Inductive definitions definition by rules

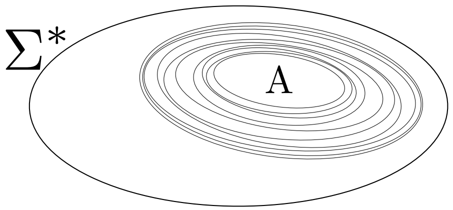
INF1a-CL lecture 18

defining an operation on languages

A^* is defined by two rules: $\dfrac{}{\texttt{""} \in A^*} \quad \dfrac{s \in A^* \quad a \in A}{s\texttt{++}a \in A^*}$

because, $\texttt{""}\texttt{++}a_1\texttt{++}a_2 \ldots \texttt{++}a_n = a_1\texttt{++}a_2 \ldots \texttt{++}a_n\texttt{++}\texttt{""}$,

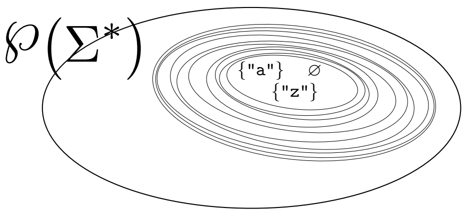
the rules above are equivalent to: $\dfrac{}{\texttt{""} \in A^*} \quad \dfrac{s \in A^* \quad a \in A}{a\texttt{++}s \in A^*}$

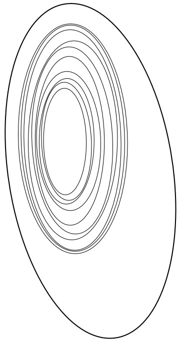


defining a set of languages

$$\dfrac{}{\varnothing \text{ is regular}} \qquad \dfrac{}{\{\texttt{"a"}\} \text{ is regular}} \qquad \dfrac{A \text{ is regular}}{A^* \text{ is regular}}$$

$$\dfrac{R \text{ is regular} \quad S \text{ is regular}}{RS \text{ is regular}} \qquad \dfrac{R \text{ is regular} \quad S \text{ is regular}}{R|S \text{ is regular}}$$





defining an operation on relations

$$\frac{p\ \texttt{friend}\ r}{p\ \texttt{foaf}\ r} \qquad \frac{p\ \texttt{foaf}\ q \quad q\ \texttt{foaf}\ r}{p\ \texttt{foaf}\ r}$$

$$\frac{p\ \texttt{friend}\ r}{p\ \texttt{foaf}\ r} \qquad \frac{p\ \texttt{friend}\ q \quad q\ \texttt{foaf}\ r}{p\ \texttt{foaf}\ r}$$

$$\frac{p\ \texttt{friend}\ r}{p\ \texttt{foaf}\ r} \qquad \frac{p\ \texttt{foaf}\ q \quad q\ \texttt{friend}\ r}{p\ \texttt{foaf}\ r}$$

$$\wp\big(A \times A\big)$$

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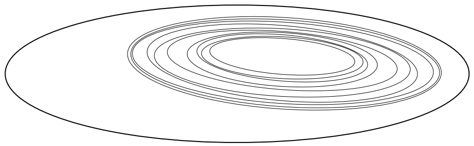
two theorems about regular languages

$$\frac{}{\varnothing \text{ is regular}} \qquad \frac{}{\{\texttt{"a"}\} \text{ is regular}} \qquad \frac{A \text{ is regular}}{A^* \text{ is regular}}$$

$$\frac{R \text{ is regular} \quad S \text{ is regular}}{RS \text{ is regular}} \qquad \frac{R \text{ is regular} \quad S \text{ is regular}}{R|S \text{ is regular}}$$

every regular language is recognised by some NFA

every regular language is recognised by some DFA



$\Gamma \vDash \Delta$ is a relation between finite sets of predicates
it satisfies the following rules:

$$\frac{}{\Gamma, a \vDash a, \Delta}\ (I)$$

$$\frac{\Gamma \vDash a, \Delta \quad \Gamma, b \vDash \Delta}{\Gamma, a \to b \vDash \Delta}\ (\to L) \qquad \frac{\Gamma, a \vDash b, \Delta}{\Gamma \vDash a \to b, \Delta}\ (\to R)$$

$$\frac{\Gamma, a, b \vDash \Delta}{\Gamma, a \wedge b \vDash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vDash a, \Delta \quad \Gamma \vDash b, \Delta}{\Gamma \vDash a \wedge b, \Delta}\ (\wedge R)$$

$$\frac{\Gamma, a \vDash \Delta \quad \Gamma, b \vDash \Delta}{a \vee b \vDash \Delta}\ (\vee L) \qquad \frac{\Gamma \vDash a, b, \Delta}{\Gamma \vDash a \vee b, \Delta}\ (\vee R)$$

$$\frac{\Gamma \vDash a, \Delta}{\Gamma, \neg a \vDash \Delta}\ (\neg L) \qquad \frac{\Gamma, a \vDash \Delta}{\Gamma \vDash \neg a, \Delta}\ (\neg R)$$

$\Gamma \vdash \Delta$ is the relation between finite sets of `Wff`s **defined by** the following rules:

$$\frac{}{\Gamma, a \vdash a, \Delta}\ (I)$$

$$\frac{\Gamma \vdash a, \Delta \quad \Gamma, b \vdash \Delta}{\Gamma, a \to b \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, a \vdash b, \Delta}{\Gamma \vdash a \to b, \Delta}\ (\to R)$$

$$\frac{\Gamma, a, b \vdash \Delta}{\Gamma, a \wedge b \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash a, \Delta \quad \Gamma \vdash b, \Delta}{\Gamma \vdash a \wedge b, \Delta}\ (\wedge R)$$

$$\frac{\Gamma, a \vdash \Delta \quad \Gamma, b \vdash \Delta}{a \vee b \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash a, b, \Delta}{\Gamma \vdash a \vee b, \Delta}\ (\vee R)$$

$$\frac{\Gamma \vdash a, \Delta}{\Gamma, \neg a \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, a \vdash \Delta}{\Gamma \vdash \neg a, \Delta}\ (\neg R)$$

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theorem $\Gamma \vdash \Delta$ iff $\Gamma \models \Delta$

the inference rules are **sound**: $\Gamma \vdash \Delta \Rightarrow \Gamma \models \Delta$

the inference rules are **complete**: $\Gamma \models \Delta \Rightarrow \Gamma \vdash \Delta$

$$\frac{}{\Gamma, a \vdash a, \Delta}\ (I)$$

$$\frac{\Gamma \vdash a, \Delta \quad \Gamma, b \vdash \Delta}{\Gamma, a \to b \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, a \vdash b, \Delta}{\Gamma \vdash a \to b, \Delta}\ (\to R)$$

$$\frac{\Gamma, a, b \vdash \Delta}{\Gamma, a \wedge b \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash a, \Delta \quad \Gamma \vdash b, \Delta}{\Gamma \vdash a \wedge b, \Delta}\ (\wedge R)$$

$$\frac{\Gamma, a \vdash \Delta \quad \Gamma, b \vdash \Delta}{a \vee b \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash a, b, \Delta}{\Gamma \vdash a \vee b, \Delta}\ (\vee R)$$

$$\frac{\Gamma \vdash a, \Delta}{\Gamma, \neg a \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, a \vdash \Delta}{\Gamma \vdash \neg a, \Delta}\ (\neg R)$$

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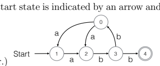
6. (a) Which of the following strings are accepted by the NFA in the diagram? (The start state is indicated by an arrow and the accepting state by a double



border.)

i. abb
ii. abbabbabbaaabb
iii. abbabbaabbabbabb
iv. abbabaabbabbabb [*3 marks*]

(b) Write a regular expression for the language accepted by this NFA. [*3 marks*]

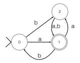
(c) Draw a DFA that accepts the same language. Label the states of your DFA to make clear their relationship to the states of the original NFA. [*10 marks*]

(d) For each of the following regular expressions, draw a non-deterministic finite state machine that accepts the language described by the regular expression.

i. x^*y
ii. $(x^*|y)$
iii. $(x^*y)^*$ [*9 marks*]

5. Each diagram shows an FSM. In each case give a regular expression for the language accepted by the FSM, make a mark in the check box against each string that it accepts (and no mark against those strings it does not accept), make a mark in the DFA check box if it is deterministic, and draw an equivalent DFA if it is not.

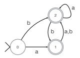
[*4 marks*]

(a) 

aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

`regex:`

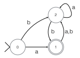
[*4 marks*]

(b) 

aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

`regex:`

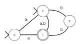
[*4 marks*]

(c) 

aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

`regex:`

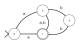
[*4 marks*]

(d) 

aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

`regex:`

[*4 marks*]

(e) 

aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

`regex:`