

Functions and Relations

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A **cartesian product** $A \times B$

is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$

Example $A = \{1, 2, 3\}$ and $B = \{b, c\}$

$$A \times B = \{(1, b), (1, c), (2, b), (2, c), (3, b), (3, c)\}$$

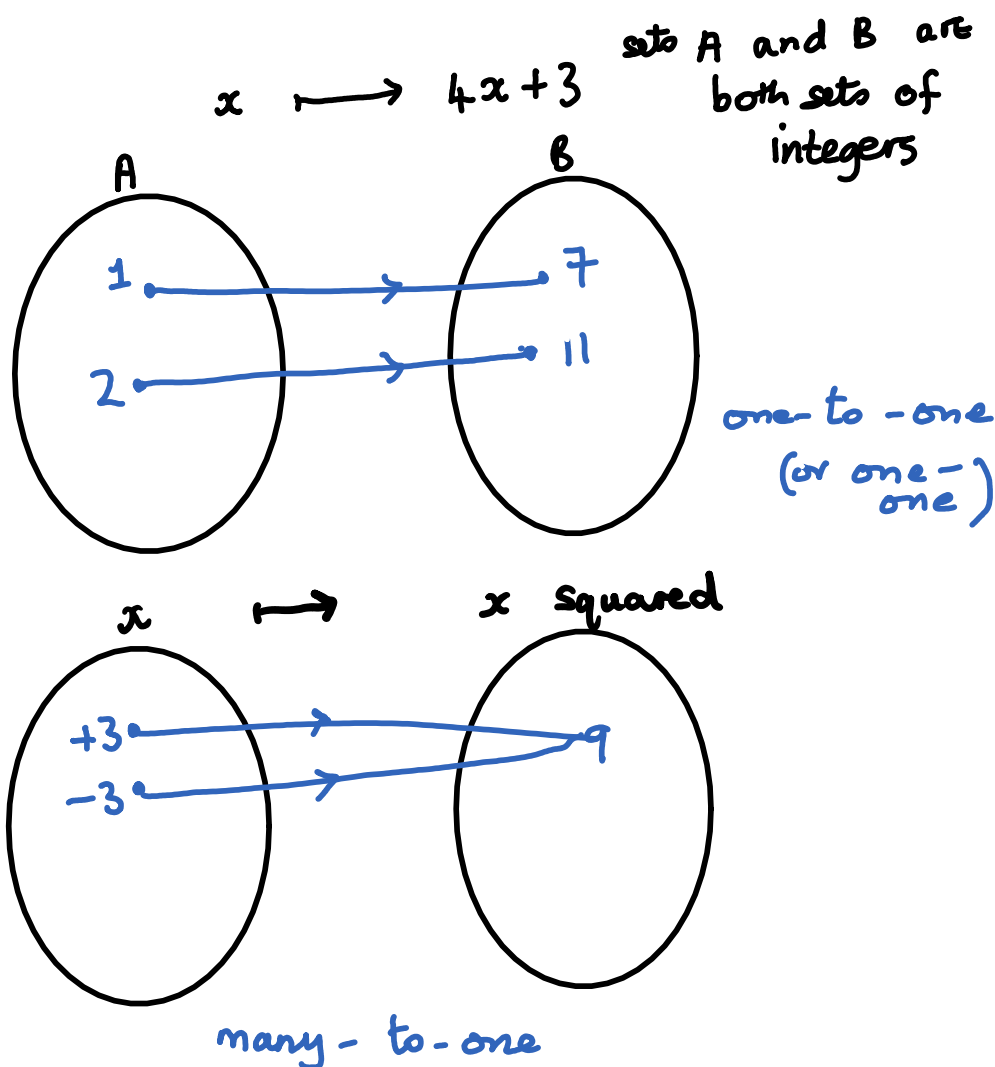
Note $(b, 1) \neq (1, b)$
so $A \times B \neq B \times A$

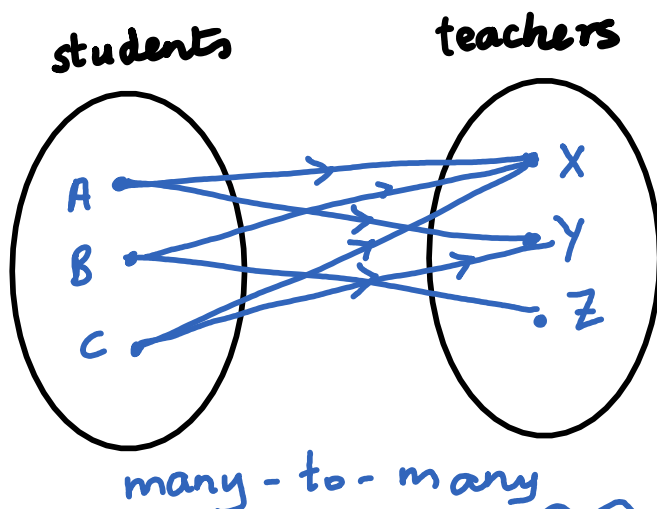
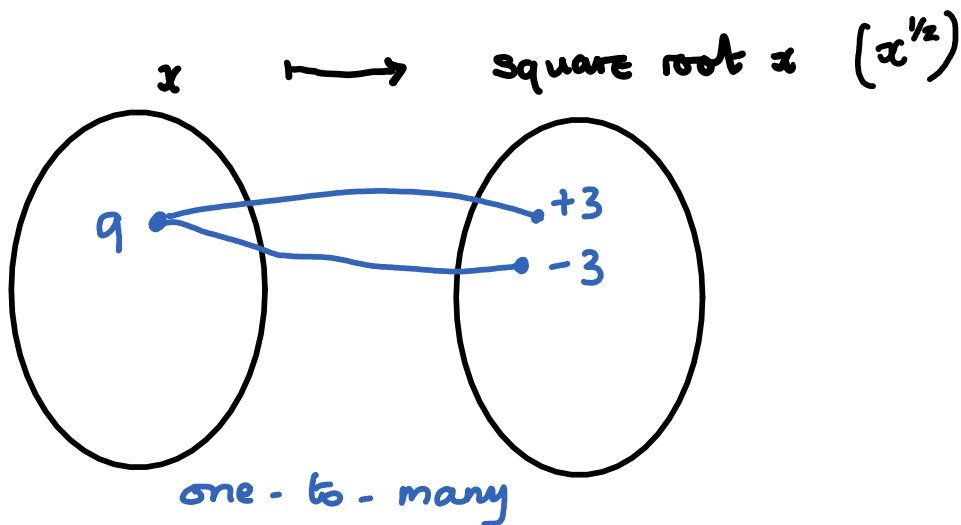
A binary relation links a member of a set A with a set B by some defined rule. We often use ordered 2-tuples to denote these. For example if $a \in A$ and $b \in B$

And a is related to b by a relation R then we can represent it by $(a, b) \in R$ or aRb .

$$R \subseteq A \times B$$

Different types of relations are possible a Type equation here. and we can explore these by using arrow diagrams (sometimes called mappings) and graphs.





Only one-to-one and many-to-one
are functions. Each input has exactly
one unique output

So $x \mapsto 4x+3$ is a function
and we can write $f(x) \mapsto 4x+3$ or
 $f: x \mapsto 4x+3$ or $f(x) = 4x+3$

$g(x) \mapsto x^2$ is also a function

Haskell $f\ x = x * x$ or
 $f\ x = x \wedge 2$ or square $x = x^2$

lambda expressions - anonymous functions

use \backslash for lambda
 $\backslash x \rightarrow (4 * x + 3)$