

Informatics 1

Functional Programming Lecture 4

More fun with recursion

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Part I

Counting

Counting

```
Prelude [1..3]
```

```
[1,2,3]
```

```
Prelude enumFromTo 1 3
```

```
[1,2,3]
```

`[m..n]` *stands for* `enumFromTo m n`

Recursion

```
enumFromTo :: Int -> Int -> [Int]
```

```
enumFromTo m n | m > n      = []
```

```
               | m <= n     = m : enumFromTo (m+1) n
```

How enumFromTo works (recursion)

```
enumFromTo :: Int -> Int -> [Int]
enumFromTo m n | m > n      = []
                | m <= n    = m : enumFromTo (m+1) n
```

```
enumFromTo 1 3
=
1 : enumFromTo 2 3
=
1 : (2 : enumFromTo 3 3)
=
1 : (2 : (3 : enumFromTo 4 3))
=
1 : (2 : (3 : []))
=
[1,2,3]
```

How enumFromTo works (recursion)

```
enumFromTo :: Int -> Int -> [Int]
enumFromTo m n | m > n      = []
                | m <= n    = m : enumFromTo (m+1) n
```

```
enumFromTo 1 0
=
[]
```

Factorial

```
Main*> factorial 3
```

Library functions

```
factorial :: Int -> Int  
factorial n = product [1..n]
```

Recursion

```
factorialRec :: Int -> Int  
factorialRec n = fact 1 n  
  where  
    fact :: Int -> Int -> Int  
    fact m n | m > n      = 1  
             | m <= n     = m * fact (m+1) n
```

How factorial works (recursion)

```
factorialRec :: Int -> Int
factorialRec n = fact 1 n
  where
    fact :: Int -> Int -> Int
    fact m n | m > n      = 1
              | m <= n    = m * fact (m+1) n
```

```
factorialRec 3
=
fact 1 3
=
1 * fact 2 3
=
1 * (2 * fact 3 3)
=
1 * (2 * (3 * fact 4 3))
=
1 * (2 * (3 * 1))
=
6
```

Counting forever!

```
Prelude [0..]  
[0,1,2,3,4,5,...  
Prelude enumFrom 0  
[0,1,2,3,4,5,...
```

[m..] *stands for* enumFrom m

Recursion

```
enumFrom :: Int -> [Int]  
enumFrom m = m : enumFrom (m+1)
```


How enumFrom works (recursion)

```
enumFrom :: Int -> [Int]
enumFrom m = m : enumFrom (m+1)
```

```
enumFrom 0
=
0 : enumFrom 1
=
0 : (1 : enumFrom 2)
=
0 : (1 : (2 : enumFrom 3))
=
...
=
[0,1,2,...    -- computation goes on forever!
```

Part II

Zip and search

Zip

```
zip :: [a] -> [b] -> [(a,b)]
zip [] ys           = []
zip xs []           = []
zip (x:xs) (y:ys)   = (x,y) : zip xs ys
```

```
zip [0,1,2] "abc"
=
(0,'a') : zip [1,2] "bc"
=
(0,'a') : ((1,'b') : zip [2] "c")
=
(0,'a') : ((1,'b') : ((2,'c') : zip [] ""))
=
(0,'a') : ((1,'b') : ((2,'c') : []))
=
[(0,'a'), (1,'b'), (2,'c')]
```

Two alternative definitions of zip

Laid back

```
zip :: [a] -> [b] -> [(a,b)]
zip [] ys           = []
zip xs []           = []
zip (x:xs) (y:ys)   = (x,y) : zip xs ys
```

Uptight

```
zipHarsh :: [a] -> [b] -> [(a,b)]
zipHarsh [] []           = []
zipHarsh (x:xs) (y:ys)   = (x,y) : zipHarsh xs ys
```

Zip with lists of different lengths

```
Prelude> zip [0,1,2] "abc"  
[(0,'a'), (1,'b'), (2,'c')]
```

```
Prelude> zipHarsh [0,1,2] "abc"  
[(0,'a'), (1,'b'), (2,'c')]
```

```
Prelude> zip [0,1,2] "abcde"  
[(0,'a'), (1,'b'), (2,'c')]
```

```
Prelude> zipHarsh [0,1,2] "abcde"  
[(0,'a'), (1,'b'), (2,'c')]*** Exception:  
Non-exhaustive patterns in function zipHarsh
```

```
Prelude> zip [0,1,2,3,4] "abc"  
[(0,'a'), (1,'b'), (2,'c')]
```

```
Prelude> zipHarsh [0,1,2,3,4] "abc"  
[(0,'a'), (1,'b'), (2,'c')]*** Exception:  
Non-exhaustive patterns in function zipHarsh
```

More fun with zip

```
Prelude> zip [0..] "word"  
[(0,'w'), (1,'o'), (2,'r'), (3,'d')]
```

```
Prelude> let pairs xs = zip xs (tail xs)  
Prelude> pairs "word"  
[('w','o'), ('o','r'), ('r','d')]
```

Zip with an infinite list

```
zip :: [a] -> [b] -> [(a,b)]
zip [] ys           = []
zip xs []           = []
zip (x:xs) (y:ys)   = (x,y) : zip xs ys
```

```
zip [0..] "abc"
=
(0,'a') : zip [1..] "bc"
=
(0,'a') : ((1,'b') : zip [2..] "c")
=
(0,'a') : ((1,'b') : ((2,'c') : zip [3..] ""))
=
(0,'a') : ((1,'b') : ((2,'c') : zip (3 : [4..]) ""))
=
(0,'a') : ((1,'b') : ((2,'c') : []))
=
[(0,'a'), (1,'b'), (2,'c')]
```

Computer can determine $(3 : [4..]) \neq []$ without computing $[4..]$.

Dot product of two lists

Comprehensions and library functions

```
dot :: Num a => [a] -> [a] -> a
dot xs ys = sum [ x*y | (x,y) <- zipHarsh xs ys ]
```

Recursion

```
dotRec :: Num a => [a] -> [a] -> a
dotRec [] [] = 0
dotRec (x:xs) (y:ys) = x*y + dotRec xs ys
```


How dot product works (comprehension)

```
dot :: Num a => [a] -> [a] -> a
dot xs ys = sum [ x*y | (x,y) <- zip xs ys ]
```

```
dot [2,3,4] [5,6,7]
=
sum [ x*y | (x,y) <- zip [2,3,4] [5,6,7] ]
=
sum [ x*y | (x,y) <- [(2,5), (3,6), (4,7)] ]
=
sum [ 2*5, 3*6, 4*7 ]
=
sum [ 10, 18, 28 ]
=
56
```

How dot product works (recursion)

```
dotRec :: Num a => [a] -> [a] -> a
dotRec [] [] = 0
dotRec (x:xs) (y:ys) = x*y + dotRec xs ys
```

```
dotRec [2,3,4] [5,6,7]
=
dotRec (2:(3:(4:[]))) (5:(6:(7:[])))
=
2*5 + dotRec (3:(4:[])) (6:(7:[]))
=
2*5 + (3*6 + dotRec (4:[]) (7:[]))
=
2*5 + (3*6 + (4*7 + dotRec [] []))
=
2*5 + (3*6 + (4*7 + 0))
=
10 + (18 + (28 + 0))
=
56
```

Search

```
Main*> search "bookshop" 'o'
[1,2,6]
```

Comprehensions and library functions

```
search :: Eq a => [a] -> a -> [Int]
search xs y = [ i | (i,x) <- zip [0..] xs, x==y ]
```

Recursion

```
searchRec :: Eq a => [a] -> a -> [Int]
searchRec xs y = srch xs y 0
  where
    srch :: Eq a => [a] -> a -> Int -> [Int]
    srch [] y i = []
    srch (x:xs) y i
      | x == y = i : srch xs y (i+1)
      | otherwise = srch xs y (i+1)
```

How search works (comprehension)

```
search :: Eq a => [a] -> a -> [Int]
search xs y = [ i | (i,x) <- zip [0..] xs, x==y ]
```

```
search "book" 'o'
=
[ i | (i,x) <- zip [0..] "book", x=='o' ]
=
[ i | (i,x) <- [(0,'b'), (1,'o'), (2,'o'), (3,'k')], x=='o' ]
=
[0|'b'=='o'] ++ [1|'o'=='o'] ++ [2|'o'=='o'] ++ [3|'k'=='o']
=
[] ++ [1] ++ [2] ++ []
=
[1,2]
```

How search works (recursion)

```
searchRec xs y = srch xs y 0
```

```
  where
```

```
  srch [] y i = []
```

```
  srch (x:xs) y i | x == y = i : srch xs y (i+1)
```

```
                  | otherwise = srch xs y (i+1)
```

```
searchRec "book" 'o'
```

```
=
```

```
srch "book" 'o' 0
```

```
=
```

```
srch "ook" 'o' 1
```

```
=
```

```
1 : srch "ok" 'o' 2
```

```
=
```

```
1 : (2 : srch "k" 'o' 3)
```

```
=
```

```
1 : (2 : srch "" 'o' 4)
```

```
=
```

```
1 : (2 : [])
```

```
=
```

```
[1,2]
```