School of Physics & Astronomy

THE UNIVERSITY of EDINBURGH

Modelling and Visualisation in Physics

PHYS10035 (SCQF Level 10)

Wednesday 28$^{\text{th}}$ April, 2021 13:00 - 16:00
(May Diet)

Please read full instructions before commencing writing.

Examination Paper Information

Answer the **ONE** question in this paper

Special Instructions

- A sheet of physical constants is supplied for use in this examination.
- This is an open book examination.
- You may use books, notes, approved electronic calculators, and passive internet resources.
- Your answers must be entirely your own work.
- You **must not** seek the assistance of any other person, organisation, or service.
- You **must not** use responsive internet tools or software resources such as programmable calculators or computer algebra packages.

Special Items

- School supplied Constant Sheets

Chairman of Examiners: Prof J Dunlop
External Examiner: Prof I Ford

ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF THIS EXAMINATION.

A phase separating mixture of magnetic fluids is described by the following two coupled partial differential equations in 2 spatial dimensions,

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= M\nabla^2 \mu, \\ \frac{\partial m}{\partial t} &= D\nabla^2 m - \left[(c - \chi\phi)m + cm^3\right]. \end{aligned} \tag{1}$$

In Eqs. (1) ϕ represents the particle concentration and m the local magnetisation; both these quantities depend on space and time. The quantities M (mobility), D (magnetisation diffusion) and c (an inverse relaxation time) are all positive constants, and their values will be specified later on. The chemical potential μ is explicitly given by

$$\mu = -a\phi + a\phi^3 - \frac{\chi}{2}m^2 - \kappa\nabla^2\phi, \tag{2}$$

where a, χ and κ are three other positive constants whose value will be specified later.

a. Using an appropriate finite difference scheme, with periodic boundary conditions in space, write a Python code to solve Eqs. (1) on a 50×50 grid, subject to the initial condition that: (i) ϕ equals a constant, ϕ_0, with some noise – a random number between -0.01 and 0.01 at each grid point – and (ii) m equals 0 with some noise – again a random number between -0.01 and 0.01 at each grid point. Your code should allow you to display both ϕ and m in real time as it is running. [Alternatively, you could have an argument to decide which field is shown in a given simulation.] **[20]**

b. Here and in what follows, you should set $a = 0.1$, $c = 0.1$, $\kappa = 0.1$, $M = 0.1$, $D = 1$, whereas χ and ϕ_0 will take variable values specified below. You can use a spatial step $\Delta x = 1$; you need to find a small enough time step Δt for the algorithm to converge. By running your code and observing the dynamics qualitatively, briefly discuss the behaviour you see and record a snapshot (or draw a sketch) for ϕ and m for (i) $\chi = 0$, $\phi_0 = 0$; (ii) $\chi = 0$, $\phi_0 = 0.5$; (iii) $\chi = 0.3$, $\phi_0 = 0$; (iv) $\chi = 0.3$, $\phi_0 = 0.5$. **[10]**

c. For the cases (iii) and (iv) in b., compute the spatial averages of ϕ and m over the whole system, respectively $\langle\phi\rangle$ and $\langle m\rangle$, as a function of time. Briefly comment on the results. **[4]**

d. A chemical reaction between the fluids in the mixture can be modelled in a simple way by adding an extra term in the equation for ϕ, as follows,

$$\frac{\partial \phi}{\partial t} = M\nabla^2\mu - \alpha(\phi - \bar{\phi}), \tag{3}$$

where α and $\bar{\phi}$ are constant. The equation for m remains the same, and the form of μ is still given by Eq. (2). Modify your code to solve the modified pair of coupled equations for ϕ and m. Set $\bar{\phi} = \phi_0 = 0.5$, $\chi = 0.3$, and show snapshot/describe the patterns of ϕ and m for: (i) $\alpha = 0.0005$, (ii) $\alpha = 0.002$, (iii) $\alpha = 0.005$. **[6]**

e. To study pattern formation in this system more quantitatively, consider the equations in d., with the same parameters, except α which will be varied. Compute the spatial average of m, $\langle m\rangle$ and the variance of m, $\langle m^2\rangle - \langle m\rangle^2$, over the system, for α between 0.0005 and 0.005, in steps of 0.0005. Plot the resulting curves, commenting on the results, and say why these curves are suggestive of a transition in the behaviour of the system. **[10]**