Nested Quantifiers

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$\forall x \forall y \ (x + y = y + x)$ $\boxed{\text{where } x, y \in \mathbb{R}}$

commutative law of addition — domain

$\forall x \exists y \ (x + y = 0)$ every real number has an additive inverse.

Ex: let $Q(x, y)$ denote "$x + y = 0$"

What are the truth values of I) $\exists y \forall x \ Q(x, y)$?
II) $\forall x \exists y \ Q(x, y)$

II) $\forall x \exists y \ Q(x, y)$ For all real numbers x there is a real number y so that $x + y = 0$ i.e. $y = -x$
So true

y can depend on x

I) $\exists y \forall x \ Q(x, y)$ There is some real number y such that for every real number x
$x + y = 0$
But there is no single real number y that works for all values of x
So false

y is a constant independent of x

So $\forall x \exists y \ Q(x, y) \not\equiv \exists y \forall x \ Q(x, y)$

However, if $\exists y \forall x \ P(x, y)$ true then $\forall x \exists y \ P(x, y)$

$\exists y \forall x \ P(x, y) \rightarrow \forall x \exists y \ P(x, y)$

$\neg \forall x \exists y \ (xy = 1) \equiv \exists x \neg \exists y \ (xy = 1)$
$\equiv \exists x \forall y \neg (xy = 1)$

$\equiv \exists x \forall y \ (xy \neq 1)$