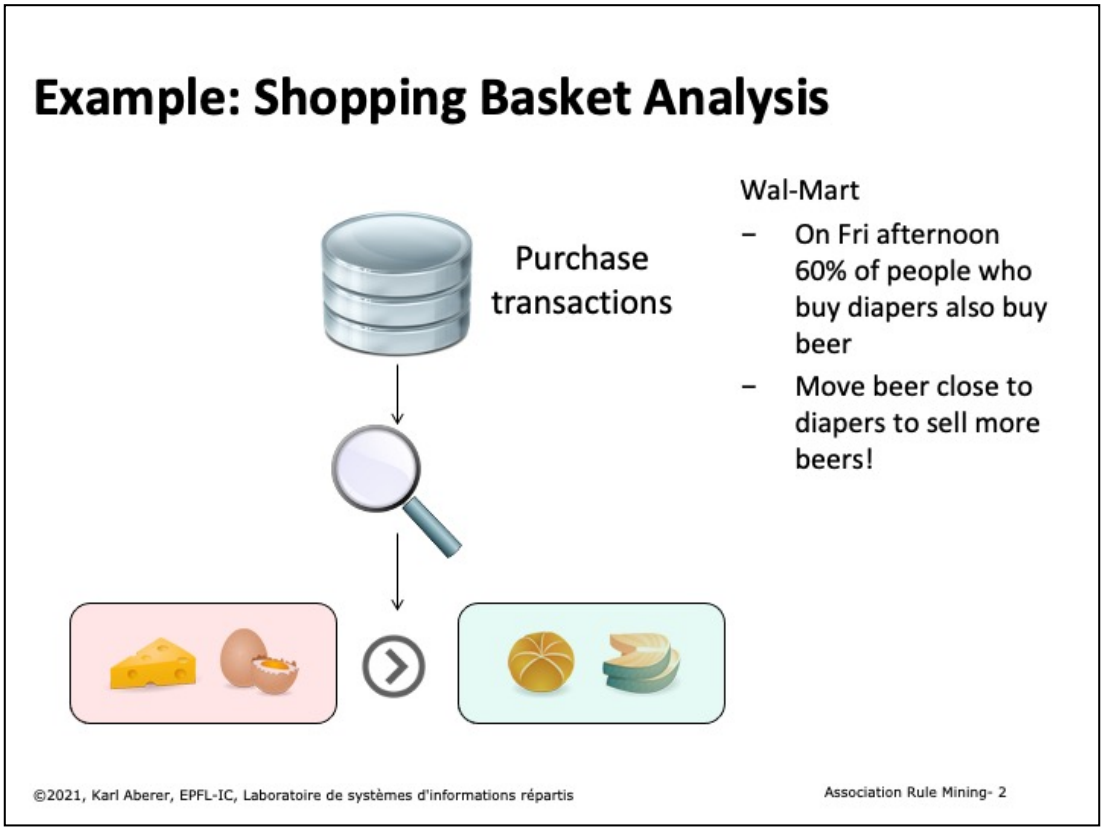
5. ASSOCIATION RULE MINING



The classical example of a data mining problem is "market basket analysis". Retail stores gather information on what items are purchased by their customers. The expectation is, by finding out what products are frequently purchased together (i.e., are associated with each other), identifying ways to optimize the sales of the products by better targeting certain groups of customers (e.g., by planning the layout of a store or planning advertisements). A well-known example was the discovery that people who buy diapers also frequently buy beers (probably exhausted fathers of small children). Therefore nowadays one finds frequently beer close to diapers in supermarkets, and of course also chips close to beer. Similarly, amazon exploits this type of associations in order to propose to their customers books that are likely to match their interests. This type of problem was the starting point for one of the best known data mining techniques: association rule mining.

Association Rules

We search association rules of the form

Body ➔ Head [support, confidence]

1. Body: predicate(*x*, {items}) — a property of *x*
2. Head: predicate(*x*, {items}) — a property likely to be implied by the Body
3. Support, confidence: measures of the validity of the rule

Example: buy(x, {diapers}) ➔ buy(x, {beer}) [0.5%, 60%]

Association rule mining is a technique for discovering unsuspected data dependencies and is one of the best known data mining techniques. The basic idea is to identify from a given database, consisting of item sets (e.g., shopping baskets), whether the occurrence of specific items, implies also the occurrence of other items with a high probability. In principle, the answer to this question could be easily found by an exhaustive exploration of all possible dependencies, which is however prohibitively expensive. Association rule mining thus solves the problem of how to search efficiently for those dependencies.

Single- and Multi-dimensional Rules

Single-dimensional rules

buy(x, {diapers}) ➔ buy(x, {beer}) [0.5%, 60%]

Multi-dimensional rules

age(x, {19-25}) ∧ buy(x, {chips}) ➔ buy(x, {coke}) [10%, 75%]

As shown in the previous example, we can have two kinds of association rules. Those that associate data for a single predicate (such as buys), which are called singe-dimensional rules, and those that use several different predicates (such as buys and age), which are called multi-dimensional rules.

From Multi- to Single-dimensional Rules

Use predicate/value pairs as items

$$age(x, \{19\text{-}25\}) \wedge buy(x, \{chips\}) \rightarrow buy(x, \{coke\})$$

$$\Downarrow$$

$$customer(x, \{age=19\text{-}25\}) \wedge customer(x, \{buy=chips\}) \rightarrow customer(x, \{buy=coke\})$$

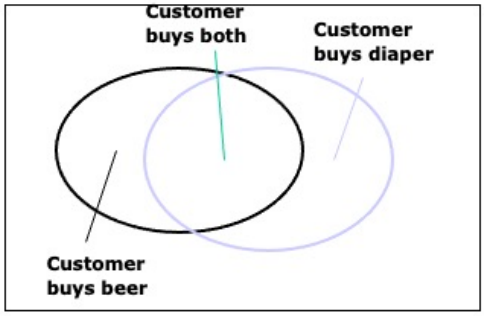
Simplified notation of single-dimensional rules

$$\{diapers\} \rightarrow \{coke\}$$

$$\{age=19\text{-}25, buy=chips\} \rightarrow \{buy=coke\}$$

However, it is straightforward to transform multi-dimensional association rules into single-dimensional rules, by considering different predicates applied to the same items as different items. Therefore in the following we will only consider single-dimensional association rules. As a result we will also use a simplified notation for the rules, where instead of using predicates we just distinguish different items.

2. Scoring Function



Transaction ID	Items Bought
2000	beer, diaper, milk
1000	beer, diaper
4000	beer, milk
5000	milk, eggs, apple

Support: probability that body and head occur in transaction
p({body, head})

Confidence: probability that if body occurs, also head occurs
p({head} | {body})

This example illustrates the basic concepts used in association rule mining. *Transactions* consist of a transaction identifier and an *itemset*. The itemset is the set of *items* that occur jointly in a transaction (e.g., the items bought). Now we can define the measures used to evaluate the quality of a rule.

Support is the number of transactions in which the association rule holds, i.e., in which all items of the rule occur (e.g., both beer and diaper). If this number is too small, probably the rule is not relevant. Confidence is the probability that in case the body of the rule (the condition) is satisfied also the head of the rule (the conclusion) is satisfied. This indicates to which degree the rule is satisfied, in those cases where it is applicable.

Support and Confidence

Transaction ID	Items Bought
2000	beer, diaper, milk
1000	beer, diaper
4000	beer, milk
5000	milk, eggs, apple

Rule1: {beer} ➔ {diaper}

p({beer, diaper})=2/4
p({diaper} | {beer})=2/3

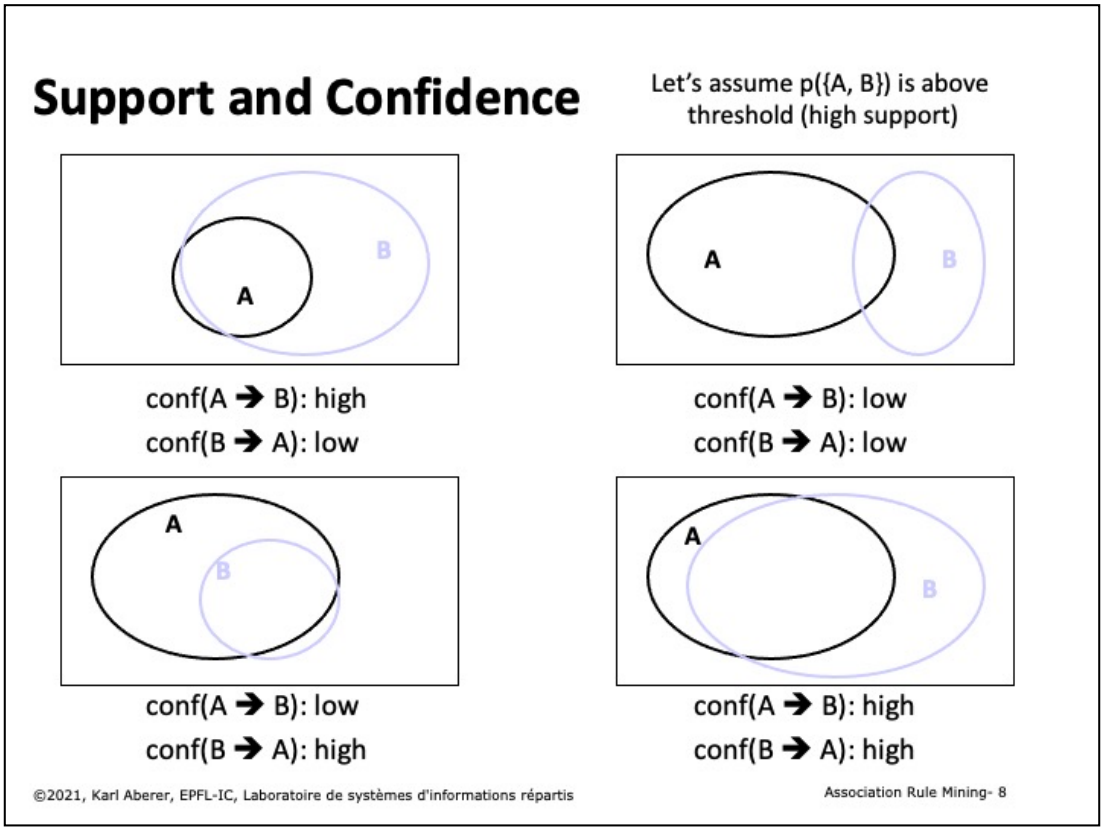
{beer} ➔ {diaper}[50%, 66%]

Rule2: {diaper} ➔ {beer}

p({diaper, beer})=2/4
p({beer} | {diaper})=2/2

{diaper} ➔ {beer}[50%, 100%]

Here we compute support and confidence for two different rules.



Here we illustrate the meaning and importance of the "direction" of an association rule. We assume that in all cases the intersection areas (i.e., the support) are above the required threshold. Then four combinations for the confidence are possible. Thus association rules not only express a high probability of co-occurrence of items, such as in the last case, where confidence is high in both directions, but also conditional dependencies among the occurrences of items (or inclusion relationships).

Definition of Association Rules

Terminology and Notation

- Set of all items I, subset of I is called **itemset**
- **Transaction** (tid, T), $T \subseteq I$ itemset, transaction identifier tid
- Set of all transactions D (**database**), Transaction $T \in D$

Association Rules $A \rightarrow B$ [s, c]

- A, B itemsets ($A, B \subseteq I$)
- $A \cap B$ empty
- $s(A \rightarrow B) = s(A \cup B) = p(A \cup B) = count(A \cup B) / |D|$ (support)
- $c(A \rightarrow B) = p(B \mid A) = s(A \cup B) / s(A)$ (confidence)

Here we summarize the basic concepts that constitute the structure of the associatino rule patterns and the measures for their evaluation that are used in association rule mining.

The support is computed as the number of occurrences of A and B together in a transaction, divided by the total number of transactions. This corresponds to the probability that a transaction contains A U B.

The confidence is computed as the support of A U B divided by the support of A, that is, the frequency of the elements in the union of A and B in a transaction divided by the frequency of A. This corresponds to the conditional probability that a transaction having A also contains B

10 itemsets out of 100 contain item A, of which 5 also contain B. The rule A ➔ B has:

A. 5% support and 10% confidence
B. 10% support and 50% confidence
C. 5% support and 50% confidence
D. 10% support and 10% confidence

10 itemsets out of 100 contain item A, of which 5 also contain B. The rule B ➔ A has:

A. unknown support and 50% confidence
B. unknown support and unknown confidence
C. 5% support and 50% confidence
D. 5% support and unknown confidence

Association Rule Mining: Problem

Problem

Given a database D of transactions (tid, T)

Find

all rules A ➔ B [s, c]

such that $s > s_{min}$ (high support)

and $c > c_{min}$ (high confidence)

The problem of mining association rules is now defined as follows:

Given a database of transactions, find all rules that correlate the presence of one set of items with that of another set of items where the support and the confidence are above a given threshold. In other words, find all rules that have high support and confidence.

Association Rule Mining Approach

Two step approach:

1. Find **frequent** itemsets
   $J \subseteq I$ such that $p(J) > s_{min}$
2. Select **pertinent** rules
   A ➔ B such that $A \cup B = J$
   and
   $p(B|A) > c_{\min}$

We will approach the problem in two steps, by first considering only the problem of finding itemsets that satisfy the condition on having a high support, which is a necessary condition for a rule to be an association rule, and then extracting from those itemsets the rules that have a high confidence.

Frequent Itemsets

Transaction ID	Items Bought
2000	beer, diaper, milk
1000	beer, diaper
4000	beer, milk
5000	milk, eggs, apple

Frequent Itemset	Support
{beer}	0.75
{milk}	0.75
{diaper}	0.5
{beer,milk}	0.5
{beer, diaper}	0.5
{milk, diaper}	0.25

1. A ➔ B can be an association rule, only if A ∪ B is a frequent itemset
2. Any subset of a frequent itemset is also a frequent itemset (**Apriori property**)
$$p(J) > s_{min} \rightarrow p(J' \subseteq J) > s_{min}$$
3. Find frequent itemsets with increasing cardinality, from 1 to k, to reduce the search space

A necessary condition for finding an association rule of the form A->B is that it has a sufficiently high support. Therefore, for finding such rules, we can first try to find itemsets within the transactions that occur sufficiently frequent. These are called *frequent itemsets*. Second we can observe that any subset of a frequent itemset is necessarily also a frequent itemset. This is called the *apriori property*. The a priori property is a central idea in the algorithm we will introduce that will be used for multiple purposes. A first use of this property is the observation that we can search for frequent itemsets by searching them by increasing cardinality: once frequent itemsets of lower cardinality are found, only itemsets of larger cardinality need to be considered that contain one of the frequent itemsets already found. This allows us to dramatically reduce the search space.

Exploiting the Apriori Property

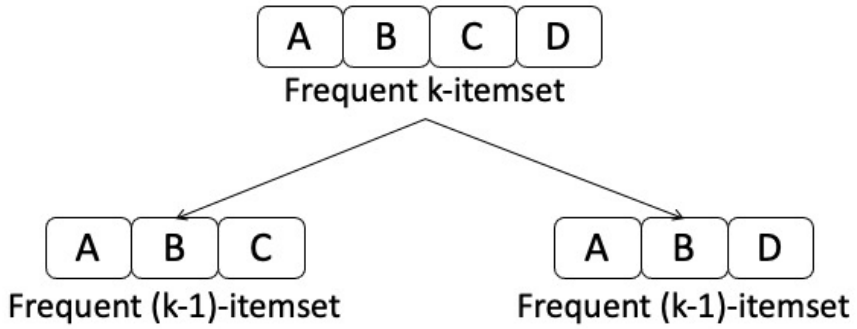
If we know the frequent (k-1)-itemsets, which is a candidate frequent k-itemset **X**?

1. Any (k-1)-itemset that is subset of **X** must be frequent
2. Any ordered (k-1)-itemset that is subset of **X** must be frequent
3. *In particular, two ordered (k-1)-itemsets that are subsets of **X** and differ only in their last position must be frequent*

Assume that we know frequent itemsets of size k-1 and want to construct a itemset of size k (a k-itemset). What properties does such a k-itemset has to satisfy?

We can definitely say, that any subset of size k-1 must be frequent. We can also order those subsets by their elements. We can that even say that two order subsets that differ only in the last position must be frequent. This gives is now a possiblity to construct k-itemsets systematically.

Exploiting the Apriori Property: Candidates



The union of two (k-1)-itemsets that differs only by one item is a **candidate** frequent k-itemset

By inverting the argument, we can see this also as a way to construct frequent k-itemsets. We take two (k-1) item sets which differ only by one item and take their union. This step is called the join step and is used to construct POTENTIAL frequent k-itemsets (called candidate k-itemset)

Example

Transaction ID	Items Bought
2000	beer, diaper, milk
1000	beer, diaper
4000	beer, milk
5000	milk, eggs, apple

Frequent Itemset	Support
{beer}	0.75
{milk}	0.75
{diaper}	0.5
{beer,milk}	0.5
{beer, diaper}	0.5
{milk, diaper}	0.25

{beer} and {milk} differ by one item, thus {beer,milk} is a candidate 2-itemset

{beer, milk} and {beer, diaper} differ by one item, thus {beer, diaper, milk} is a candidate 3-itemset

Note that the candidate itemsets generated by the join step are not necessarily frequent itemsets, as the example illustrates. The condition we use is necessary, but not sufficient.

Exploiting the Apriori Property: JOIN step

Given the frequent (k-1)-itemsets, L_{k-1}, we can construct a candidate set C_k by joining two (k-1)-itemsets that differ by exactly 1 item in the last position

Algorithm (**JOIN**)

1. Sort the itemsets in L_{k-1}
2. Find all pairs with the same first k-2 items, but different k-1th item
3. Join the two itemsets in all pairs and add to C_k

The construction of potential k-itemsets, so-called candidates, from (k-1)-itemsets, can be organized in a way that minimizes the number of candidates being generated. By sorting the items only items that are the highest in the order are the ones being combined from different (k-1)-itemsets. For the remaining k-2 items we have just to consider just those pairs of (k-1)-itemsets that coincide on all of them.

We only combine the itemsets that have the first k-2 identical elements in order to avoid the generation of duplicate candidates. For example if we have

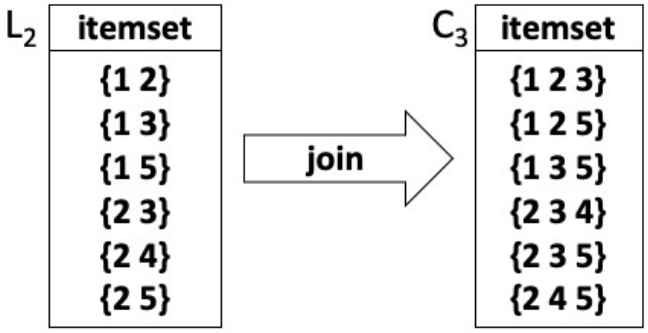
ABC

ACD

BCD

If we combine ACD and BCD (which also differ by one item) we would get ABCD. But ABCD have been already generated by combining ABC and ABD.

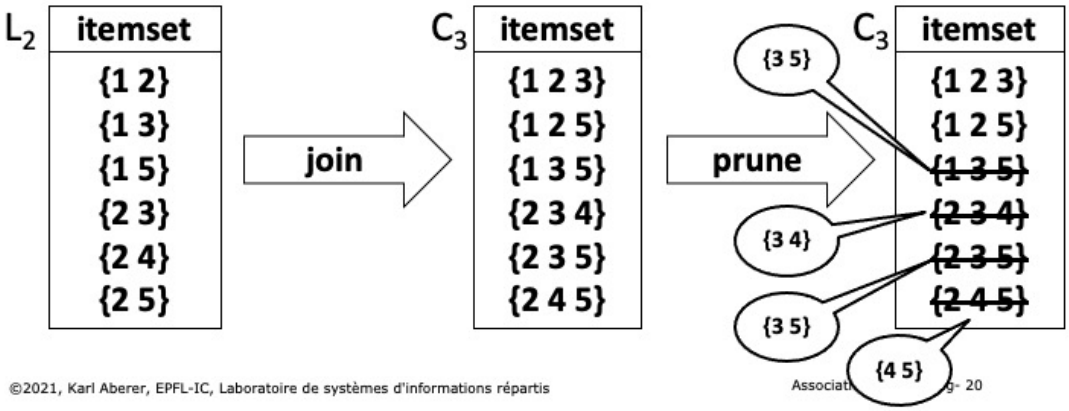
JOIN step: Example



Exploiting the Apriori Property: PRUNE Step

A k-itemset in the candidate set C_k might still contain (k-1)-itemsets that are not frequent (k-1)-itemsets

Eliminate them (**PRUNE**)



The k-itemsets constructed in the join step are not necessarily frequent k-itemsets. One possible reason is that they contain some subsets of items which are not frequent. These are next eliminated in a prune step, by checking if every (k-1) itemset of the candidate itemset is indeed a frequent (k-1) itemset.

Final Step

For the remaining k-itemset in the candidate set C_k eliminate those that are not frequent by counting how often they occur in the database

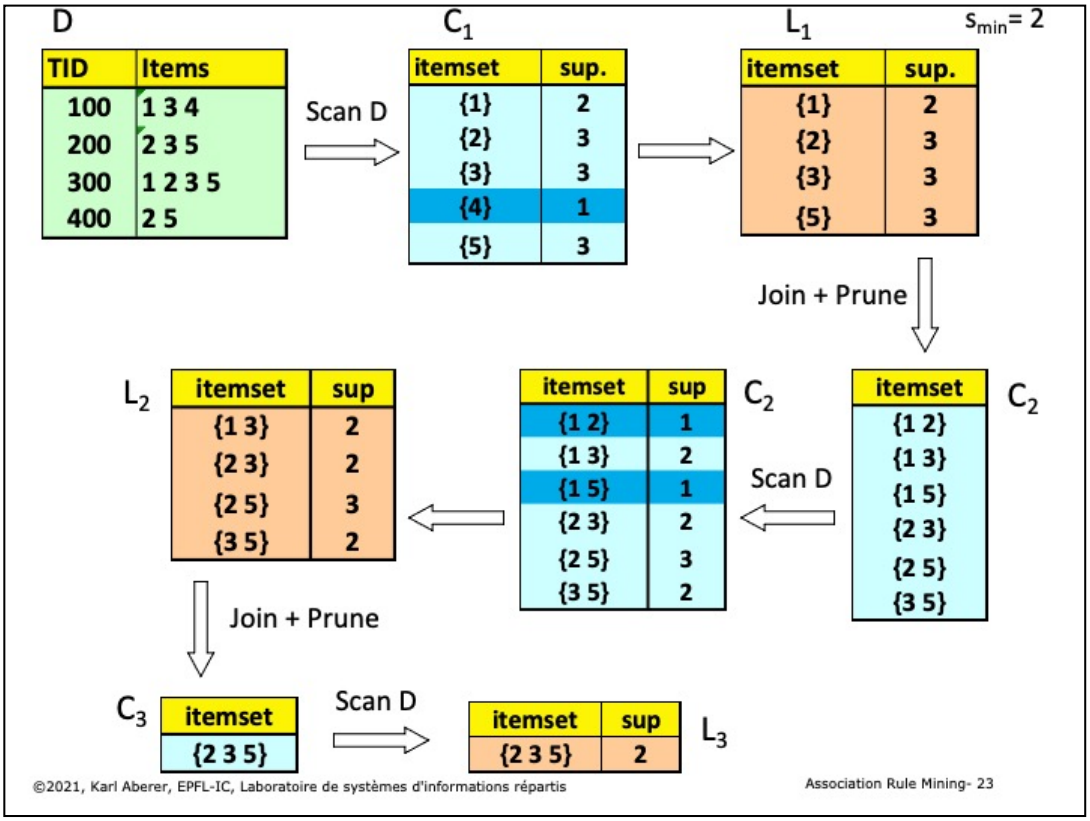
- The final step is the most expensive (full access to database D)
- Advantage: Performed only for a smaller number of candidate itemsets, reduced by **JOIN** and **PRUNE**

After this step is completed, it needs to be checked which of the candidate itemsets constructed so far are indeed frequent. For this purpose the frequency of these itemsets needs to be determined by accessing the database. The important aspect of the apriori algorithm is that this last step is the most expensive one, since it requires access to the complete database, but needs to be performed for a smaller number of itemsets, since many possibilities have been eliminated in the join and prune step.

Apriori Algorithm

$k := 1$, $L_k := \{ J \subseteq I : |J|=1 \wedge p(J) > s_{min} \}$ //frequent items
while $L_k \neq \emptyset$
    $C'_{k+1} := JOIN(L_k)$ // join itemsets in L_k that differ by one element
    $C_{k+1} := PRUNE(C'_{k+1})$ // remove non-frequent itemsets
    for all (id,T) ∈ D // compute the frequency of candidate
        for all $J \in C_{k+1}$, $J \subseteq T$
            count(J)++
        end for
    end for
    $L_{k+1} := \{ J \subseteq C_{k+1} : p(J) > s_{min} \}$ // add candidate frequent itemsets
    k := k+1
end while
return $\cup_k L_k$

This is the summary of the complete apriori algorithm.



Here we give a complete example for the apriori algorithm. Notice, in this example, how the scan steps (when determining the frequency with respect to the database) eliminate certain itemsets. Pruning does not lead to any elimination of itemsets in this example. The algorithm built 9+3 (frequent + non frequent) = 12 itemsets out of 2^5 = 32 total possible itemsets

Select pertinent rules

```
R := ∅                                   // initial set of rules
L := Apriori(D)                          // set of frequent itemsets
for all J ∈ L
        for all A ⊆ J, A != ∅
                r := A ➔ J \ A           // create candidate rule
                if c(A ➔ J \ A) = s(J)/s(A)> c_min
                        R := R ∪ r
                end if
        end for
end for
```


Once the frequent itemsets are found with Apriori, the derivation of pertinent association rules is straightforward.

One checks for every frequent itemset whether there exists a subset A that can occur as the body of a rule. For doing that, the support count, i.e., the frequency of the itemset in the database, which was obtained during the execution of the Apriori algorithm, is used to compute the confidence as a conditional probability. Note that also L\A is a frequent itemset, and therefore the support count is available for that set from the Apriori algorithm.

Note that $c(A \rightarrow J \setminus A) = s(J \setminus A \cup A)/s(A) = s(J)/s(A)$

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2
{2 3 5}	2

c_{min}= 0.75

J={1,3}

A={1}, J\A={3} {1} ➔ {3} c = sup({1,3})/sup({1}) = 1 OK

A={3}, J\A={1} {3} ➔ {1} c = sup({1,3})/sup({3}) = 0.66 KO

J={2,3,5}

A={3,5}, J\A={2} {3,5} ➔ {2} c = sup({2,3,5})/sup({3,5}) = 1 OK

A={2}, J\A={3,5} {2} ➔ {3, 5} c = sup({2,3,5})/sup({2}) = 0.66 KO

Notice, in this example, of how the scan steps (when determining the frequency with respect to the database) eliminate certain itemsets. Pruning does not lead to any elimination of itemsets in this example. The algorithm built 9+3 (frequent + non frequent) = 12 itemsets out of 2^5 = 32 total possible itemsets

Given the frequent 2-itemsets {1,2}, {1,4}, {2,3} and {3,4}, how many 3-itemsets are generated and how many are pruned?

A. 2, 2

B. 1, 0

C. 1, 1

D. 2, 1

After the join step, the number of k+1-itemsets ...

A. is equal to the number of frequent k-itemsets
B. can be equal, lower or higher than the number of frequent k-itemsets
C. is always higher than the number of frequent k-itemsets
D. is always lower than the number of frequent k-itemsets

If rule {A,B} ➔ {C} has confidence c_1 and rule {A} ➔ {C} has confidence c_2, then ...

A. $c_2 >= c_1$
B. $c_1 > c_2$ and $c_2 > c_1$ are both possible
C. $c_1 >= c_2$

Interesting Association Rules

Not all high-confidence rules are interesting

	$Coffee$	$\overline{Coffee}$	
Tea	150	50	200
$\overline{Tea}$	650	150	800
	800	200	1000

{Tea} ➔ {Coffee} has support 0.15 and confidence 0.75

- But 80% of people drink coffee anyway
- Drinking tea decreases the probability to drink coffee!

High confidence and support to not necessarily imply that a rule is interesting or useful. We illustrate this by the following example. The matrix indicates the number of items that like / do not like coffee or tea. If we consider the rule {Tea} –> {coffee} we will find that it has support 0.15 and confidence 0.75, which we may consider as very high. So at the first glance this looks like a good rule. Looking more carefully we will realize that actually 80% of the people drink coffee. In view of this, the rule just holds, because people in general drink a lot of coffee. Even worse, considering that 80% of people drink coffee, among the people drinking tea the propensity to drink coffee is less pronounced, only 75%. So drinking tea has a negative impact on drinking coffee which is the opposite of what the rule suggests. This motivates the need for other measures to evaluate whether a rule is interesting.

Alternative Measures of Interest

Added Value (A, B itemsets)

$$AV(A \rightarrow B) = confidence(A \rightarrow B) - support(B)$$

Interesting rules are those with high positive or negative interest values (usually above 0.5)

Alternative: $Lift(A \rightarrow B) = \frac{confidence(A \rightarrow B)}{support(B)}$

Example:

- AV({Tea} ➔ {Coffee}) = 0.75 – 0.8 = -0.05
- Lift({Tea} ➔ {Coffee}) = 0.75 / 0.8 = 0.9375

Only if the probability of finding item B when item A has been found is greater than the probability of finding item B at all can we say that A implies B.

Quantitative Attributes

Transforming quantitative (numeric ordered values) into categorical ones

- *Static discretisation* into predefined bins
- *Dynamic discretisation* based on the distribution of the data

The rules depend on the chosen discretisation!!

(1) age{*x*, [18,19]} ∧ live{*x*, Lausanne} ➔ work{*x*, student}

(2) age{*x*, [20,21]} ∧ live{*x*, Lausanne} ➔ work{*x*, student}

vs.

(1) age{*x*, [18,21]} ∧ live{*x*, Lausanne} ➔ work{*x*, student}

For quantitative attributes the search for association rule is more complex. A simple approach is to statically or dynamically discretize quantitative attributes into categorical attributes.

However, the rules that can be found depend on the discretization chosen. It may happen that the bins are for example too fine-grained, and a rule that could be expressed in a compact form at a coarser granularity is split into multiple rules.

For example: if age is discretized into steps of 2 years we would probably find rules

Age(X, 18..19) and lives(X, Lausanne) -> profession(X, student)

Age(X, 20..21) and lives(X, Lausanne) -> profession(X, student)

This could be also expressed as a rule

Age(X, 18..21) and lives(X, Lausanne) -> profession(X, student)

which is more compact but requires a different discretization.

Improving Apriori for Large Datasets

1. Transaction reduction
   - A transaction that does not contain any frequent k-itemset is useless in subsequent scans
2. Sampling
   - Mining on a sampled subset of DB, with a lower support
3. Partitioning (SON algorithm)
   - Any itemset that is potentially frequent in a DB must be frequent in at least one of the partitions of the DB

Though the basic Apriori algorithm is designed to work efficiently for large datasets, there exist a number of possible improvements:

•Transactions in the database that turn out to contain no frequent k-itemsets can be omitted in subsequent database scans.

•The sampling method selects samples from the database and searches for frequent itemsets in the sampled database using a correspondingly lower threshold for the support.

•One can try to identify first frequent itemsets in partitions of the database, that fits in memory. This method is based on the assumption that if an itemset is not frequent in one of the partitions at least (local frequent itemset) then it will also not be frequent in the whole database.

Sampling

Approach

1. Randomly sample transactions with probability p
2. Detect frequent itemsets with support p*s
3. Eliminate **false positives** by counting frequent itemsets on complete data after discovery

Refinements

- If we assume that the m transactions are randomly sorted, we can just choose the first p*m ones
- **False negatives** can be reduced by choosing a lower threshold, e.g. 0.9 p*s

Random sampling has to consider two issues: false positives and false negatives. False positives can be dealt with by verifying the candidate itemsets on the complete transaction set. For false negative no such possibility exists. By decreasing the support threshold, the number of false negatives can be reduced, at the cost of testing more candidate itemsets when scanning the complete dataset.

A further optimization can be achieved, when we know that the transactions occur in random order. Then sampling can be replaced by simple using the first p*m elements of the database.

Partitioning

Approach

1. Divide transactions in 1/p partitions and repeatedly read partitions into main memory
2. Detect in-memory algorithm to find all frequent itemsets with support threshold p*s
3. An itemset becomes a candidate if it is found to be frequent in **at least** one partition
4. On a **second pass**, count all the candidate itemsets and determine which are frequent in the entire set of transactions

With partitioning we are not sampling the dataset, but processing the complete transaction set piece by piece. If an itemset is frequent in one partition (with a correspondingly adapted threshold), then it becomes a candidate. At this stage it is not sure that it is also frequent for the whole transaction set. Therefore in a second pass itemsets that are frequent are determined.

Partitioning – Why it works

Key "monotonicity" idea

- an itemset cannot be frequent in the entire set of transactions unless it is frequent in at least one subset
- Proof: If for all partitions support is below p*s, the total support is less than (1/p) p*s = s!

The second pass is needed, since the condition of being frequent in at least one partition is necessary, but not sufficient.

Partitioning – Distributed Version

Partitioning lends itself to distributed data mining

Transactions distributed among many nodes

- Compute frequent itemsets at each node
- Distribute candidates to all nodes
- Accumulate the counts of all candidates

MapReduce implementation!

Partitioning is an embarrassingly parallel algorithm, and can therefore be naturally executed in a distributed environment, e.g. using Map-Reduce

A false negative in sampling can only occur for itemsets with support smaller than …

A. the threshold s
B. p*s
C. p*m
D. None of the above

FP Growth

Frequent itemset discovery without candidate itemset generation

Proceeds in 2 steps:

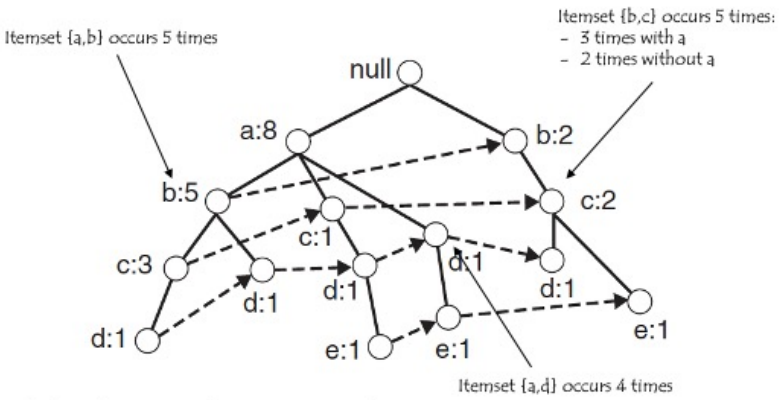
1. Build a data structure, called the FP-tree
   – Requires 2 passes over the dataset
2. Extract frequent itemsets directly from the FP-tree

Though Apriori is as the original method the most popular rule mining algorithm, and also frequently used, other algorithms have been devised that attempt to provide better performance (under certain circumstances). FP Growth is one example of such an algorithm that is mainly aiming at main memory implementations (and thus less suitable for distributed implementation).

FP-Tree Data Structure

Transaction Data Set

TID	Items
1	{a,b}
2	{b,c,d}
3	{a,c,d,e}
4	{a,d,e}
5	{a,b,c}
6	{a,b,c,d}
7	{a}
8	{a,b,c}
9	{a,b,d}
10	{b,c,e}



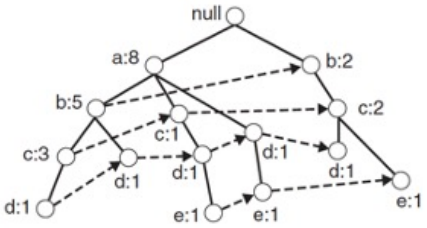
- Items are sorted by decreasing support
- Nodes correspond to single items
- Counters indicate frequency of itemset from root to node
- Different nodes for the same item are connected by a linked list

FP Growth is based on the construction of a data structure, called FP-Tree. The core structure of and FP-Tree as similar to a trie. In order to construct the FP-Tree, first items in the itemsets are sorted. The nodes in the tree correspond to items, and paths from the root correspond to itemsets. Itemsets sharing the same prefix, share also the same path prefix in the tree. Counters at the nodes indicate how frequent the itemset corresponding to the path from the root to the node is. If we consider the left most path, we see that item a occurs 8 times, itemset ab 5 times, abc 3 times, and abcd once. Since the same item can occur in the tree in multiple places, these different occurrences are connected as linked list. For itemsets that occur in different paths (e.g. bc, which occurs together with a and without a), the frequencies can be computed by following the linked lists of the last item in the itemset and testing whether the other items are present in the path.

FP-Tree: Item Sorting

Transaction Data Set

TID	Items
1	{a,b}
2	{b,c,d}
3	{a,c,d,e}
4	{a,d,e}
5	{a,b,c}
6	{a,b,c,d}
7	{a}
8	{a,b,c}
9	{a,b,d}
10	{b,c,e}



Item frequencies
a: 8
b: 7
c: 6
d: 5
e: 3

FP-Tree Size: The FP-Tree is more compact the more common prefixes the itemsets share

- Sorting items by decreasing support increases this probability

Sorting items in itemsets by decreasing support is an important heuristics to keep the FP-Tree compact, as more frequent items tend to be on the top of the tree which increases the likelihood that different itemsets share a common prefix. In the example we had sorted the items this way. If it is not the case they are resorted according to their frequencies.

Step 1: FP-Tree Construction

Requires 2 passes over the dataset

Pass 1: Compute support for each item

- Pass over the transaction set
- Sort items in order of decreasing support

Pass 2: Construct the FP-Tree data structure

- Tree is expanded one itemset at a time

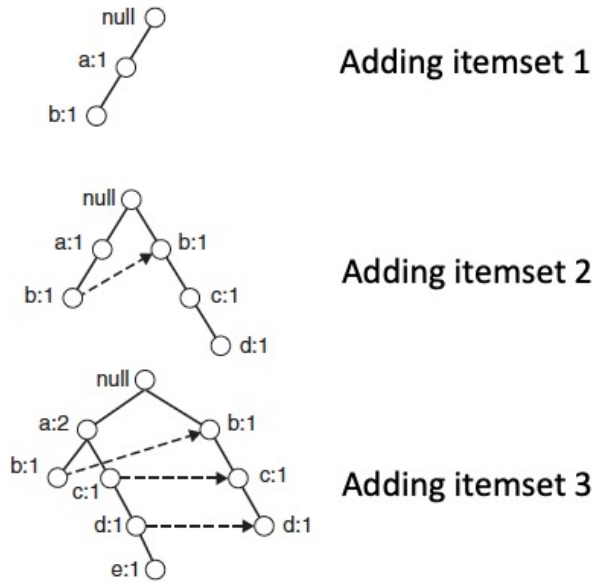
To construct the FP-Tree one proceeds in two passes (not to confuse with the two steps of the overall FP-Growth algorithm mentioned in the beginning). In a first pass the transactions are scanned to compute the support for each single item. The items are then sorted in decreasing order. This order is being used to sort itemsets as required by the specification of the FP-Tree.

In the second pass the FP-Tree structure itself is constructed by adding one itemset at a time.

Step 1: FP-Tree Construction

Transaction Data Set

TID	Items
1	{a,b}
2	{b,c,d}
3	{a,c,d,e}
4	{a,d,e}
5	{a,b,c}
6	{a,b,c,d}
7	{a}
8	{a,b,c}
9	{a,b,d}
10	{b,c,e}

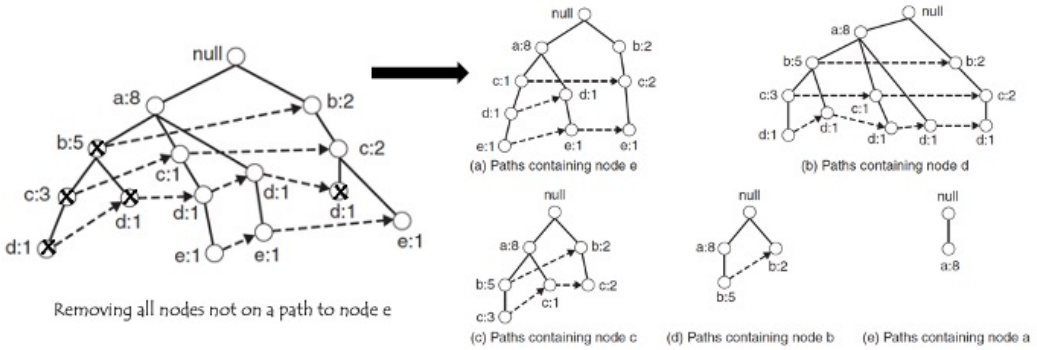


Constructing the FP-Tree proceeds in a way analogous to constructing a trie structure. The items in the itemset are processed in their order. If for the current item there exists already a node, no new node is created. If it does not exist, a new branch is created for the items. The number of occurrences is updates by 1 at each node that is traversed (since each subset of the itemset is also an itemset). Finally links are introduced between nodes with the same labels.

Step 2: Frequent Itemset Extraction

Bottom-up approach

– For each item extract the tree with paths ending in the item



In the second step frequent itemsets are extracted. For each item first a subtree is extracted from the full FP-Tree that contains only the paths with the item. For extracting those subtrees the linked lists are helpful. One needs to traverse just the corresponding list and retain the paths from the traversed nodes to the root.

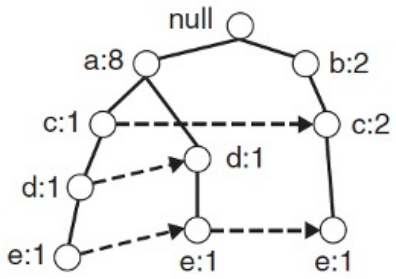
The figures illustrates the five resulting subtrees.

Step 2: Divide and Conquer Strategy

Start with item with lowest support (e.g. item *e*) and extract it's tree

Check whether the item has sufficient support (e.g. 2)

- Add up counts along the list of the item
- E.g. item *e* has support 3



Then start processing for the item with the lowest support. First we verify that the item has sufficient support. This we can simply do by following the list and adding up all the counts found. In the example, item e would this have support 3. If the support is above threshold processing proceeds, otherwise we know that no itemset containing e has sufficient support.

Step 2: Divide and Conquer Strategy

If item has sufficient support, check for itemsets ending in the item

- Example: if item *e* is frequent check whether itemsets *de, ce, be, ae* are frequent
- Again in order of lowest support

In order to check whether these itemsets are frequent compute a **conditional FP-Tree**

If we find that the item at hand (e.g. item e) has sufficient support, then we check next whether the itemsets consisting of two items and ending in the item have also sufficient support. For doing so we derive now a conditional FP-Tree from the tree that we have constructed before for the item. The conditional tree is constructed in three steps:

- First the support counts in the tree are updated such that only the number of itemsets containing the current item is considered. This can be simply performed traversing the paths from the bottom to the root.
- Then the nodes corresponding to the current item are removed
- Finally all nodes that have an insufficient support count are also removed from the conditional FP Tree

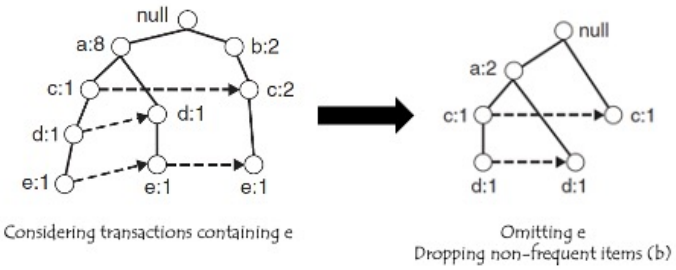
Conditional FP-Tree

The FP-Tree that would be built if we only consider transactions containing a particular itemset and omitting this itemset (e.g. {e})

- Removing that itemset from the tree and dropping items that are not frequent

Transaction Data Set

TID	Items
1	{a,b}
2	{b,c,d}
3	{a,c,d,e}
4	{a,d,e}
5	{a,b,c}
6	{a,b,c,d}
7	{e}
8	{a,b,e}
9	{a,b,d}
10	{b,c,e}



Considering transactions containing e

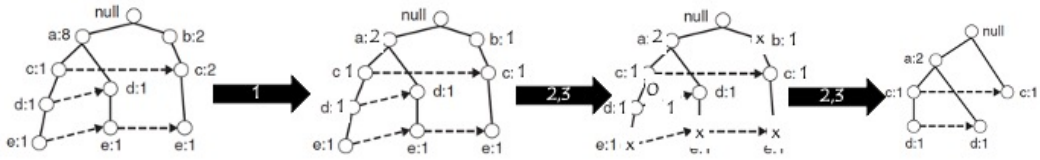
Omitting e
Dropping non-frequent items (b)

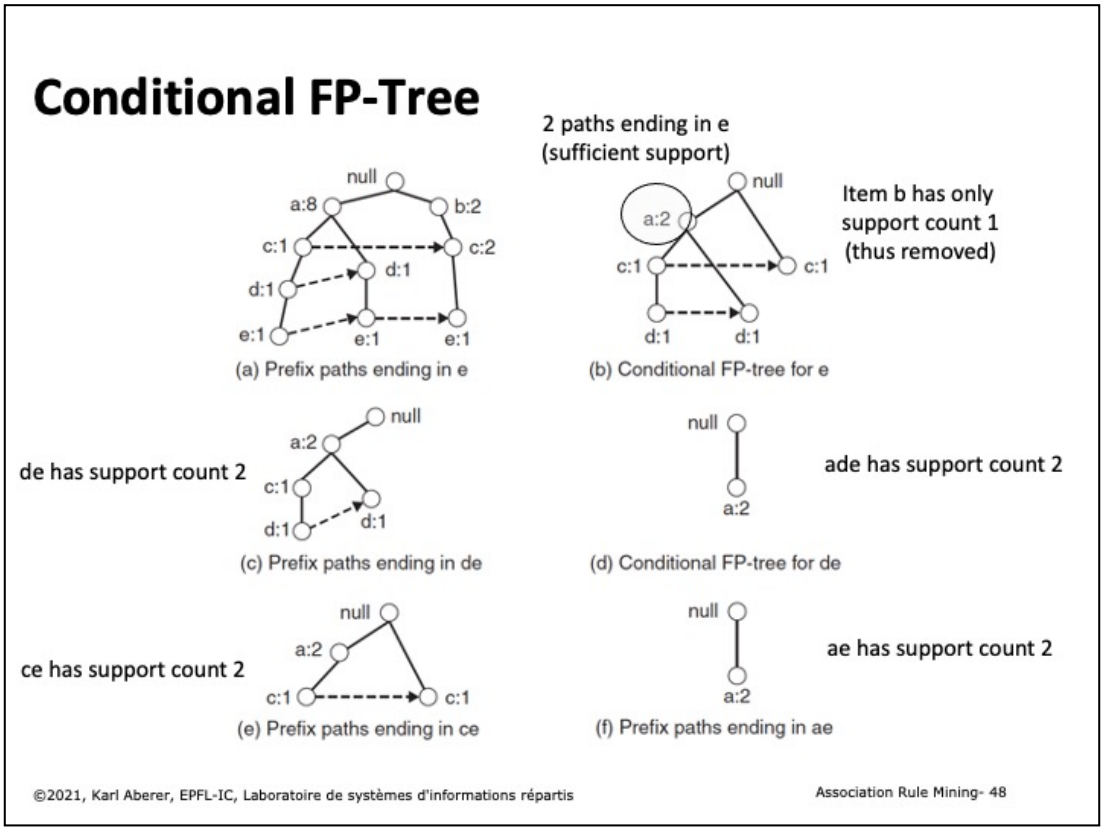
One way to understand the construction of the conditional FP Tree is that it is the FP Tree that would be constructed when we only consider the transactions that consider the curent item (or itemset) and then remove e and any non-frequent item from those itemsets. In the example this would be item e that we consider, and item b would be eliminated, since not frequent in the remaining transactions.

Deriving Conditional FP-Tree

The conditional FP-Tree can be derived from extracted tree

1. Update support counts to itemsets containing the item
2. Remove the nodes of the item
3. Remove nodes with insufficient support count





Here we show all conditional FP Trees that would be constructed from 2-itemsets ending in item e. Once the conditional FP Trees have been constructed, we can read off the support for all 2-itemsets, by traversing the lists at the leaf level (as we did first for item e itself). Once this step is finished, the algorithm continues in the same way for the remaining 2-itemsets. For example, for itemset de we would now construct a conditional FP Tree, by first extracting from tree (b) the tree with paths ending in de, resulting in tree (c), and then extracting the conditional FP Tree for itemset de (tree (d)).

Result

Frequent itemsets detected in this order

Suffix	Frequent Itemsets
e	{e}, {d,e}, {a,d,e}, {c,e},{a,e}
d	{d}, {c,d}, {b,c,d}, {a,c,d}, {b,d}, {a,b,d}, {a,d}
c	{c}, {b,c}, {a,b,c}, {a,c}
b	{b}, {a,b}
a	{a}

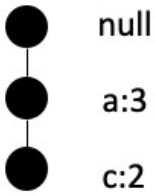
This table summarizes the frequent itemsets in the order they are detected, i.e., starting from the items with the lowest support.

In the first pass over the database of the FP Growth algorithm

A. Frequent itemsets are extracted
B. A tree structure is constructed
C. The frequency of items is computed
D. Prefixes among itemsets are determined

The FP tree below is …

A. not valid, b is missing
B. not valid, since count at leaf level larger than 1
C. possible, with 2 transactions {a}
D. possible, with 2 transactions {a,c}



Summary FP Growth

Advantages

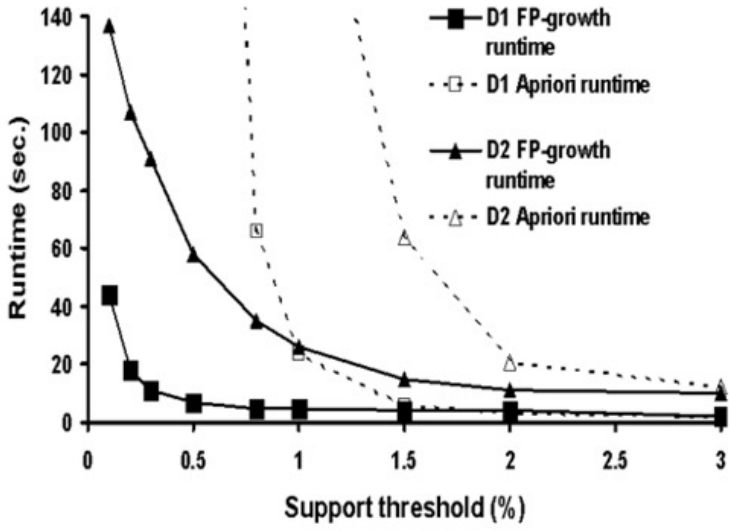
- Only 2 passes over the dataset
- Compresses dataset
- (Generally) much faster than Apriori

Disadvantages

- Works less efficiently for high support thresholds
- Has to run in main memory
- Difficult to find distributed implementation

FP Growth Works less efficiently for high support thresholds, because it prunes only single items.

Performance Comparison



Components of Data Mining Algorithms

1. Pattern structure/model representation
   - **association rules**
2. Scoring function
   - **support, confidence**
3. Optimisation and search
   - **JOIN, PRUNE**
   - **FP-Tree, ordering of items**
4. Data management
   - **transaction reduction, partitioning, sampling**

References

Textbook

- http://www.mmds.org/mmds/v2.1/ch06-assocrules.pdf
- Jiawei Han, *Data Mining: concepts and techniques*, Morgan Kaufman, 2000, ISBN 1-55860-489-8
- Pang-Ning Tan, Michael Steinbach, Vipin Kumar: *Introduction to Data Mining*, Addison-Wesley. Chapter 6: Association Analysis: Basic Concepts and Algorithms

Some relevant research literature

- R. Agrawal, T. Imielinski, and A. Swami. *Mining association rules between sets of items in large databases*. SIGMOD'93, 207-216, Washington, D.C.
- J. Han, J. Pei and Y. Yin. *Mining Frequent patterns without candidate generation*. SIGMOD 2000, 1–12.