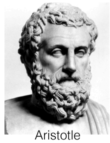
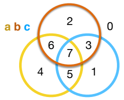
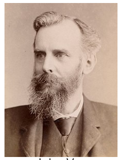
Aristotelian Syllogisms
Venn diagrams



Aristotle
384-322 BC

inf1a-cl 26/09/19
Michael Fourman





John Venn
1834–1923

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more common sense: more contraposition

When can you buy drinks in a shop?
In Scotland alcohol can be sold between the hours of 10am and 10pm.
In some other countries you can buy alcohol 24/7.
In others you can never buy alcohol (legally).
(For this discussion we assume you are of age to buy alcohol in Scotland.)

$$\frac{\text{In Scotland} \quad \text{Time is between 10am and 10pm}}{\text{Can legally buy alcohol.}}$$

$$\frac{\text{In Scotland} \quad \text{Cannot legally buy alcohol.}}{??}$$

$$\frac{\text{Time is between 10am and 10pm.} \quad \text{Cannot legally buy alcohol.}}{??}$$

Contraposition

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$$\frac{\text{Time is between 10am and 10pm.} \quad \text{Cannot legally buy alcohol.}}{\text{Not in Scotland.}}$$

Contraposition

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(For this discussion we assume you are of age to buy alcohol in Scotland.)

$$\frac{\text{In Scotland} \quad \text{Time is between 10am and 10pm}}{\text{Can legally buy alcohol.}}$$

$$\frac{\text{In Scotland} \quad \text{Cannot legally buy alcohol.}}{\text{Time is after 10pm and before 10am; be patient} \ldots}$$

$$\frac{\text{Time is between 10am and 10pm.} \quad \text{Cannot legally buy alcohol.}}{\text{Not in Scotland.}}$$

From each syllogism we get two new ones.

Contraposition

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}\ \text{barbara}$$

$$\frac{b \vDash c \quad a \nvDash c}{??}$$

What can we deduce in each case?

$$\frac{a \vDash b \quad a \nvDash c}{??}$$

What does this mean?

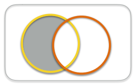
$$a \nvDash c$$

Contraposition

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}\ \text{barbara}$$

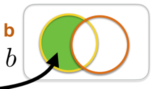
$$\frac{b \vDash c \quad a \nvDash c}{a \nvDash b}\ \text{baroco}$$

$a \vDash b$
every **a** is **b**



$$\frac{a \vDash b \quad a \nvDash c}{b \nvDash c}\ \text{bocardo}$$

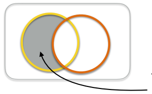
some **a** is not **b**
$a \nvDash b$



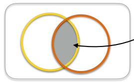
This region is *inhabited*

Venn diagrams

every **a** is **b**
$a \models b$

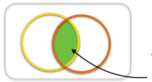


no **a** is **b**
$a \models \neg b$

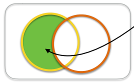


These regions are empty

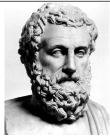
some **a** is **b**
$a \not\models \neg b$



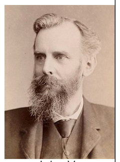
some **a** is not **b**
$a \not\models b$



These regions are inhabited

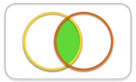
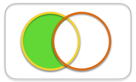
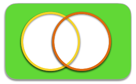
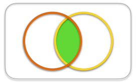
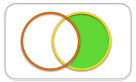
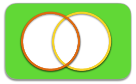


Aristotle
384–322 BC

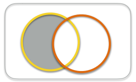
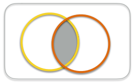
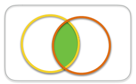
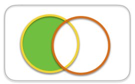


John Venn
1834–1923

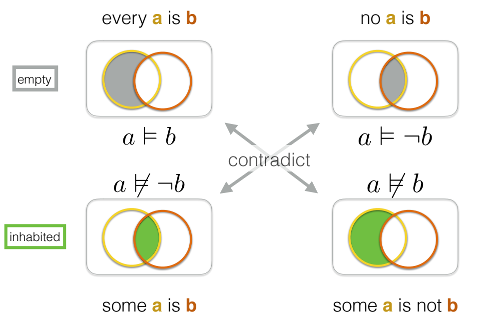
more local contraposition

some **a** is **b**	some **a** is not **b**	some not **a** is not **b**
		
$\dfrac{a \not\models \neg b}{b \not\models \neg a}$	$\dfrac{a \not\models b}{\neg b \not\models \neg a}$	$\dfrac{\neg a \not\models b}{\neg b \not\models a}$
		
some **b** is **a**	some not **b** is **a**	some not **b** is not **a**

Aristotle's Categorical Propositions

	all **a** is **b**	no **a** is **b**
empty		
inhabited		
	some **a** is **b**	some **a** is not **b**

Aristotle's Categorical Propositions



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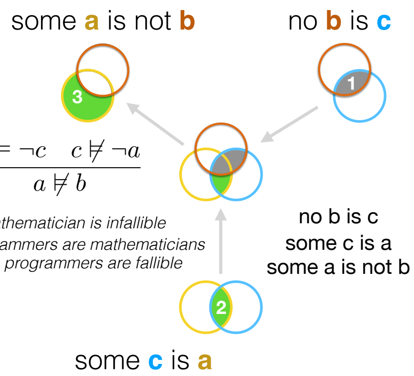


$$\frac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$$

No mathematician is infallible
Some programmers are mathematicians
∴ Some programmers are fallible

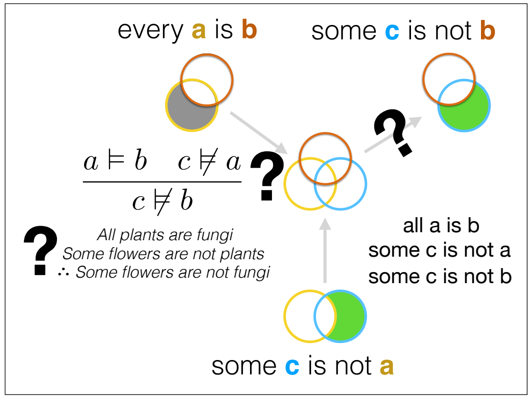
no b is c
some c is a
some a is not b

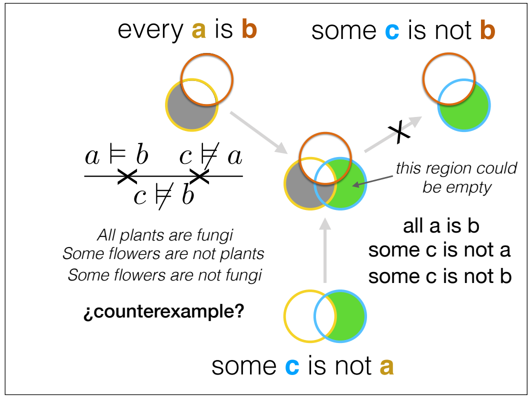
?

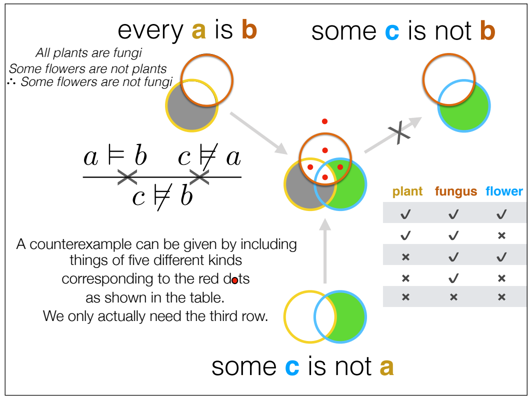
All plants are fungi
Some flowers are not plants
∴ Some flowers are not fungi

Is this a valid argument?

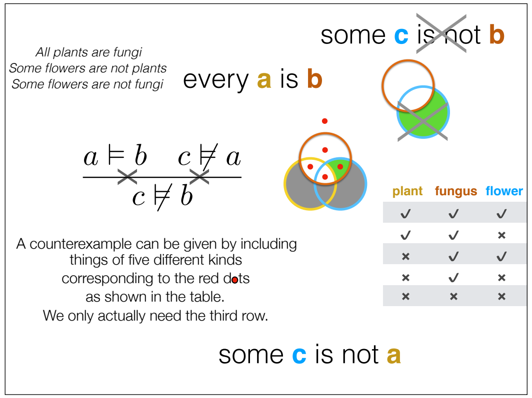
Give it as a syllogism, and use Venn diagrams
either to show it is valid,
or to produce a counterexample.



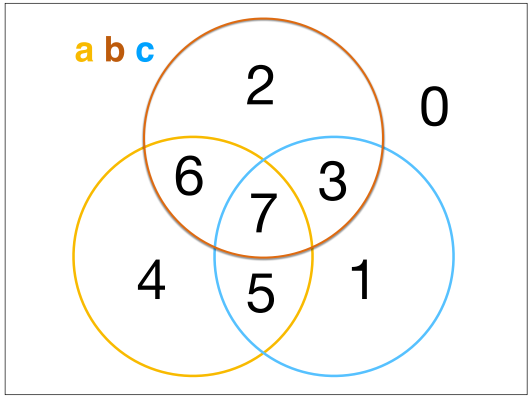


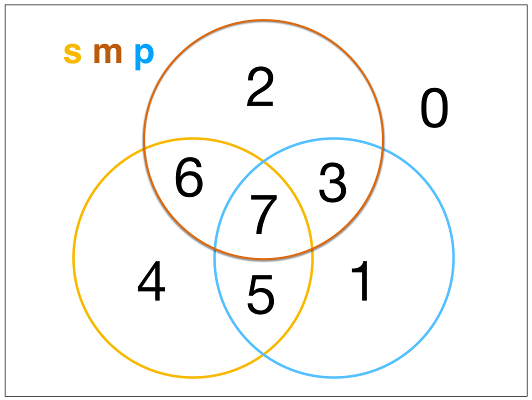


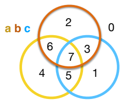
plant	fungus	flower
✓	✓	✓
✓	✓	×
×	✓	✓
×	✓	×
×	×	×



plant	fungus	flower
✓	✓	✓
✓	✓	×
×	✓	✓
×	✓	×
×	×	×







$$\frac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$$

fresison

no b is c
some c is a
some a is not b

no b is c means both 3 and 7 are empty
some c is a means at least one of 7 and 5 is inhabited
— since 7 is empty, 5 is inhabited.
This shows that the conclusion is valid since,
some a is not b means at least one of 4 and 5 is inhabited.

Showing a rule is valid by deriving it from a valid rule

ferio
$$\frac{c \vDash \neg b \quad a \nvDash \neg c}{a \nvDash b}$$

substitute $\neg b$ for b

$$\frac{c \vDash b \quad a \nvDash \neg c}{a \nvDash \neg b}$$

take the contrapositive of the conclusion

dimatis
$$\frac{c \vDash b \quad a \nvDash \neg c}{b \nvDash \neg a}$$

local contraposition

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a	e	i	o
$a \vDash b$	$a \vDash \neg b$	$a \nvDash \neg b$	$a \nvDash b$

barbara $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$ bocardo $\dfrac{a \vDash b \quad a \nvDash c}{b \nvDash c}$ baroco $\dfrac{b \vDash c \quad a \nvDash c}{a \nvDash b}$

celarent $\dfrac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$ disamis $\dfrac{a \vDash b \quad a \nvDash \neg c}{b \nvDash \neg c}$ festino $\dfrac{b \vDash \neg c \quad a \nvDash \neg c}{a \nvDash b}$

cesare $\dfrac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}$ datisi $\dfrac{a \vDash b \quad a \nvDash \neg c}{c \nvDash \neg b}$ ferio $\dfrac{c \vDash \neg b \quad a \nvDash \neg c}{a \nvDash b}$

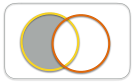
camestres $\dfrac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}$ ferison $\dfrac{a \vDash \neg b \quad a \nvDash \neg c}{c \nvDash b}$ fresison $\dfrac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$

calemes $\dfrac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a}$ darii $\dfrac{a \vDash b \quad c \nvDash \neg a}{c \nvDash \neg b}$ dimatis $\dfrac{c \vDash b \quad a \nvDash \neg c}{b \nvDash \neg a}$

predicates
a b c

Euler diagram

Venn diagram



entailment
a relation
between predicates
a ⊨ b

sound rules

$$\frac{a \models b \quad b \models c}{a \models c}$$

negation
¬¬a = a

$$\frac{a \models b}{\neg b \models \neg a}$$

$$\frac{a \models b \quad a \not\models c}{b \not\models c}$$

counterexamples

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a	e	i	o
$a \models b$	$a \models \neg b$	$a \not\models \neg b$	$a \not\models b$

barbara $\dfrac{a \models b \quad b \models c}{a \models c}$ bocardo $\dfrac{a \models b \quad a \not\models c}{b \not\models c}$ baroco $\dfrac{b \models c \quad a \not\models c}{a \not\models b}$

celarent $\dfrac{a \models b \quad b \models \neg c}{a \models \neg c}$ disamis $\dfrac{a \models b \quad a \not\models \neg c}{b \not\models \neg c}$ festino $\dfrac{b \models \neg c \quad a \not\models \neg c}{a \not\models b}$

cesare $\dfrac{a \models b \quad c \models \neg b}{a \models \neg c}$ datisi $\dfrac{a \models b \quad a \not\models \neg c}{c \not\models \neg b}$ ferio $\dfrac{c \models \neg b \quad a \not\models \neg c}{a \not\models b}$

camestres $\dfrac{a \models b \quad c \models \neg b}{c \models \neg a}$ ferison $\dfrac{a \models \neg b \quad a \not\models \neg c}{c \not\models b}$ fresison $\dfrac{b \models \neg c \quad c \not\models \neg a}{a \not\models b}$

calemes $\dfrac{a \models b \quad b \models \neg c}{c \models \neg a}$ darii $\dfrac{a \models b \quad c \not\models \neg a}{c \not\models \neg b}$ dimatis $\dfrac{c \models b \quad a \not\models \neg c}{b \not\models \neg a}$

The first row is derived from Barbara by contraposition

The second row is derived from the first by substituting $\neg c$ for c .

You should derive each of the following rows from those above, using substitutions and local contraposition.

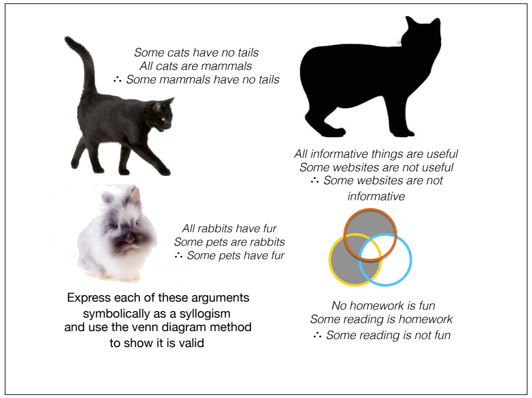
$$\frac{r \models f \quad p \not\models \neg r}{p \not\models \neg f}$$

All rabbits have fur
Some pets are rabbits
∴ Some pets have fur

Here is a little proof to show this argument is sound

$$\frac{p \models \neg f \qquad \dfrac{r \models f}{\neg f \models \neg r}}{p \models \neg r}$$

This shows that, if $r \models f$ *and* $p \models \neg f$ *then* $p \models \neg r$, *but we assume that* $r \models f$ *and* $p \not\models \neg r$, *so* $p \not\models \neg f$.



a	e	i	o
$a \models b$	$a \models \neg b$	$a \not\models \neg b$	$a \not\models b$

barbara $\dfrac{m \models p \quad s \models m}{s \models p}$	bocardo $\dfrac{m \not\models p \quad m \models s}{s \not\models p}$	baroco $\dfrac{p \models m \quad s \not\models m}{s \not\models p}$
celarent $\dfrac{m \models \neg p \quad s \models m}{s \models \neg p}$	disamis $\dfrac{m \not\models \neg p \quad m \models s}{s \not\models \neg p}$	festino $\dfrac{p \models \neg m \quad s \not\models \neg m}{s \not\models p}$
cesare $\dfrac{p \models \neg m \quad s \models m}{s \models \neg p}$	datisi $\dfrac{m \models p \quad m \not\models \neg s}{s \not\models \neg p}$	ferio $\dfrac{m \models \neg p \quad s \not\models \neg m}{s \not\models p}$
camestres $\dfrac{p \models m \quad s \models \neg m}{s \models \neg p}$	ferison $\dfrac{m \models \neg p \quad m \not\models \neg s}{s \not\models p}$	fresison $\dfrac{p \models \neg m \quad m \not\models \neg s}{s \not\models p}$
calemes $\dfrac{p \models m \quad m \models \neg s}{s \models \neg p}$	darii $\dfrac{m \models p \quad s \not\models \neg m}{s \not\models \neg p}$	dimatis $\dfrac{p \not\models \neg m \quad m \models s}{s \not\models \neg p}$

This is the traditional presentation of the syllogisms.
For each syllogism, the conclusion is a categorical proposition relating a *subject* s to a a *predicate* p The assumptions are categorical propositions relating p and s to a middle predicate, m.
The three aeio vowels in the name of each syllogism (each name includes 3 of these vowels)
signify the forms of the three categorical propositions.

The names are in a code that tells how the syllogism in question is derived from one of the following four syllogisms: barbara, celarent, darii, ferio. The first letter of the name of each syllogism matches the name of the syllogism it is derived from.
When one of the consonants smc follows one of the vowels aeio, it tells us how the corresponding proposition should be changed:
The c in bocardo and baroco corresponds to our contrapositive construction of these rules
The s in festino relates it to ferio via a local contraposition, and the two occurrences of s in fresison show it is derived from ferio using two local

contrapositions, ferison again uses one local contraposition.

The letter m means that we swap s and p — observe that each name with an m ends with s, which represents the contraposition required to put s and p back in the correct order. When we swap s and p we also have to change the order of the premises, but first we must apply any the further contrapositions required if there is another s in the name.

No reptiles have fur
All snakes are reptiles
∴ Some snakes have no fur

$$\frac{m \models \neg p \quad s \models m}{s \not\models p}\ \textit{cesaro}$$

$$\frac{p \models m \quad s \models \neg m}{s \not\models p}\ \textit{camestros}$$

All horses have hooves.
No humans have hooves.
∴ Some humans are not horses.

No flowers are animals.
All flowers are plants.
∴ Some plants are not animals.

$$\frac{m \models \neg p \quad m \models s}{s \not\models p}\ \textit{felapton}$$

$$\frac{m \models p \quad m \models s}{s \not\models \neg p}\ \textit{darapti}$$

All squares are rectangles.
all squares are rhombuses.
∴ Some rhombuses are rectangles.

These forms are not sound syllogisms.
In each case, we can give a counterexample
by making one of the predicates empty.

Syllogisms as a logical system

three predicates
four kinds of assertion
16 rules — but they can all be derived from barbara by simple reasoning
the modern notation helps a lot in making patterns visible

The meaning of a categorical proposition is defined by saying some region of a Venn diagram is empty or inhabited.
Are there some ideas we can't expresses categorical propositions?

We can say that any region of a two predicate Venn diagram is inhabited or void.
What about every region of a four-predicate Venn diagram?

In general, What is a predicates?
What operations are there on predicates?

We consider a finite universe U
— a collection of things.

Any subset $a \subseteq U$ can be treated as a predicate:
$a\ x$ is true iff $x \in a$
$a \models b$ iff $a \subseteq b$ and barbara is sound
$x \in \neg a$ iff $x \notin a$ $a \models b$ iff $\neg b \models \neg a$