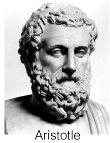
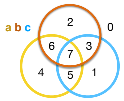
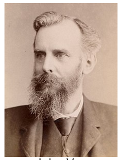
Aristotelian Syllogisms
Venn diagrams



Aristotle
384-322 BC

inf1a-cl 24/09/19
Michael Fourman





John Venn
1834–1923

It's all greek to me!

-λογῐ́ᾱ • (-logíā) f (genitive -λογῐ́ᾱς);

1. *Base for nouns denoting the study of something, or the branch of knowledge of a discipline.*

The suffix -ology is commonly used in the English language to denote a field of study.
Wikipedia gives hundreds of examples.

— here is a small selection of those starting with a

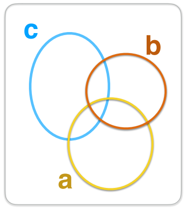
acarology
The study of mites and ticks.
acridology
The study of grasshoppers and locusts
aerolithology
The study of meteorites.
agathology
The science or theory of the good or goodness.
agrology
The science and art of agriculture.
anesthesiology
The study of anesthesia and anesthetics.
arachnology
The scientific study of spiders and related animals.
aristology
The art or study of cooking and dining.

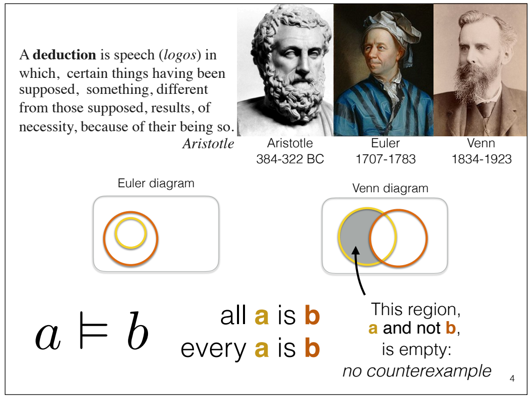
If we abstract away from the discipline to find universal laws of reasoning,
logic is what remains.

```
p :: U -> Bool
```

A predicate **p** is a function.
For each thing **x**, in our universe,
p x is Boolean value, **True** or **False**

Each predicate **p** defines a subset of the universe
{ x | p x }
we draw these sets, labelled with the name of the property it represents, to picture relations between predicates





Here we explain the meaning of an entailment between two predicates.

In English we can say “every a is b” “every man is mortal”

or “all a is b” “all apples are fruit”.

Beware of using “any” it behaves strangely.

Consider the question,

“Can you solve any of those problems?”

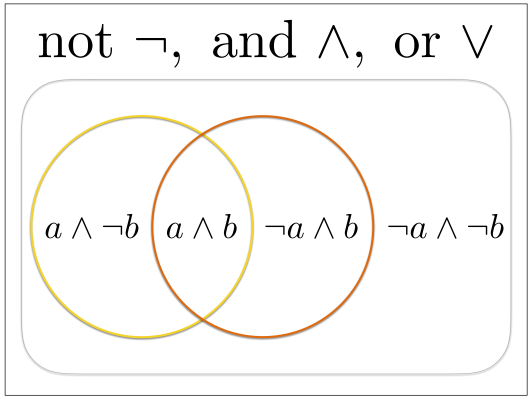
Are the following answers equivalent?

“Yes, I can solve the last one.”

“Yes, I can solve any of those problems.”

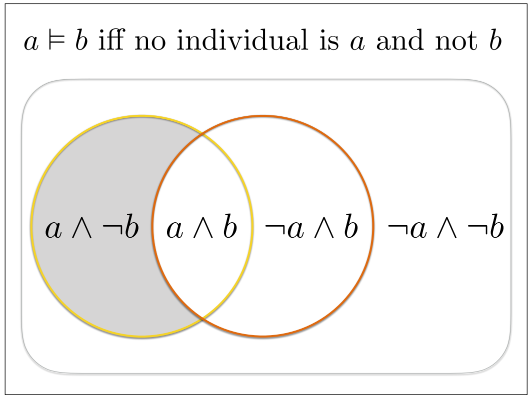
(Assume that there are several problems.)

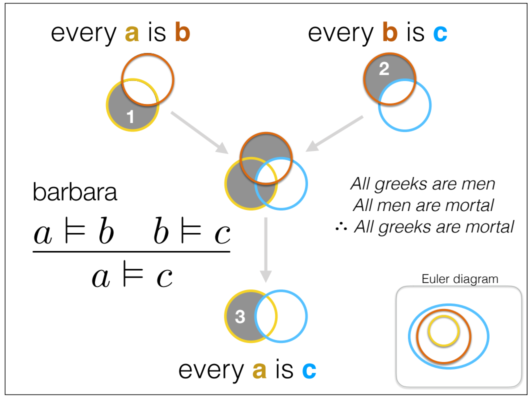
If I just answer, “Yes”, what does it mean?



This Venn diagram shows things in four regions: those things that satisfy both a and b; those that satisfy a but not b; those that satisfy b but not a and those that satisfy neither.

every a is b iff no a is not b

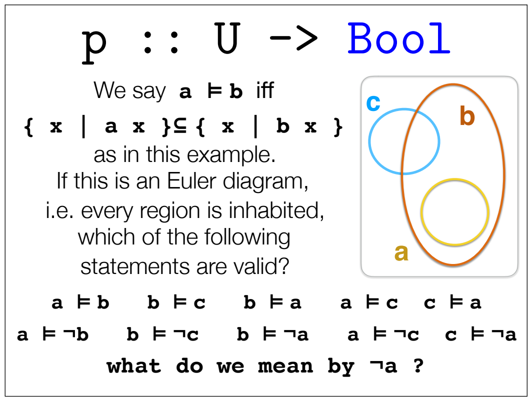




If region 1 is empty and region 2 is empty then region 3 is empty, since 3 is covered by 1 and 2 — if there were anything in 3 it would have to be in either 1 or 2.

So we have a *sound* rule of deduction.

This tells us how to combine two entailments to move an argument forward.

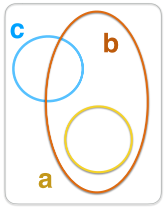


`p :: U -> Bool`

We say **a ⊨ b** iff

{ x | a x }⊆{ x | b x }

as in this example.
If this is an Euler diagram,
i.e. every region is inhabited,
which of the following
statements are valid?



a ⊨ b **b ⊭ c** **b ⊭ a** **a ⊭ c** **c ⊭ a**

a ⊭ ¬b **b ⊭ ¬c** **b ⊭ ¬a** **a ⊨ ¬c** **c ⊨ ¬a**

where, **(¬a) x = ¬(a x)**

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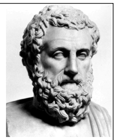
barbara $\dfrac{a \models b \quad b \models c}{a \models c}$

We say this rule is *sound:*
if a b c are *any predicates,* in *any universe,* and the *premises*, above the line, are valid then the *conclusion*, below the line, is *valid*.

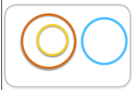
$$\frac{a \models b \quad b \models c}{a \models c}$$



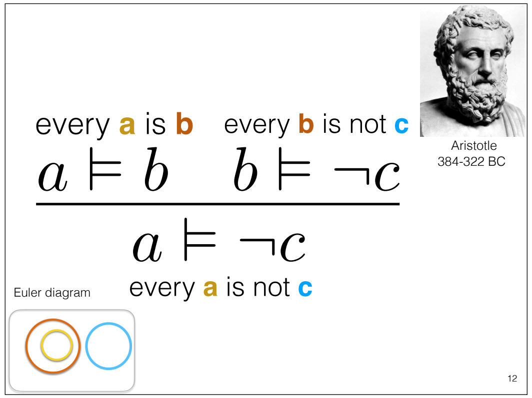
Aristotle
384-322 BC

since this works for *any* properties, a,b,c, it works for a, b, ¬c
the following rule must also be sound

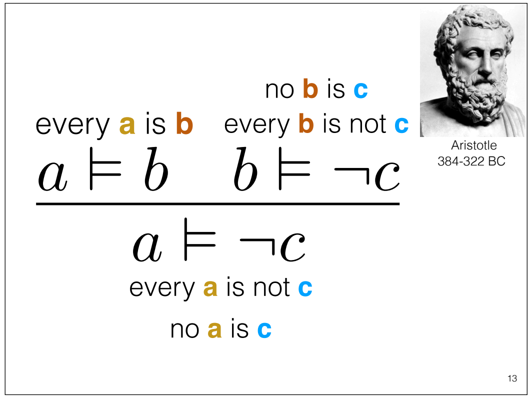
Euler diagram



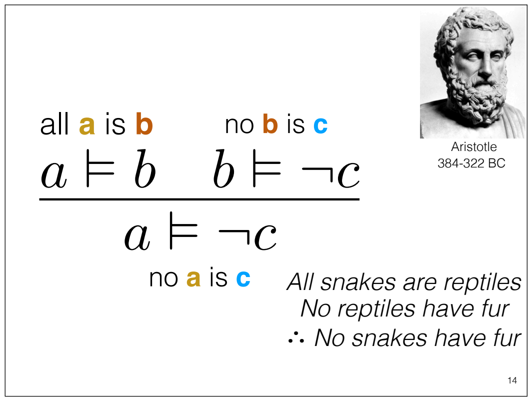
$$\frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

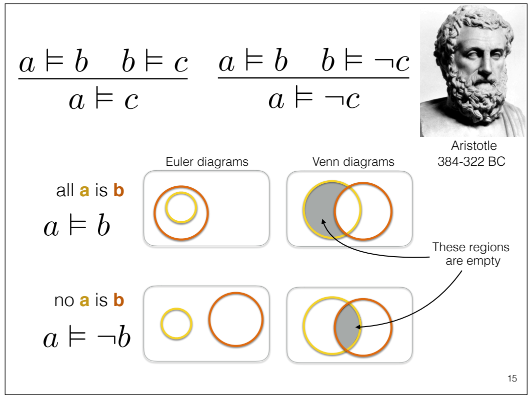


Now we put this back into words.

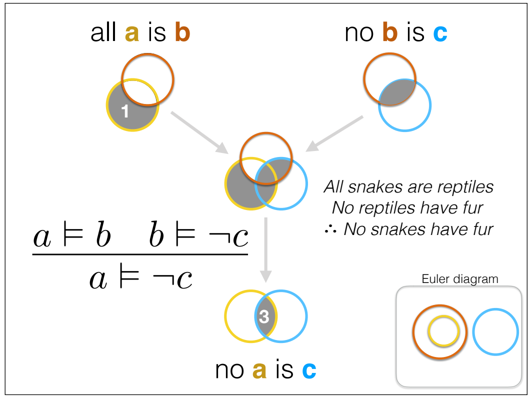


There is more than one way of saying things in english.

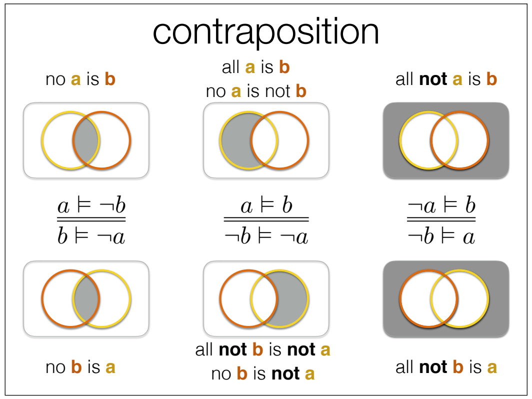




Now we look at this syllogism in Venn diagrams.



The argument looks bit different.



Looking at the diagrams we can see that several rules are valid for negation.

If we agree that ¬¬a = a , then these reduce to a single rule.

The first rule of boolean algebra

$$\neg\neg a = a$$

The second rule of boolean logic
the first is *barbara*

$$\frac{\dfrac{\dfrac{a \models b}{\neg b \models \neg a}}{\neg\neg a \models \neg\neg b}}{a \models b} \qquad \frac{a \models b}{\neg b \models \neg a} \qquad \frac{a \models b}{\neg b \models \neg a}$$

contraposition

Here we derive the 2-way rule from the single rule.

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barbara $\dfrac{a \models b \quad b \models c}{a \models c}$

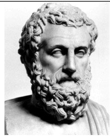
celarent $\dfrac{a \models b \quad b \models \neg c}{a \models \neg c}$

cesare $\dfrac{a \models b \quad c \models \neg b}{a \models \neg c}$

camestres $\dfrac{a \models b \quad c \models \neg b}{c \models \neg a}$

calemes $\dfrac{a \models b \quad b \models \neg c}{c \models \neg a}$

take contrapositive



Aristotle
384-322 BC

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barbara $\dfrac{a \models b \quad b \models c}{a \models c}$

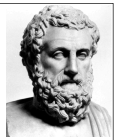
logic is just common sense in symbolic form

celarent $\dfrac{a \models b \quad b \models \neg c}{a \models \neg c}$

cesare $\dfrac{a \models b \quad c \models \neg b}{a \models \neg c}$

camestres $\dfrac{a \models b \quad c \models \neg b}{c \models \neg a}$

calemes $\dfrac{a \models b \quad b \models \neg c}{c \models \neg a}$



Aristotle
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More sound rules

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}\ \text{celarent}$$

all snakes are reptiles
no reptiles have fur
∴ no snakes have fur

all humans are mammals
no reptiles are mammals
∴ no humans are reptiles

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}\ \text{cesare}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}\ \text{camestres}$$

all humans are mammals
no reptiles are mammals
∴ no reptiles are humans

all humans are mammals
no mammals are reptiles
∴ no reptiles are humans

$$\frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a}\ \text{calemes}$$

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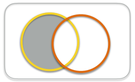
predicates
a b c

Venn diagram



entailment
a relation
between predicates
$a \vDash b$

valid rules

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

negation
$\neg\neg a = a$

$$\frac{a \vDash b}{\neg b \vDash \neg a}$$ contraposition

Syllogisms are not arguments — to make an argument we have to apply a syllogism to particular predicates in a particular universe. The arguments we used to derive sound syllogisms using contraposition are not arguments about the world they are meta-arguments, arguing about arguing.

In some ways this is like chess. Chess is not about a particular board — although most of us require a board to keep track of a game, there are people who keep the game in their head, and don't need a physical board. Each physical board can be used to play out the same game — winning is not a matter of moving the pieces, but rather of knowing how the prices should be moved. So cheating at darts or croquet is quite different from cheating at chess.

So, what is a rule — that might, or might not be sound?

more contraposition

When can you buy drinks in a shop?
In Scotland alcohol can be sold between the hours of 10am and 10pm.
In some other countries you can buy alcohol 24/7.
In others you can never buy alcohol (legally).
(For this discussion we assume you are of age to buy alcohol in Scotland.)

$$\frac{\text{In Scotland} \quad \text{Time is between 10am and 10pm}}{\text{Can legally buy alcohol.}}$$

$$\frac{\text{In Scotland} \quad \text{Cannot legally buy alcohol.}}{??}$$

$$\frac{\text{Time is between 10am and 10pm.} \quad \text{Cannot legally buy alcohol.}}{??}$$