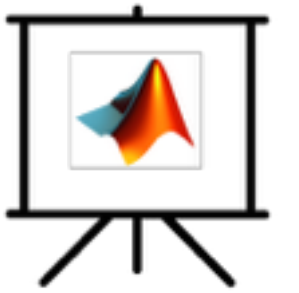
# Course content

- Get familiarised:
  - MATLAB syntax and functions
  - Basics of programming
- Learn to:
  - Import, visualise, explore and export data
  - Build, simulate and explore mathematical models of biological systems with MATLAB

# MATLAB

- MATLAB is a language for technical computing
- MATLAB (Matrix Laboratory) is based on matrix (**array**) operations
- It integrates computation, visualisation, and programming
- Interactive environment
- MATLAB is a numerical software





# MATLAB Windows, variables and output format



# MATLAB Windows, variables and output format

exercise 2.1 in pdf

# Arrays

MATLAB was originally written to ease dealing with tools of linear algebra – vectors and matrices.

**Array** - is a multi dimensional grid of data.

**Single number**: is a 1 x 1 array.

**Column vector**: a m x 1 array.

$$\mathbf{x} = \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} \begin{array}{c} \uparrow \\ m \\ \downarrow \end{array} \begin{array}{c} \leftarrow 1 \rightarrow \end{array}$$

**Row vector**: a 1 x n array.

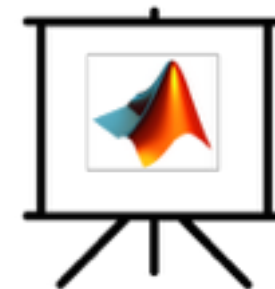
$$\mathbf{y} = (y_1 \ y_2 \ y_3 \ y_4 \ y_5) \begin{array}{c} \uparrow \\ 1 \\ \downarrow \end{array} \begin{array}{c} \leftarrow n \rightarrow \end{array}$$

**Matrix**: a m x n array.

$$\mathbf{A} = \begin{pmatrix} -3 & 0 & 2 & 3 & -3 \\ -5 & -1 & 3 & 0 & -3 \\ 2 & 3 & -5 & 2 & 2 \\ -1 & 0 & 2 & -1 & -2 \\ 4 & -3 & -1 & -2 & 0 \end{pmatrix} \begin{array}{c} \uparrow \\ m \\ \downarrow \end{array} \begin{array}{c} \leftarrow n \rightarrow \end{array}$$

**MATLAB stores data all in arrays**

So working with arrays is fundamental to working with MATLAB



# Arrays

4x4 array

column

row

|       |       |       |       |
|-------|-------|-------|-------|
| {1,1} | {1,2} | {1,3} | {1,4} |
| {2,1} | {2,2} | {2,3} | {2,4} |
| {3,1} | {3,2} | {3,3} | {3,4} |
| {4,1} | {4,2} | {4,3} | {4,4} |

3x3x2 array

A(2,3,2)

|   |   |   |
|---|---|---|
| 6 | 8 | 3 |
| 4 | 3 | 6 |
| 5 | 9 | 2 |

A(:, :, 1) =

|   |    |   |
|---|----|---|
| 1 | 0  | 3 |
| 4 | -1 | 2 |
| 8 | 2  | 1 |

A(:, :, 2) =

|   |   |   |
|---|---|---|
| 6 | 8 | 3 |
| 4 | 3 | 6 |
| 5 | 9 | 2 |

page

column

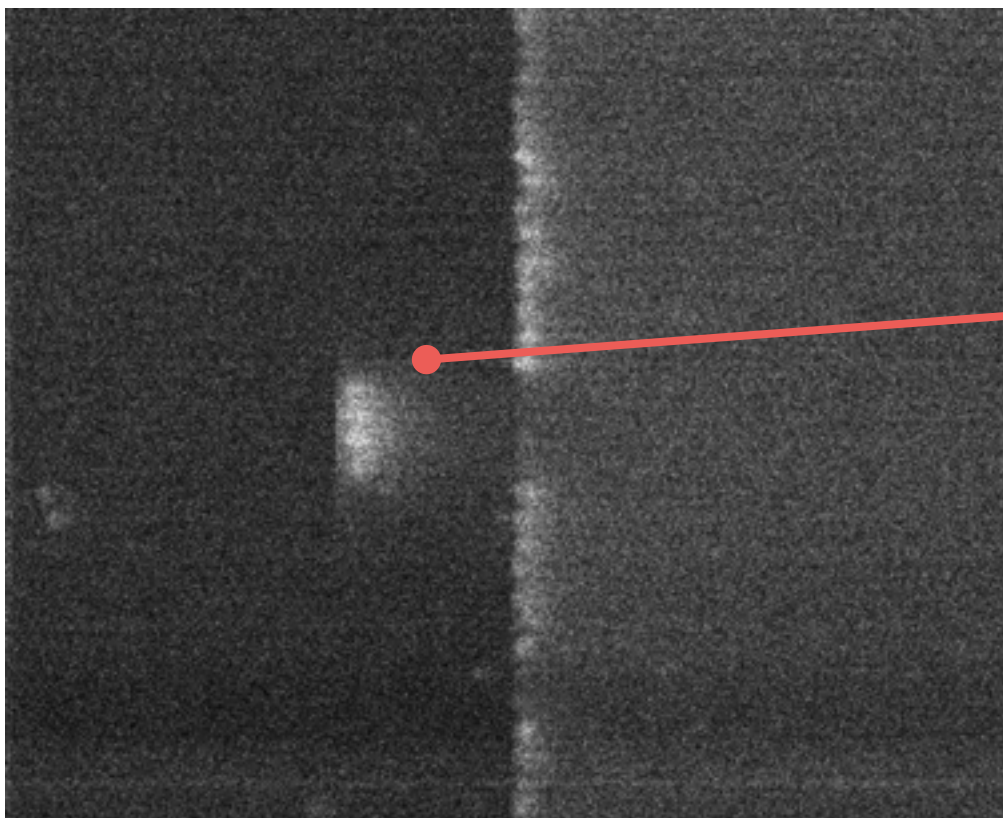
row

|         |         |         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| (1,1,1) | (1,2,1) | (1,3,1) | (1,4,1) | (1,1,2) | (1,2,2) | (1,3,2) | (1,4,2) | (1,1,3) | (1,2,3) | (1,3,3) | (1,4,3) |
| (2,1,1) | (2,2,1) | (2,3,1) | (2,4,1) | (2,1,2) | (2,2,2) | (2,3,2) | (2,4,2) | (2,1,3) | (2,2,3) | (2,3,3) | (2,4,3) |
| (3,1,1) | (3,2,1) | (3,3,1) | (3,4,1) | (3,1,2) | (3,2,2) | (3,3,2) | (3,4,2) | (3,1,3) | (3,2,3) | (3,3,3) | (3,4,3) |
| (4,1,1) | (4,2,1) | (4,3,1) | (4,4,1) | (4,1,2) | (4,2,2) | (4,3,2) | (4,4,2) | (4,1,3) | (4,2,3) | (4,3,3) | (4,4,3) |

# Arrays - Example 1

## Image storage

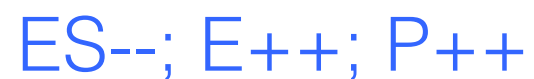
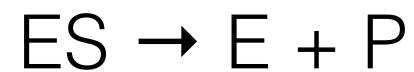
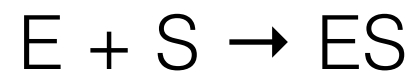
**Image:** set of data that is real-valued, ordered, represents color and intensity



|     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 73  | 61  | 55  | 27  | 137 | 112 | 121 | 197 | 239 |
| 9   | 25  | 131 | 55  | 124 | 147 | 173 | 133 | 135 |
| 32  | 42  | 86  | 76  | 144 | 68  | 143 | 94  | 178 |
| 26  | 76  | 57  | 78  | 91  | 87  | 51  | 176 | 148 |
| 42  | 60  | 95  | 90  | 95  | 36  | 150 | 158 | 122 |
| 38  | 26  | 84  | 65  | 51  | 49  | 106 | 66  | 119 |
| 32  | 48  | 78  | 28  | 24  | 19  | 94  | 127 | 62  |
| 61  | 128 | 88  | 92  | 55  | 99  | 110 | 126 | 127 |
| 55  | 70  | 57  | 63  | 59  | 101 | 118 | 90  | 88  |
| 54  | 26  | 38  | 31  | 67  | 78  | 31  | 127 | 107 |
| 67  | 57  | 81  | 70  | 83  | 142 | 143 | 99  | 88  |
| 103 | 48  | 78  | 119 | 61  | 55  | 120 | 139 | 201 |

# Arrays - Example 2

## Stoichiometry matrix

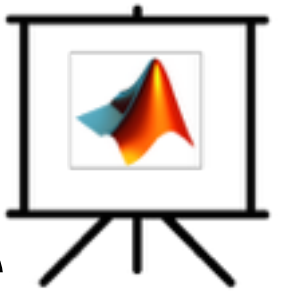


|            | E  | S  | ES | P |
|------------|----|----|----|---|
| E + S → ES | -1 | -1 | 1  | 0 |
| ES → E + S | 1  | 1  | -1 | 0 |
| ES → E + P | 1  | 0  | -1 | 1 |

# Arrays - Other examples

- Can you think of other examples?

# Operations on Arrays





# Help





# Arrays

exercise 2.2 in pdf

# Data types/classes





# Data types

exercise 2.3 in pdf

# In-built Functions

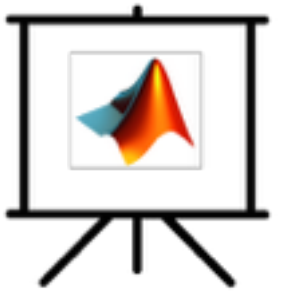


# In-built Functions



exercise 2.4 in pdf

# Scripts, logical operators and control of flow



# Relational/Logical Operator



|    |                          |
|----|--------------------------|
| <  | Less than                |
| <= | Less than or equal to    |
| >  | Greater than             |
| >= | Greater than or equal to |
| == | Equal to                 |
| ~= | Not equal to             |
| &  | Logical AND              |
|    | Logical OR               |





# Control flow

|                         | Description  | Language Synthanx   | Use for:  |
|-------------------------|--|---|---|
| <b>if, elseif, else</b> | Execute statements if condition is true                | if logical operator (true)<br>statements<br>elseif logical operator (true)<br>statements<br>else<br>statements<br>end | <ul style="list-style-type: none"><li>• Conditional assignment</li><li>• Compare arrays</li><li>• Test for equality</li><li>• Evaluate various conditions</li></ul> |
| <b>for</b>              | Execute statements a specified number of times         | for condition<br>statements<br>end  | <ul style="list-style-type: none"><li>• Assign matrix values</li><li>• Decrement values</li><li>• Execute statements for specificed values</li></ul>                |
| <b>while</b>            | Repeat execution of statements while condition is true | while logical operator (true)<br>statements<br>end  | <ul style="list-style-type: none"><li>• Repeat statements until expression is false</li></ul>   |

- Loops have always been slow in MATLAB, so avoid loops if possible.; Vectorisation still speeds things up.

# Scripts, logical operators and control of flow



exercises 2.5 and 4.3 in pdf

# Array operations vs loops



# Functions

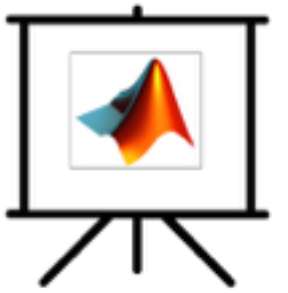




# Functions

exercise 4.1 in pdf

# Scripts vs Functions

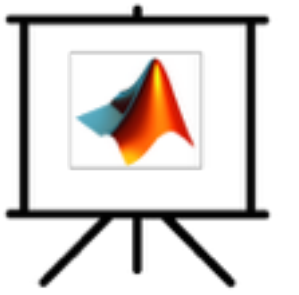


# Scripts vs Functions



exercise 4.2 in pdf

# Debugging tools





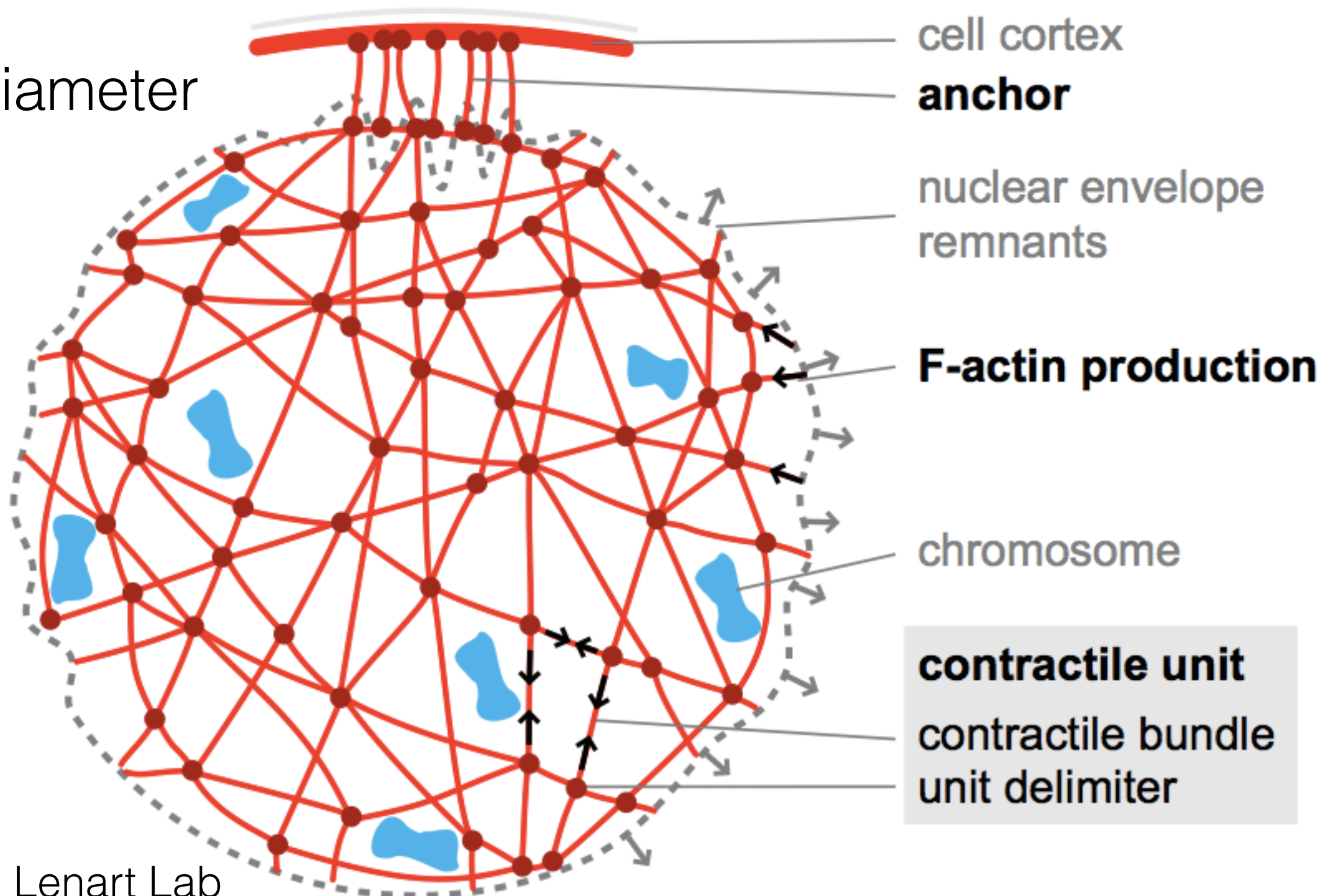


# Debugging

exercise 4.4 in pdf

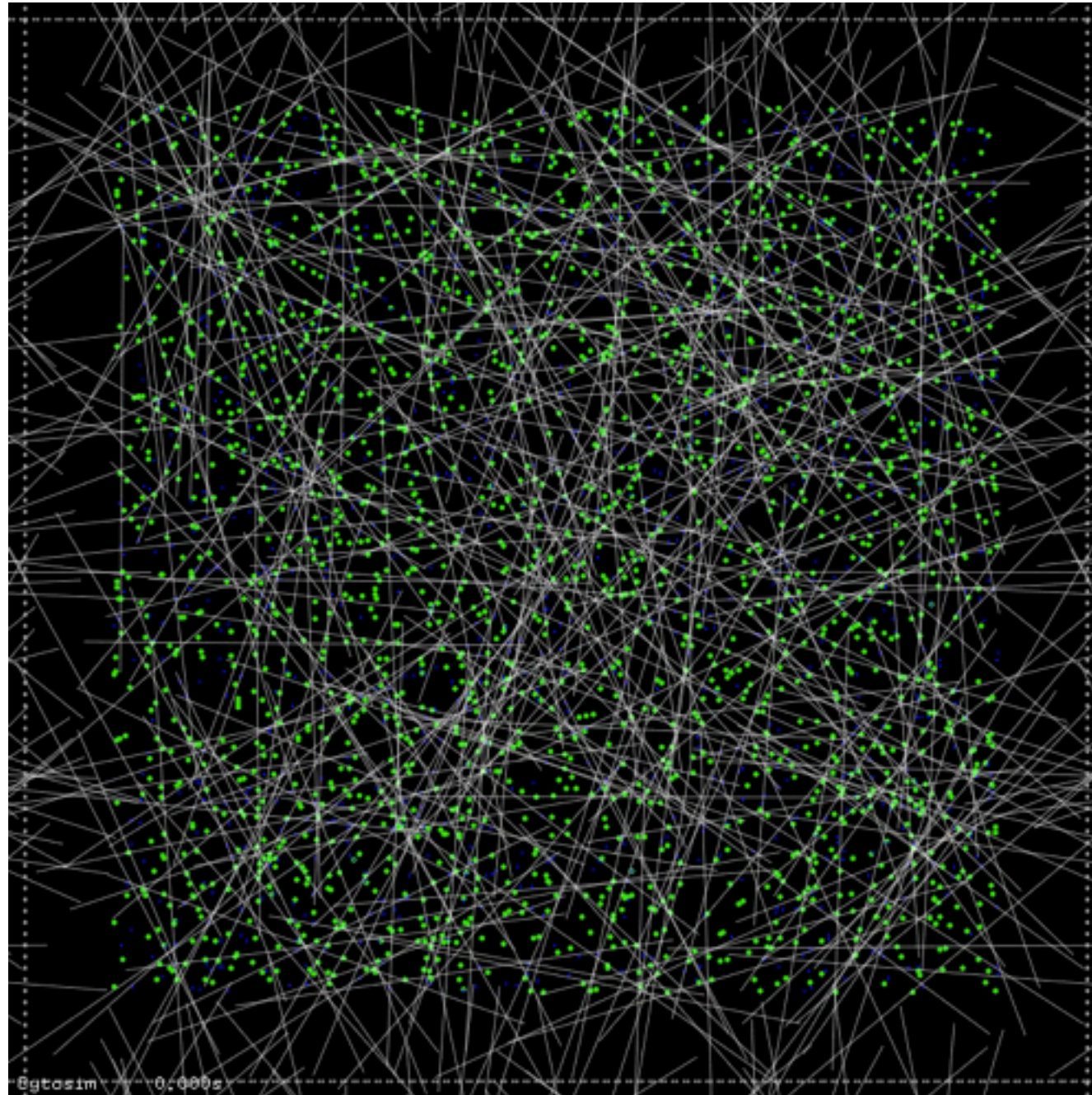
# Contractile actin network

80 $\mu$ m in diameter



# Contractile actin network

The  
nucleus  
is (0,0)



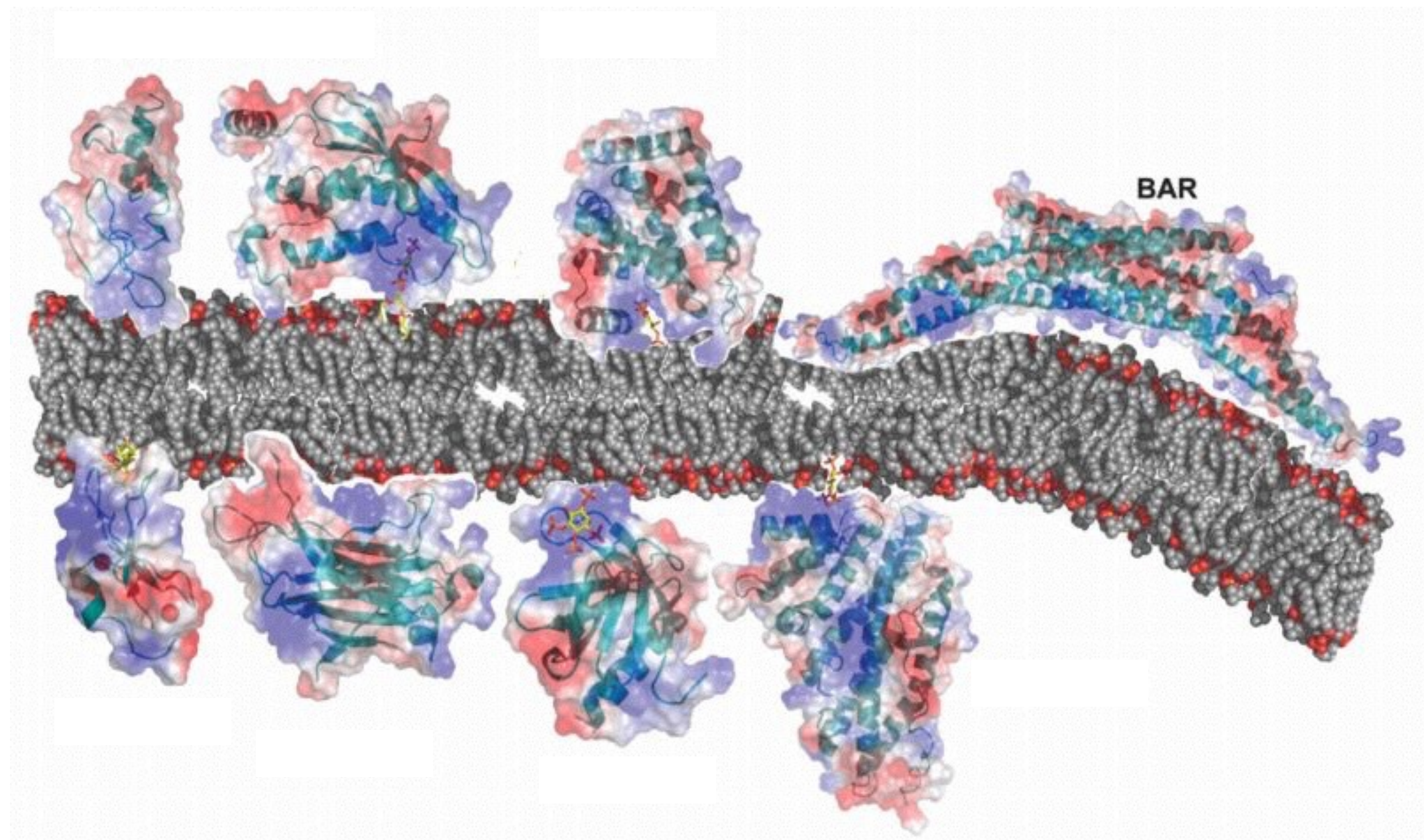
Q3. Is the  
network  
contracting?

Q4. Can you  
quantify the  
behaviour?

Cytosim



# Lipid-mediated recruitment of proteins to specific membranes



# Lipid-mediated recruitment of proteins to specific membranes

Measured membrane-binding properties of some of the most common phosphoinositide-binding targets

We want to answer two questions

Q1. Do we observe cooperativity between lipids?

Q2. Is cooperativity inhibitory or enhancing for protein recruitment?



Importing data  
interactively/programmatically

# Importing data

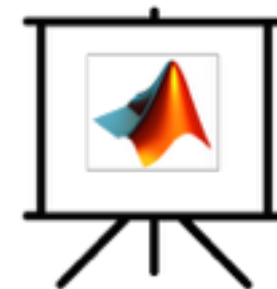


exercise 6.1 in pdf

Plotting interactively/  
programmatically







# Plotting

|                                       |  |
|---------------------------------------|--|
| plot(x, y)                            | Plots graph with x and y being vecotors with the coordinates in the x and y axes |
| xlabel('X')<br>ylabel('Y')            | names the axes   |
| title('Plot')                         | Names the figure   |
| xlabel('range')<br>ylabel('variable') | Names the axes   |
| legend('name1'...)                    | Graph legend for lines and patches   |
| hold on<br>hold off                   | Retains current graph in figure<br>Transcribes current graph in figure           |
| figure (1)                            | Creates figure 1   |
| saveas(h,<br>filename)                | Saves figure with handle h to file filename                                      |

| Color specifier | Colors  | Line specifier |                      | Marker specifier |           |
|-----------------|---------|----------------|----------------------|------------------|-----------|
| r               | Red     | '-'            | Solid line (default) | '+'              | Plus sign |
| g               | Green   | '--'           | Dashed line          | 'o'              | Circle    |
| b               | Blue    | '.'            | Dotted line          | '*'              | Asterisk  |
| c               | Cyan    | '-.'           | Dash-dot line        | '.'              | Point     |
| m               | Magenta |                |                      | 'x'              | Cross     |
| y               | Yellow  |                |                      | 'square' or 's'  | Square    |
| k               | Black   |                |                      | 'diamond' or 'd' | Diamond   |
| w               | White   |                |                      |                  |           |

# Plotting

## (interactively/programmatically)



exercise 6.2 in pdf

# More practice!

Mathematical descriptions of  
the time behaviour of a spatially  
homogenous chemical system

Image processing

Mathematical descriptions of  
the time behaviour of a spatially  
homogenous chemical system

# What is a mathematical model?

Wikipedia (April 17th 2013): “A mathematical model is a description of a **system** using **mathematical** concepts and language.”

## variables

$[x]$

Vmax

Kd

EC<sub>50</sub>

length

$t_{1/2}$

## relationships

$$K_d = \frac{[A] \cdot [B]}{[AB]}$$

$$d[X]/dt = k \cdot [Y]^2$$

$$\sum_i [X]_i - F(t) = 0$$

$$k(t) \sim N(k, \sigma^2)$$

If  $\text{mass}_t > \text{threshold}$   
then  $\text{mass}_{t+\Delta t} = 0.5 \cdot \text{mass}$

## constraints

$$[x] > 0$$

Energy conservation

Boundary conditions  
( $v < \text{upper limit}$ )

Objective functions  
(maximise ATP)

**Initial conditions**

Different types: Dynamical models, logical models, rule-based models, multi-agent models, statistical models, etc.

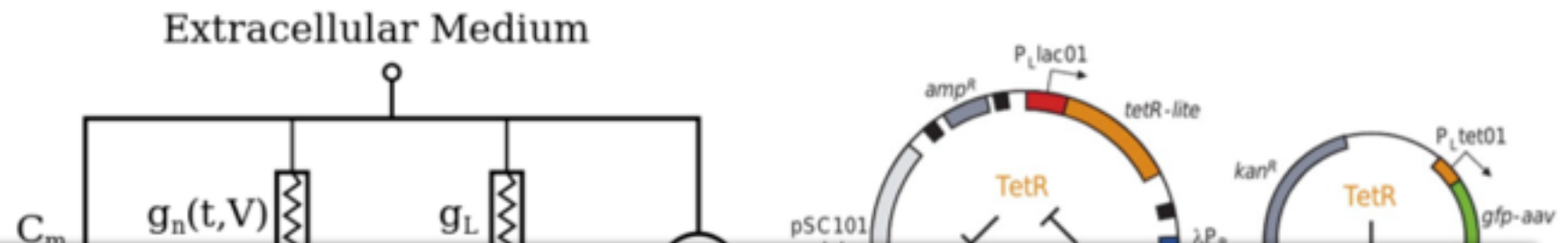
# Why using mathematical models?

**Describe**

**Explain**

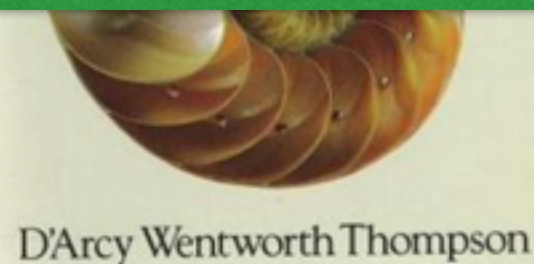
**Predict**

ON GROWTH  
AND FORM

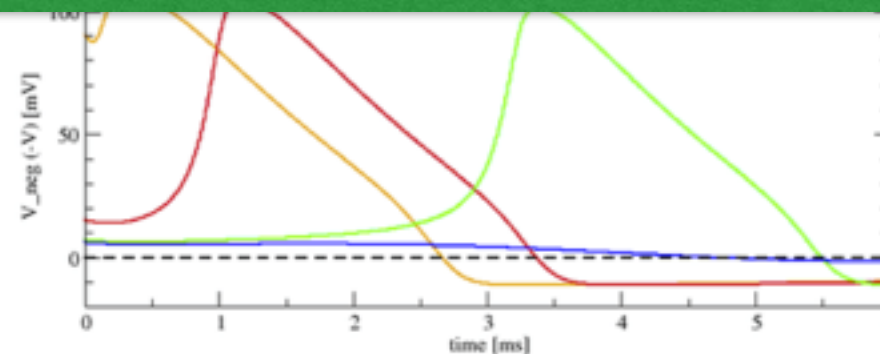


"Not unique! Several system structures may generate the same properties."

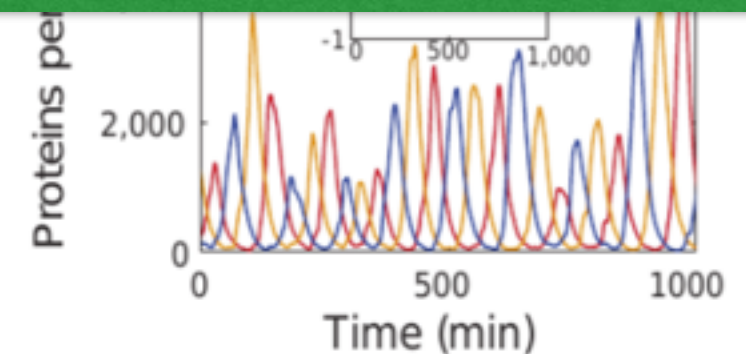
Michael Stumpf (ICL)



**1917**



**1952**



**2000**



# Modelling the spread of HIV in a society

## Necessary constituents?

- ✓ Susceptible - S
- ✓ Infected/infective - I
- ✓ Recovered/no longer susceptible - R

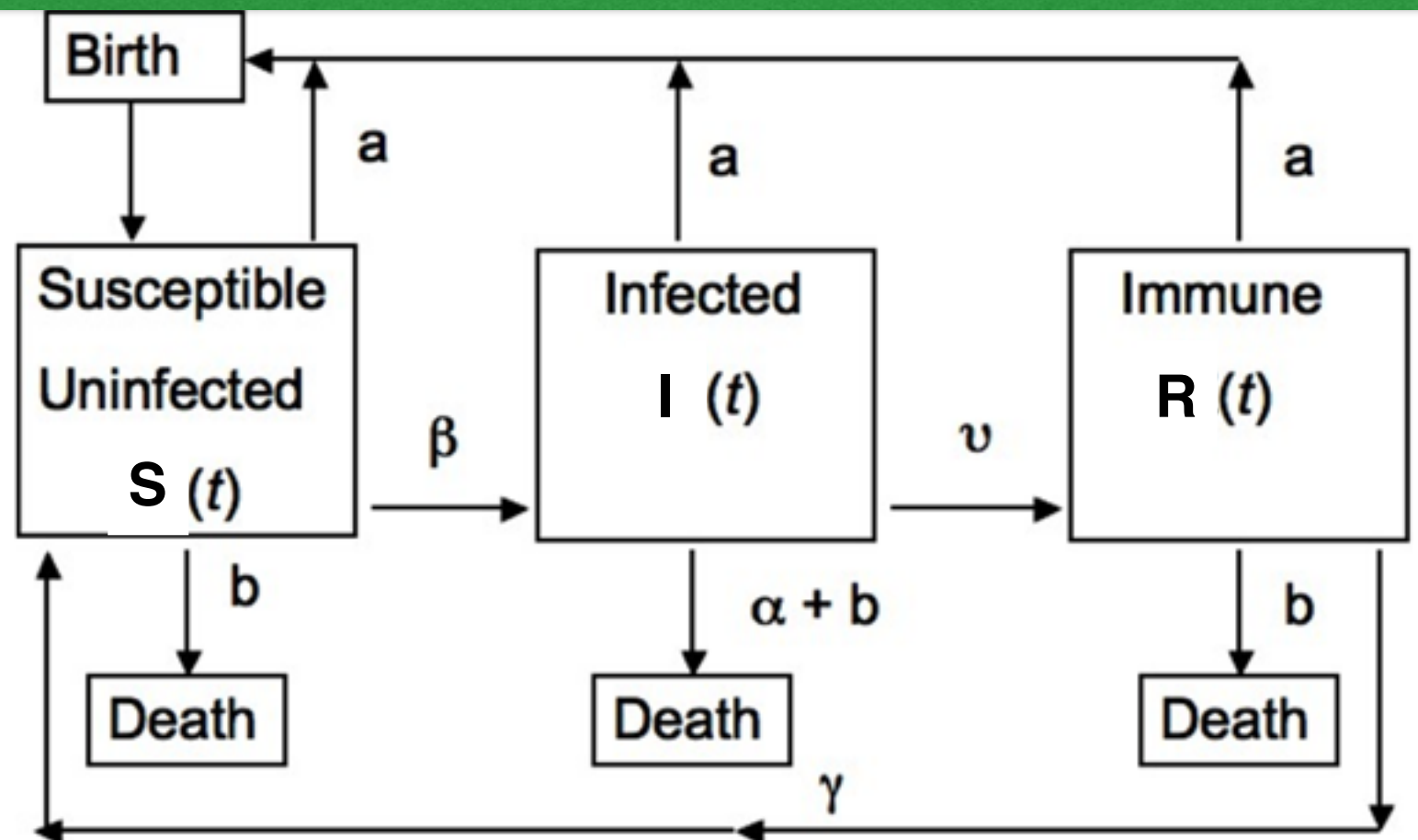
## Emerging properties



Naming variables carefully is very important!!

## Relationships

$$\begin{aligned}\frac{dS}{dt} &= -\beta \frac{SI}{N} \\ \frac{dI}{dt} &= \beta \frac{SI}{N} - \gamma I \\ \frac{dR}{dt} &= \gamma I \\ N &= S + I + R\end{aligned}$$



# Why using mathematical models?

Mathematical modelling can be very helpful for discovering and understanding biological processes and organisation principles, because:

- it forces the investigator to formulate hypotheses and insights in a clear-cut and formal way
- it may allow for the representation and evaluation of system compounds that are experimentally not accessible
- it allows to explore many scenarios or parameter values in less time and cheaper than in experiments
- it may help to extract structural dependencies or mathematical and physical relation that are hard to find by biological intuition, e.g. + and - loops



# **Deterministic** mathematical description of the time behaviour of a spatially homogenous chemical system

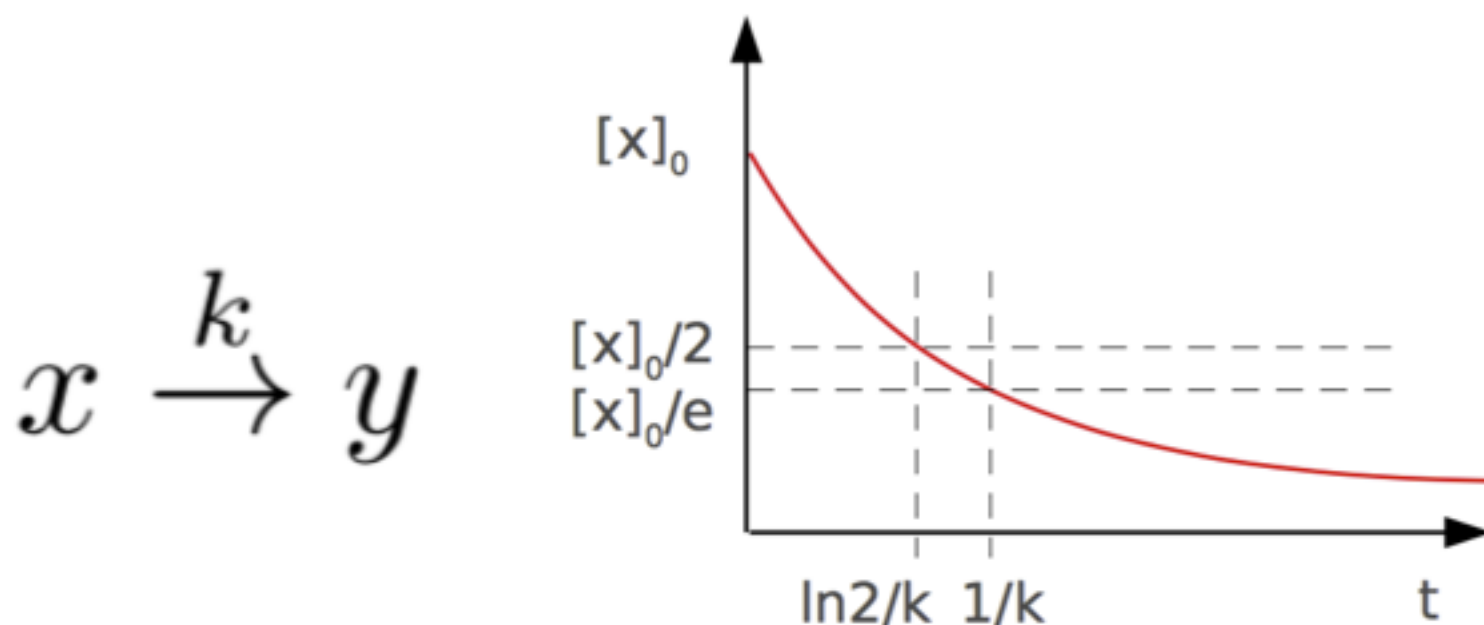
Assumptions:

1. chemical reactions are **continuous** rate processes
2. chemical reactions are a **predictable** process, based only on the initial conditions
  - ➔ **governed** by a set of coupled, ordinary differential equations (**ODEs**)

# Differential Equations

**A differential equation is a mathematical equation that relates some function with its derivatives.**

(In biology) the **functions** represent **physical quantities**, the **derivatives** represent their **rates of change**, and the **equation** defines a **relationship between the two**.



$$\frac{d[x]^{(t)}}{dt} = -1 \cdot k \cdot [x]^{(t)}$$

# Law of Mass Action

Waage and Guldberg (1864)



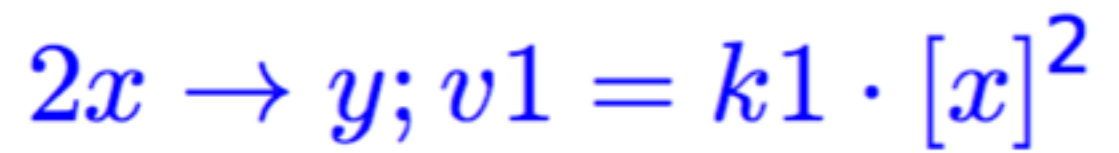
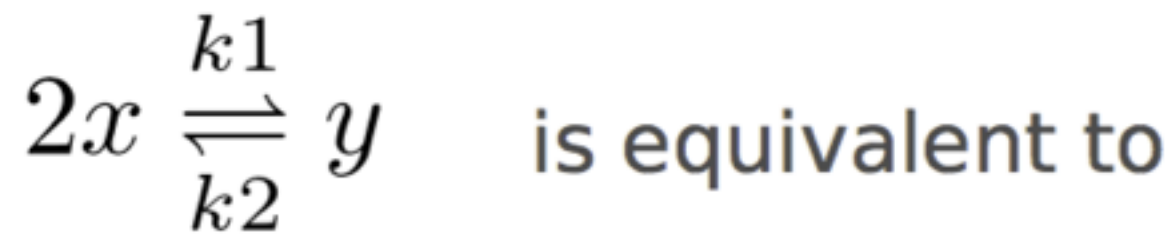
$$v = k \cdot \prod_i [X_i]^{n_i}$$

Diagram illustrating the Law of Mass Action equation:

- $v$ : velocity
- $k$ : rate-constant
- $\prod_i$ : activity
- $[X_i]$ : stoichiometry
- $n_i$ : stoichiometry

When the numbers of catalyst and substrate molecules are usually in the same order of magnitude, using mass action kinetics would make sense, since the reaction depends on those concentrations.

# Reversible reaction



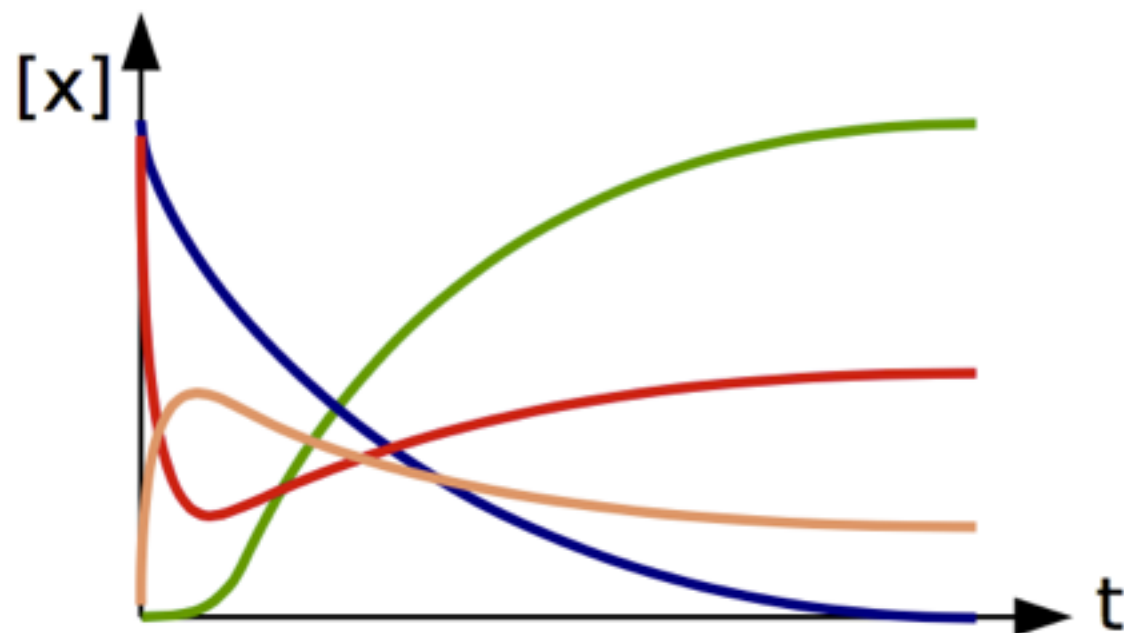
$$\frac{d[x]}{dt} =$$

$$\frac{d[y]}{dt} =$$

# Example of an enzymatic reaction



$$\begin{aligned} d[E]/dt &= -k_1[E][S] + k_2[ES] + k_3[ES] \\ d[S]/dt &= -k_1[E][S] + k_2[ES] \\ d[ES]/dt &= +k_1[E][S] - k_2[ES] - k_3[ES] \\ d[P]/dt &= +k_3[ES] \end{aligned}$$



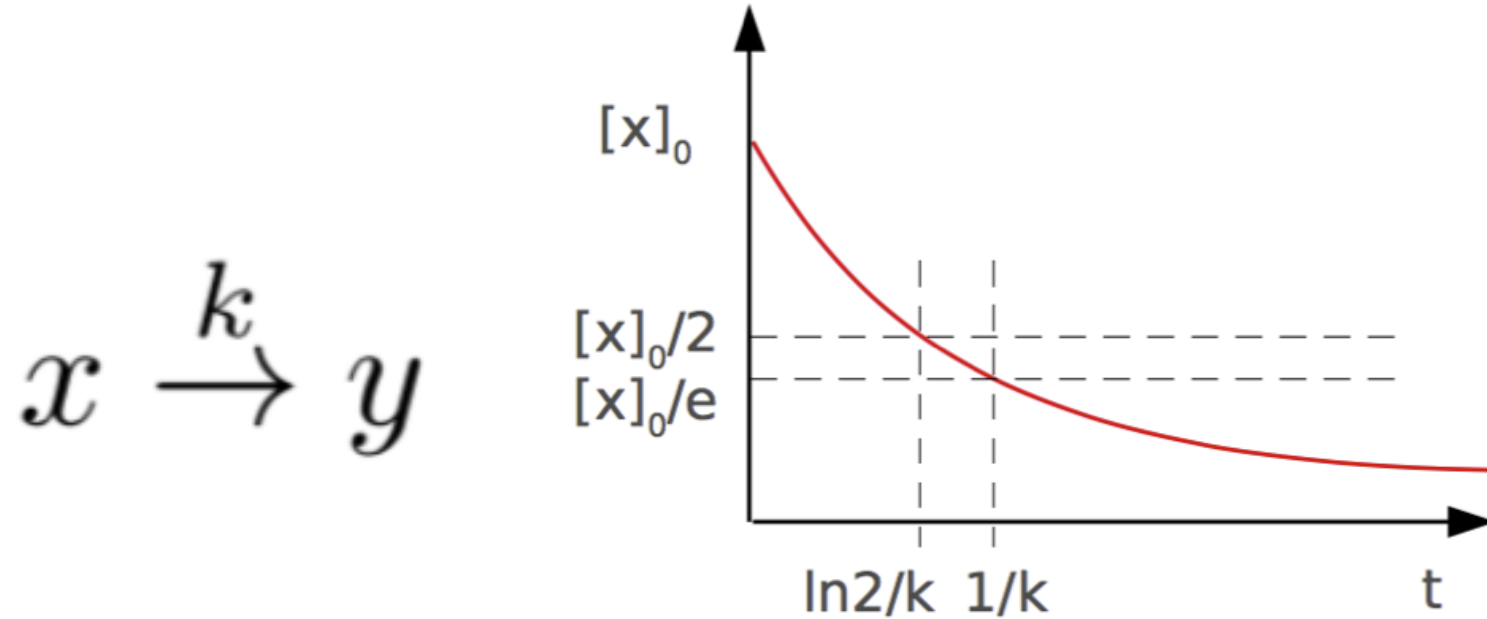
Not feasible in general



Numerical integration

(In biology) the **functions** represent **physical quantities**, the **derivatives** represent their **rates of change**, and the **equation** defines a **relationship between the two**.

Describe



Explain

$$\frac{d[x](t)}{dt} = -1 \cdot k \cdot [x](t)$$

Predict

$$x(t) =$$

?

Analytical (exact) solution

$$x(t) = [x]_0 \cdot e^{-kt}$$

Numerical approximation  
to solution

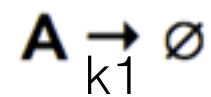
Others



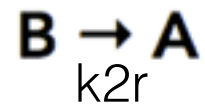
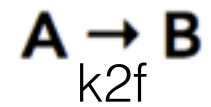
# Exercises



**Decay:**



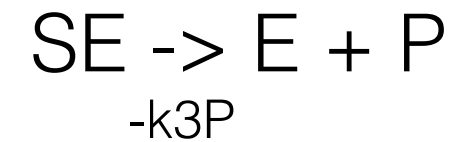
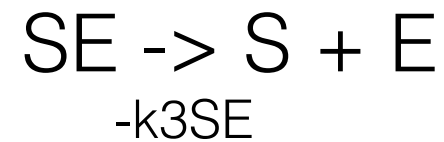
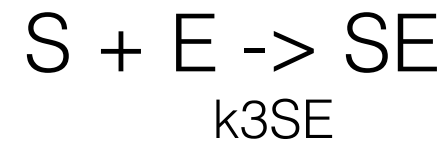
**Equilibrium:**



**Exercises:**

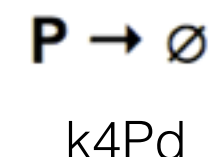
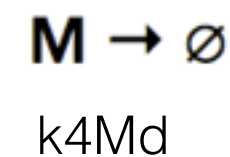
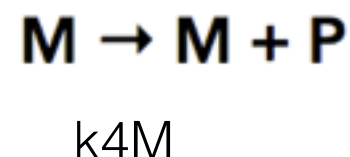
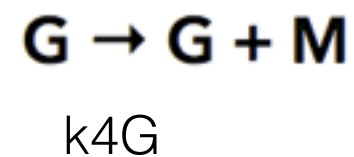
1. Write an ODE model of each of these systems
  - use mass action
2. Simulate the model in python

**Enzymatic reaction:**



**Gene expression (central dogma):**

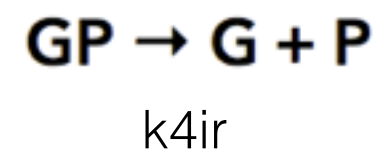
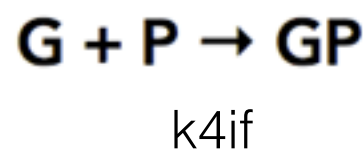
Reactions:



**Gene regulation:**

Add a feedback in which the Protein inhibits transcription by binding.

Additional reactions:



(GP is inactive)

**Challenges:**

Conceive an oscillatory system involving gene expression.

Conceive an oscillatory system involving the phosphorylation of proteins.

Develop a program which takes a list of reactions, and parameters as arguments.



# Image processing



exercise 8.2 in pdf

# Next steps

- Difference between decimal numbers and floating point numbers.
- **Algorithmic analysis:** How to define efficiency of an algorithm and why does this matter.
- **Performance tuning:** Useful Matlab tools for understanding your code: `mlint`, `tic/toc`, `profile on/off/report`.
- Learn about **numerical approaches for searching** (e.g. exhaustive enumeration, bisection search) and **numerical methods for approximations** to equations (e.g. Euler method, Newton method).
- Learn how to write **recursive programs** and why they might be useful. Use examples such as solving the problem of the Tower of Hanoi and calculating the Fibonacci numbers.

# Final comments

- For MATLAB, think \*ARRAYS\*
- Carefully name variables and files; choose names that already explain the program/variable
- Comment! (spend almost as much time as programming).
- Save, save save!
- More tips: <http://www.matlabtips.com>

# Final comments

- **Other apps:** <http://uk.mathworks.com/discovery/matlab-apps.html>
- **If you don't know how to do something:**
  - Find previous examples (Google) and edit them
  - See the MATLAB documentation and videos
  - See MATLAB examples  
<http://www.mathworks.com/examples/matlab>  
MATLAB Central
  - For more mathematical questions see/ask in [mathoverflow.net](http://mathoverflow.net)
- **Alternatives:**
  - SciLab
- **Feedback - Please fill in the online form!**