Quantifiers

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Create a proposition from a propositional function $P(x)$ by assigning values to variables we can use "some", "for all", "many," "none" x

Universal quantifier $\forall$

$P(x)$ [true] for all values of x in the domain

written as $\forall x \, P(x)$

Ex Let $P(x)$ be the propositional function "$x + 2 > x$"
What is the truth value of $\forall x \, P(x)$ where the domain is $x \in \mathbb{R}$? [T]

Note: generally the domain of discourse is non-empty [True as no element for which $P(x)$ false]

Can disprove by a counterexample?
If $P(x)$ is $x < 2$ and the domain is $\mathbb{Z}^+$
$\forall x \, P(x)$ is false when $x = 3$

Q1 $P(x): x^2 \geq x$ What is the truth value of $\forall x \, P(x)$?
DEPENDS ON THE DOMAIN – must specify
if $x \in \mathbb{R}$, false eg $x = \frac{1}{2}$
if $x \in \mathbb{Z}$ true [no integers $0 < x < 1$]

The Existential Quantifier $\exists$

There exists an element of x in the domain such that $P(x)$ true
written as $\exists x \, P(x)$

for some
for at least one
there is

Domain $x \in \mathbb{R}$

Ex I) $P(x). \; x > 3$ $\exists x \, P(x)$? T
II) $Q(x): x = x + 2$ $\exists x \, Q(x)$? F
III) $R(x): x^2 > 20$ $\exists x \, R(x)$? $x = 6$ ✓ T

$\exists! \, x \, P(x)$
↑ uniqueness quantifier : one and only one eg $P(x): x - 1 = 0$
$x \in \mathbb{Z}$

Precedence of Quantifiers.

$\forall, \exists$ have higher precedence than all logical operators from propositional calculus

$\forall x\, P(x) \vee Q(x)$ is the disjunction of $\forall x\, P(x)$ and $Q(x)$

$\equiv (\forall x\, P(x)) \vee Q(x)$

*. Note $\forall x\, (P(x) \wedge Q(x)) \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$ $\quad \forall x\, P(x) \vee \forall x\, Q(x) \not\equiv \forall x\, (P(x) \vee Q(x))$

Similarly $\exists x\, (P(x) \vee Q(x)) \equiv \exists x\, S(x) \vee \exists x'\, R(x)$

but $\exists x\, (P(x) \wedge Q(x)) \not\equiv \exists x\, P(x) \wedge \exists x\, Q(x)$

Bound variables

If a quantifier is used on variable x then it is bound

if a variable has no quantifier it is free

All variables in a propositional function must be bound or set to a particular value to make it a proposition

Ex. a) $\exists \overset{\text{bound}}{x}\, (x + \overset{\text{free}}{y} = 2)$

b) $\exists x \underbrace{(P(x) \wedge Q(x))}_{\text{scope of } \exists} \vee \forall x \underbrace{R(x)}_{\text{scope of } \forall}$ all bound

could also write as $\exists x\, (P(x) \wedge \underbrace{Q(x)) \vee \forall z}_{\text{scopes do not overlap}} R(z)$