

| | |
|-------------------|-------------------------------------|
| $p \rightarrow q$ | If it is raining then I get the bus |
|-------------------|-------------------------------------|

Converse
 $q \rightarrow p$

If I get the bus
then it is raining

Contrapositive
 $\neg q \rightarrow \neg p$

If I do not get
the bus then it
is not raining

Inverse
 $\neg p \rightarrow \neg q$

If it is not
raining then I
do not get the bus

$$q \rightarrow p$$

| |
|---|
| T |
| T |
| F |
| T |

logically
equivalent

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$\neg p \rightarrow \neg q \equiv \neg \neg q \rightarrow \neg \neg p$$

$$\neg p \rightarrow \neg q \equiv q \rightarrow p$$

| p | q | $\neg p$ | $\neg q$ | $\neg q \rightarrow \neg p$ | $p \rightarrow q$ | $\neg p \rightarrow \neg q$ |
|---|---|----------|----------|-----------------------------|-------------------|-----------------------------|
| T | T | F | F | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | T | F |
| F | F | T | T | T | T | T |

Biconditionals

$$p \leftrightarrow q$$

same as $(p \rightarrow q) \wedge (q \rightarrow p)$

p "if and only if" q

p is necessary and sufficient for q
If p then q and conversely

p iff q

| p | q | $p \rightarrow q$ | $q \rightarrow p$ | $(p \rightarrow q) \wedge (q \rightarrow p)$ |
|---|---|-------------------|-------------------|--|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

If you use your umbrella it is raining
and

If it is raining you use your umbrella

So $p \leftrightarrow q$
is the same as
 $p = q$
or $p \equiv q$

Q Show $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent

| p | q | r | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|--------------|-----------------------|------------|------------|--------------------------------|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

✓

✓

Distributive law

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$