Negating Expressions with Quantifiers

08 October 2019 23:45

"Every student in first year UG SoI takes Inf 1A"

let $P(x)$ be the statement "student $x$ takes Inf 1A"

then $\forall x \ P(x)$ with domain of students in first year UG SoI.

## Negating $\forall$

Negation: "It is not the case that every student in first year UG SoI takes Inf 1A"

$\equiv$ "There is (or are) a (some) student(s) in first year UG SoI who do(es) not take Inf 1A" $\exists x \neg P(x)$

So $\neg \forall x P(x) \equiv \exists x \neg P(x)$

## Negating $\exists$

"There is (exists) a student in first year UG SoI who takes Inf 1A"

$\exists x \ P(x)$

Negation "It is not the case that there is a student in first year UG SoI who takes Inf 1A"

$\equiv$ "Every student in first year UG SoI does not take Inf 1A" $\forall x \neg P(x)$

So $\neg \exists x \ P(x) \equiv \forall x \neg P(x)$

$$\neg \forall x P(x) \equiv \neg \left( P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n) \right)$$

domain with $n$ elements $x_1, x_2, \dots, x_n$

$$\equiv \neg P(x_1) \vee \neg P(x_2) \vee \dots \vee \neg P(x_n)$$

$$\equiv \exists x \neg P(x)$$

De Morgan's Law

Q: Show $\neg \forall x (P(x) \to Q(x))$ and $\exists x (P(x) \wedge \neg Q(x))$ are logically equivalent

$$\neg \forall x \left( P(x) \to Q(x) \right)$$

$$\equiv \exists x \left( \neg \left( P(x) \to Q(x) \right) \right)$$

$$\equiv \exists x \left( \neg \left( \neg P(x) \vee Q(x) \right) \right)$$

$$\equiv \exists x \left( P(x) \wedge \neg Q(x) \right)$$

remember $P(x) \to Q(x) \equiv \neg P(x) \vee Q(x)$