Control System Design

DC motor and brake

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Introduction

Context

We have been tasked to design a numeric controller for a given system which has to meet some requirements. The system is called *DC motor and brake* and it consists of two current controlled motors connected to a brake trough a shaft.

Our objective is to send the best signal to the motors to ensure that:

- the shaft can reach any reasonnable speed in less than half a second
- the shaft's speed in maintained as smooth as possible
- the system can reject a step disturbance as fast as possible
- the shaft's speed can follow a feasible speed reference of $4Hz$ from any feasible speed

In addition to this, we could implement a strategy to reject various brake disturbances profile and magnitude, while maintaining the speed variations as smooth as possible.

Plant description

As explained in the introduction, the plant consists of two motors and one brake acting on the same shaft. There are three sensors available: one speed sensor and two position sensors.

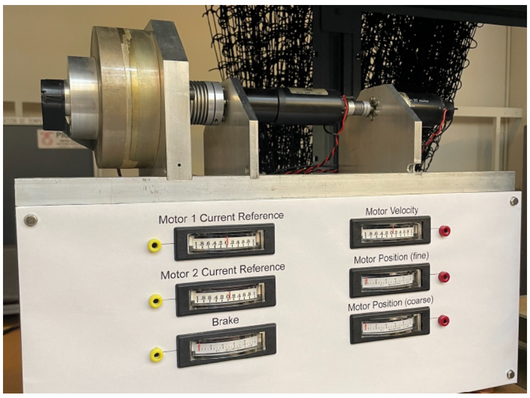

Figure 1: Picture of the plant

1 System identification

Based on the mechanical equations of the DC motor, the form of the expected transfer function was already known:

$$G(s) = \frac{A_0}{\tau s + 1} \tag{1.1}$$

Because it has no pole in 0 (or equivalently it is a non-integrator system), the response of the system to a step command is enough to find both A_0 and τ.

As the plant is not perfect (the motors have some friction variations depending on the temperature, there is noise, ...), the objective is to establish a linearized model of it. In the following section, the determination of a transfer function for motor 1 will be explained in detail. This transfer function will link the speed of the shaft to the voltage applied to motor 1. We only study the speed and not the position because all of the requirements are on the speed, meaning that there is no point of studying the position.

1.1 Step response

The starting point of the step response experiment is the static characteristic of the motor:

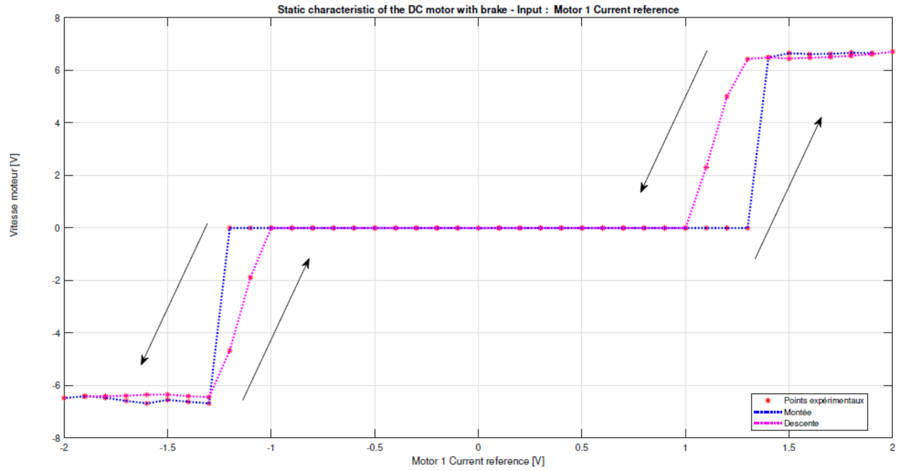

Figure 1.1: Static characteristic of the system driven by the motor 1

By analyzing figure 1.1, a first obvious deduction to be made is that the velocity cannot reach a value greater than approximately $6.5V$[1] (and $-6.5V$ is minimum value). This clarifies the use of " *feasible*" in the requirements as a speed of $7V$ could never been achieved, no matter what the input is.

What is really interesting to understand is that the static characteristic gives the speed of the shaft after a sufficient amount of time (in theory, after an infinite amount of time bt in practice, once the transient response vanishes). By looking at it, it is clear that to have a speed that is different from $0V$ and that is outside of the saturated region, a high command should be send to reach saturation and it must then be lowered to a point where the speed is no longer saturating (a command between $1.1V$ and $1.3V$ approximately).

With this in mind, the following step response has been recorded:

[1] Any speed will be expressed in volts (V) as speed is measured by a sensor that returns a voltage

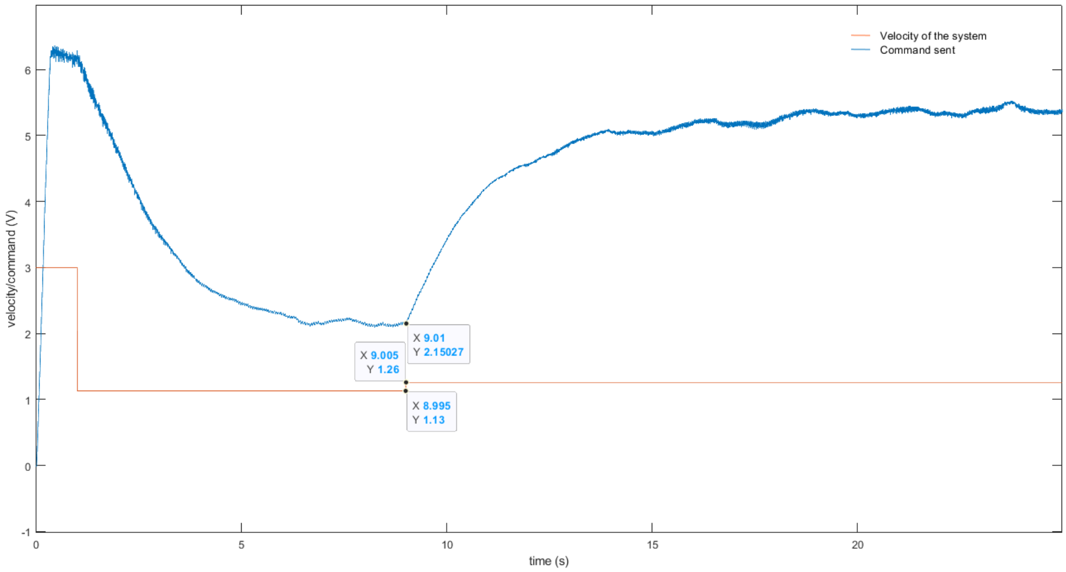

Figure 1.2: Step response of the system when the motor 1 has a step as command

Where the command is first set at a high enough voltage ($\geq 1.5V$, based on 1.1). After the velocity saturates, it lowers to a value where the velocity will stabilize ($1.13V$). This operating point OP_{1+} will correspond to the transfer function $G(s)$ that we are trying to estimate here. Then the step is introduced (with a magnitude of $0.13V$).

$$OP_{1+} = \begin{bmatrix} \text{command} = 1.13V \\ \text{velocity} = 2.15V \end{bmatrix} \tag{1.2}$$

With an coordinates change for easier visualization, the data plot 1.3 is used for the determination of A_0 and τ. It is indeed known that for a transfer function in the form of 1.1, the parameter A_0 is equal to the asymptotic value of the step response divided by the amplitude of the step. τ on the other hand is equal to the time after which the step response reaches $1/e$ (≈ 0.63) times the asymptotic step response. This gives as transfer function:

$$G_{1+}(s) = \frac{24.88}{1.915s + 1} \tag{1.3}$$

The name $G_{1+}(s)$ has been chosen because the "1" indicates that it corresponds to the motor 1 and the "+" indicates that it is used for a positive speed. As shown on the static characteristic 1.1, the operating point at which the system will be operating to have a negative speed (OP_{1-}) is quite far from OP_{1+}, which means that another model will be needed to describe the behavior of the system in this zone (see discussion in section 1.2).

With the model 1.3, the simulated step response can be computed and as shown in figure 1.3 it can be seen that it matches quite well with the experimental results.

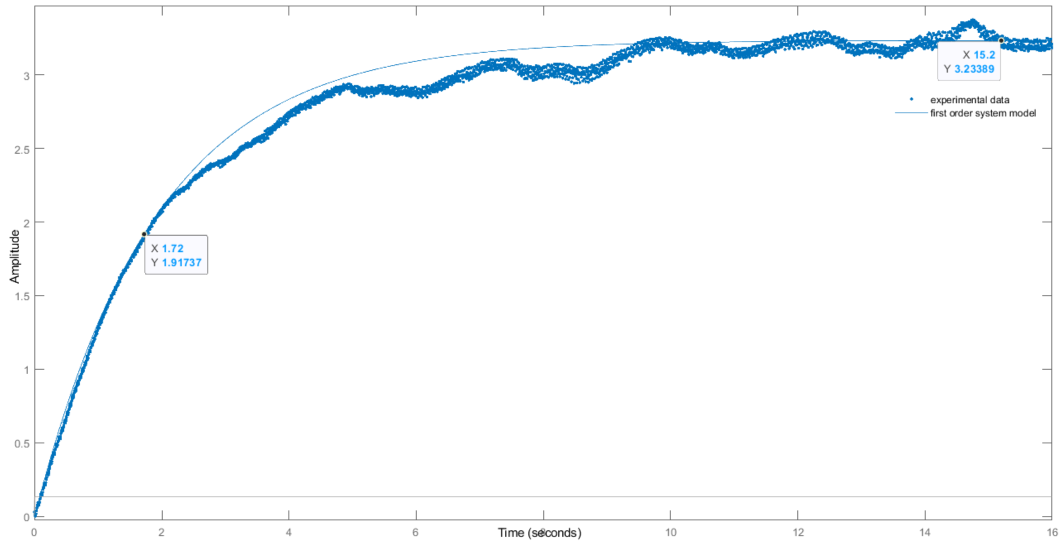

Figure 1.3: Estimation of the step response compared to the real one

1.2 Validation

The estimated transfer function established in section 1 must yet be tested. Indeed, as it is the result of the linearization of the real plant around the equilibrium point OP_{1+} we have to ensure that it has the same behaviour as the plant even when the system is not in OP_{1+}.

The system has been controlled by a step similar to the one in figure 1.2 with slightly modified values. It started at $1.2V$ and the downward step has been chosen with an amplitude of $0.05V$. Figure 1.4 shows the step command with the real plant response and the response computed based on the transfer function 1.3.

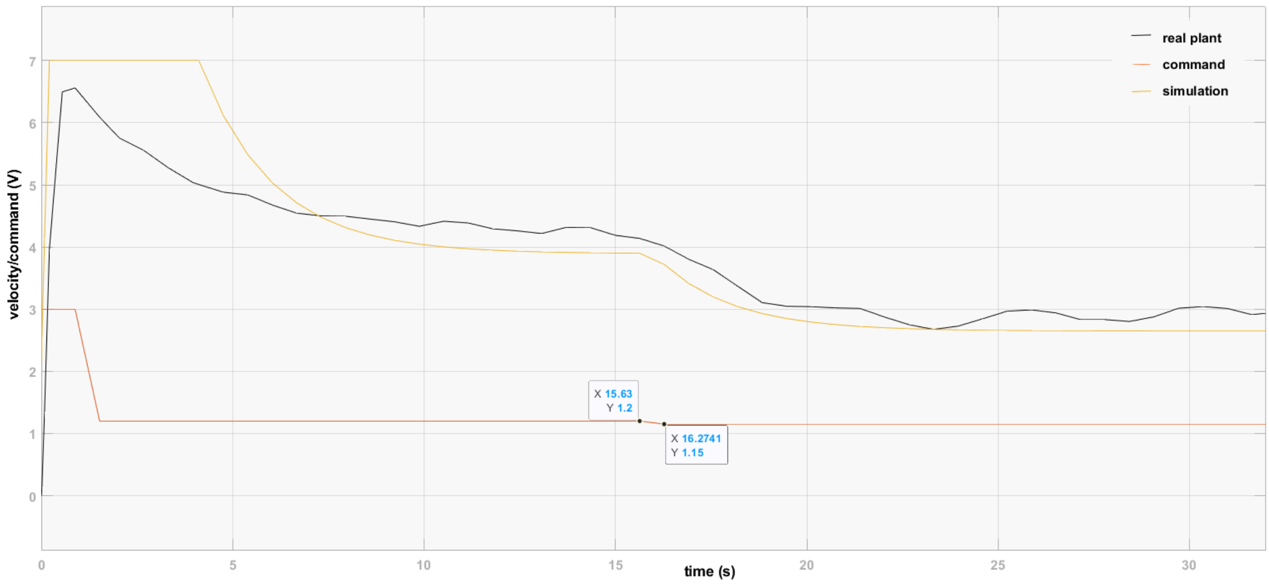

Figure 1.4: Validation of the model $G_{1+}(s)$

It appears quite clearly that the simulated response is not corresponding to the real one. Note that the first part of the response (until $\sim 10s$) is not interesting as the model don't have to take the velocity saturation into account. Based on the observation that the asymptotic value of the velocity was not corresponding between the real plant and the simulation, we tried changing A_0 to get a better matching. With $A_0 = 30$, the model was close to the reality as shown in figure 1.5, which leads to the transfer function:

$$\tilde{G}_{1+}(s) = \frac{30}{1.915s+1} \tag{1.4}$$

The conclusion that can be drawn from this whole section is that depending on the operating point, the transfer function needs to be modified *by a multiplicative factor* but the position of the pole don't change. This means that $G_{1+}(s)$ can be used if we keep in mind that the numerator can vary a little.

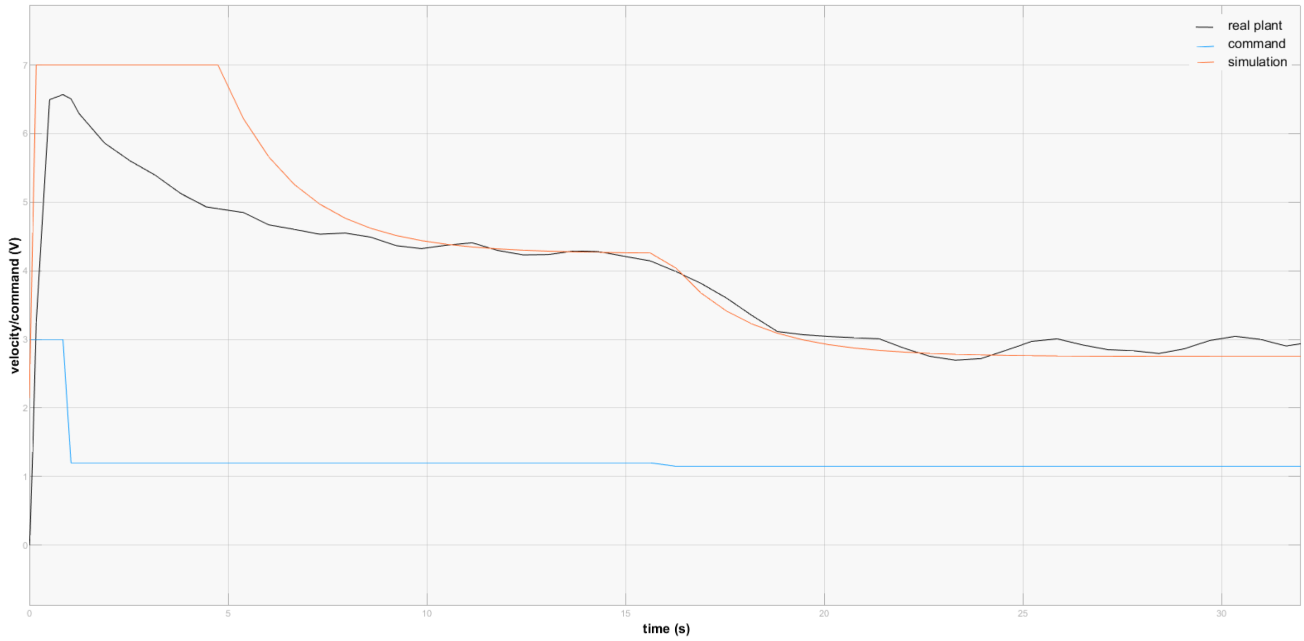

Figure 1.5: Validation of the modified model $\tilde{G}_{1+}(s)$

1.3 Other transfer functions

Based on the same experiment, we determined the other transfer functions[2]:

$$G_{1-}(s) = \frac{A_0}{\tau s + 1} \qquad OP_{1-} = \begin{bmatrix} \text{command} & = & \alpha V \\ \text{velocity} & = & \beta V \end{bmatrix} \tag{1.5}$$

$$G_{2+}(s) = \frac{19.51}{1.7s + 1} \qquad OP_{2+} = \begin{bmatrix} \text{command} & = & 1.1V \\ \text{velocity} & = & 2.4V \end{bmatrix} \tag{1.6}$$

$$G_{2-}(s) = \frac{A_0}{\tau s + 1} \qquad OP_{2-} = \begin{bmatrix} \text{command} & = & \alpha V \\ \text{velocity} & = & \beta V \end{bmatrix} \tag{1.7}$$

[2]With the naming convention mentioned before

2

Design of the controller

2.1 Requirements analysis

The first step of the controller design is to analyse the requirements that have been given in the introduction:

- the shaft can reach any reasonnable speed in less than half a second
- the shaft's speed in maintained as smooth as possible
- the system can reject a step disturbance as fast as possible
- the shaft's speed can follow a feasible speed reference of $4Hz$ from any feasible speed

There is one reference tracking objective, one disturbance rejection objective and two dynamical response properties. The design of the controller will follow the following path:

1. Rejecting a step disturbance (with each motor separately)
2. Following a $4Hz$ reference (with each motor separately)
3. Merge the two controllers into a single digital one
4. Reach any speed in $< 0.5s$

The method used here is to design the controller in there Laplace domain in the first time. Once the continuous time controller has the desired behaviour, it is discretized using the Tustin method (and without forgetting to take the ZOH into account).

2.2 Disturbance rejection

From theory, it is known that a controller is able to asymptotically reject a disturbance if it has at the denominator of its transfer function the denominator of the disturbance.

As a step disturbance is of the form:

$$D(s) = \frac{Ae^{-\tau s}}{s} \tag{2.1}$$

The controller will need a pole in 0. This can be easily done by using a PI controller, which looks like:

$$C_{PI}(s) = k_p \left(1 + \frac{1}{T_i s}\right) \tag{2.2}$$

$$= k_p \left(\frac{T_i s + 1}{T_i s}\right) \tag{2.3}$$

The k_p will be chosen once the dynamic response characteristics will have to be met. However, T_i can already be chosen based on a simple criteria. As the open-loop transfer function equals the product $C_{PI}(s) \times G_{1+}(s)$, a solid choice is to use T_i to cancel the pole in the system. This way, the position of the poles of the closed-loop will be entirely based on the controller.
Of course this will be true in the case of a perfect model but as proved in the section 1.2, the parameter τ of the transfer function (which fixes the pole of it) don't seems to move a lot depending on the operating point. We can conclude from this that the best choice is:

$$T_i = \tau \tag{2.4}$$

for each controller, which leads to:

$$C_{PI}(s) = k_p \left(\frac{\tau s + 1}{\tau s}\right) \tag{2.5}$$

2.3 Reference tracking

For tracking purposes, the method used to reject disturbances can also be applied. Indeed, a reference with $D(s)$ at its denominator will be perfectly followed if the controller denominator contains $D(s)$.

$$R(s) = \mathscr{L}\{A\sin(8\pi t)\} \tag{2.6}$$

$$= \frac{8A\pi}{s^2 + (8\pi)^2} \tag{2.7}$$

A second controller is needed to ensure the tracking of a $4Hz$ sine:

$$C_{\sin}(s) = \frac{1}{s^2 + (8\pi)^2} \tag{2.8}$$

This controller has no parameter that can be moved as we chose a minimalist controller.

2.4 Merging and discretization of the controller

Merging

The total continuous controller is the product of the two previously built controllers because they are put in series, which gives a the following structure:

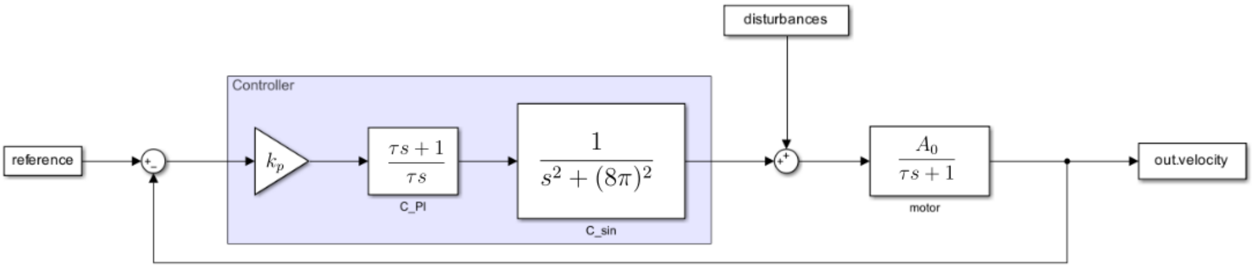

Figure 2.1: Structure of the combined *continuous time* controller

Meaning that the transfer function of the whole controller is:

$$C(s) = k_p \times \frac{\tau s+1}{\tau s} \times \frac{1}{s^2+(8\pi)^2} \tag{2.9}$$

$$= k_p \times \frac{\tau s+1}{\tau s^3+(8\pi)^2 s} \tag{2.10}$$

Discretization

The continuous transfer function is obtained by using the Tustin approximation[1] on the continuous time transfer function of the controller 2.10. This then leads to the recurrence equation which will be implemented in the digital controller.

$$C(z) = k_p \times \frac{(2\tau+T_s)T_s^2 z^3 + \big(2(2\tau+T_s)T_s^2 + (T_s-2\tau)T_s^2\big)z^2 + \big((2\tau+T_s)T_s^2 + 2(T_s-2\tau)T_s^2\big)z + (T_s-2\tau)T_s^2}{\big(8\tau+2(8\pi)^2T_s^2\big)z^3 + \big(-24\tau+2(8\pi)^2T_s^2\big)z^2 + \big(24\tau-2(8\pi)^2T_s^2\big)z + (-8\tau-2(8\pi)^2T_s^2)} \tag{2.11}$$

$$u[n] = -\frac{a_1}{a_0}u[n-1] - \frac{a_2}{a_0}u[n-2] - \frac{a_3}{a_0}u[n-3] + \frac{b_0}{a_0}e[n] + \frac{b_1}{a_0}e[n-1] + \frac{b_2}{a_0}e[n-2] + \frac{b_3}{a_0}e[n-3] \tag{2.12}$$

The following has to be discussed with the TA as it doesn't look useful to me

It is known that the discretization of the system will add a sampler and a holder before the system. This zero order holder (ZOH) has to be taken into account at this step of the design. This implies that the discretized

[1] $s \leftrightarrow \frac{2}{T_s} \times \frac{z-1}{z+1}$

transfer function of the system is not simply the Z-transform of its continuous time counterpart but rather:

$$G(z) = \frac{z}{z-1} \times Z\left\{\mathscr{L}^{-1}\left\{\frac{G(s)}{s}\right\}\right\} \tag{2.13}$$

$$= \frac{z}{z-1} \times Z\left\{1 - e^{-t/\tau}\right\} \tag{2.14}$$

Conclusion