Control System Design

DC motor and brake

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Context

We have been tasked to design a numeric controller for a given system which has to meet some requirements. The system is called *DC motor and brake* and it consists of two current controlled motors connected to a brake trough a shaft.

Our objective is to send the best signal to the motors to ensure that:

- the shaft can reach any reasonnable speed in less than half a second
- the shaft's speed in maintained as smooth as possible
- the system can reject a step disturbance as fast as possible
- the shaft's speed can follow a feasible speed reference of $4Hz$ from any feasible speed

In addition to this, we could implement a strategy to reject various brake disturbances profile and magnitude, while maintaining the speed variations as smooth as possible.

Plant description

As explained in the introduction, the plant consists of two motors and one brake acting on the same shaft. There are three sensors available: one speed sensor and two position sensors.

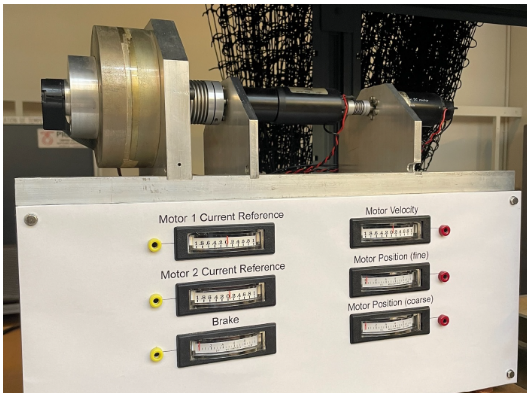

Figure 1: Picture of the plant

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System identification

Based on the mechanical equations of the DC motor, the form of the expected transfer function was already known:

$$G(s) = \frac{A_0}{\tau s + 1} \tag{1.1}$$

Because it has no pole in 0 (or equivalently it is a non-integrator system), the response of the system to a step command is enough to find both A_0 and τ.

As the plant is not perfect (the motors have some friction variations depending on the temperature, there is noise, ...), the objective is to establish a linearized model of it. In the following section, the determination of a model for motor 1 will be explained in detail.

1.1 Step response

The starting point of the step response experiment is the static characteristic of the motor:

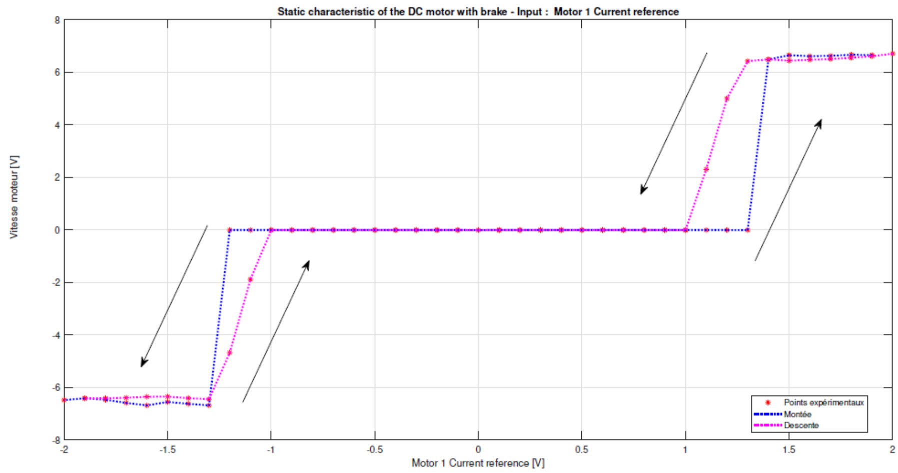

Figure 1.1: Static characteristic of the system driven by the motor 1

By analyzing figure 1.1, a first obvious deduction to be made is that the velocity cannot reach a value greater than approximately $6.5V$[1] (and $-6.5V$ is minimum value). This clarifies the use of " *feasible*" in the requirements as a speed of $7V$ could never been achieved, no matter what the input is.

What is really interesting to understand is that the static characteristic gives the speed of the shaft after a sufficient amount of time (in theory, after an infinite amount of time bt in practice, once the transient response vanishes). By looking at it, it is clear that to have a speed that is different from $0V$ and that is outside of the saturated region, a high command should be send to reach saturation and it must then be lowered to a point where the speed is no longer saturating (a command between $1.1V$ and $1.3V$ approximately).

With this in mind, the following step response has been recorded:

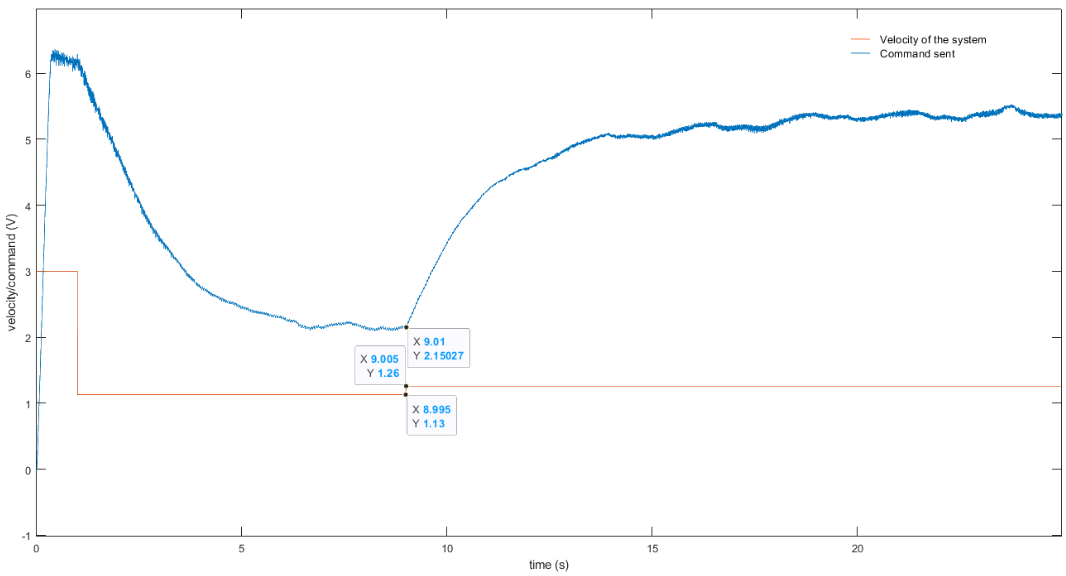

Figure 1.2: Step response of the system when the motor 1 has a step as command

Where the command is first set at a high enough voltage ($\geq 1.5V$, based on 1.1). After the velocity saturates, it lowers to a value where the velocity will stabilize ($1.13V$). This operating point OP_{1+} will correspond to the transfer function $G(s)$ that we are trying to estimate here. Then the step is introduced (with a magnitude of $0.13V$).

$$OP_{1+} = \begin{bmatrix} \text{command} = 1.13V \\ \text{velocity} = 2.15V \end{bmatrix} \tag{1.2}$$

With an coordinates change for easier visualization, the data plot 1.3 is used for the determination of

[1]Any speed will be expressed in volts (V) as speed is measured by a sensor that returns a voltage

A_0 and τ. It is indeed known that for a transfer function in the form of 1.1, the parameter A_0 is equal to the asymptotic value of the step response divided by the amplitude of the step. τ on the other hand is equal to the time after which the step response reaches $1/e$ (≈ 0.63) times the asymptotic step response. This gives as transfer function:

$$G_{1+}(s) = \frac{24.88}{1.915s + 1} \tag{1.3}$$

The name $G_{1+}(s)$ has been chosen because the "1" indicates that it corresponds to the motor 1 and the "+" indicates that it is used for a positive speed. As shown on the static characteristic 1.1, the operating point at which the system will be operating to have a negative speed (OP_{1-}) is quite far from OP_{1+}, which means that another model will be needed to describe the behavior of the system in this zone (see discussion in section 1.2).

With the model 1.3, the simulated step response can be computed and as shown in figure 1.3 it can be seen that it matches quite well with the experimental results.

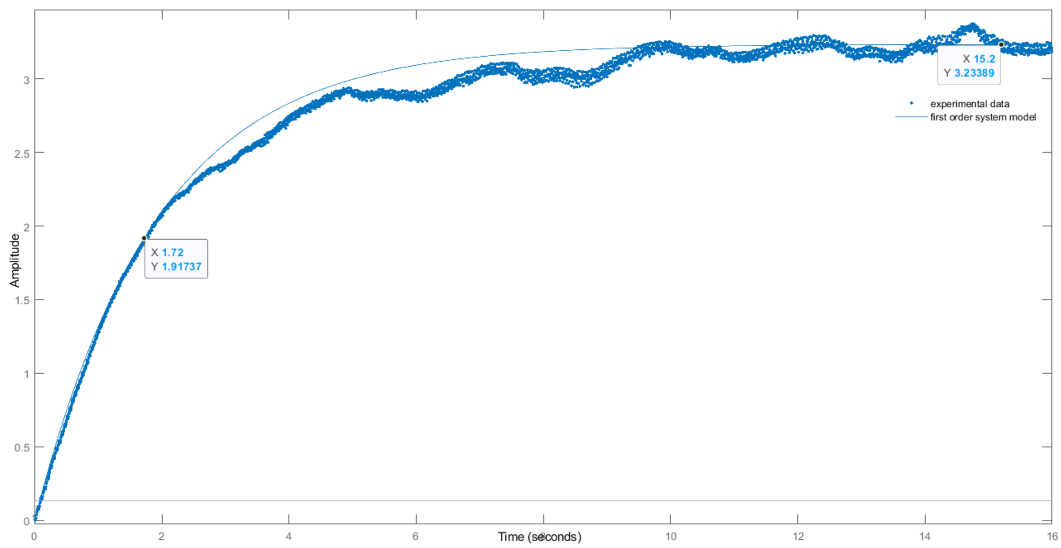

Figure 1.3: Estimation of the step response compared to the real one

1.2 Validation

The estimated transfer function established in section 1 must yet be tested. Indeed, as it is the result of the linearization of the real plant around the equilibrium point OP_{1+} we have to ensure that it has the same behaviour as the plant even when the system is not in OP_{1+}.

The system has been controlled by a step similar to the one in figure 1.2 with slightly modified values. It started at $1.2V$ and the downward step has been chosen with an amplitude of $0.05V$. Figure 1.4 shows the step command with the real plant response and the response computed based on the transfer function 1.3.

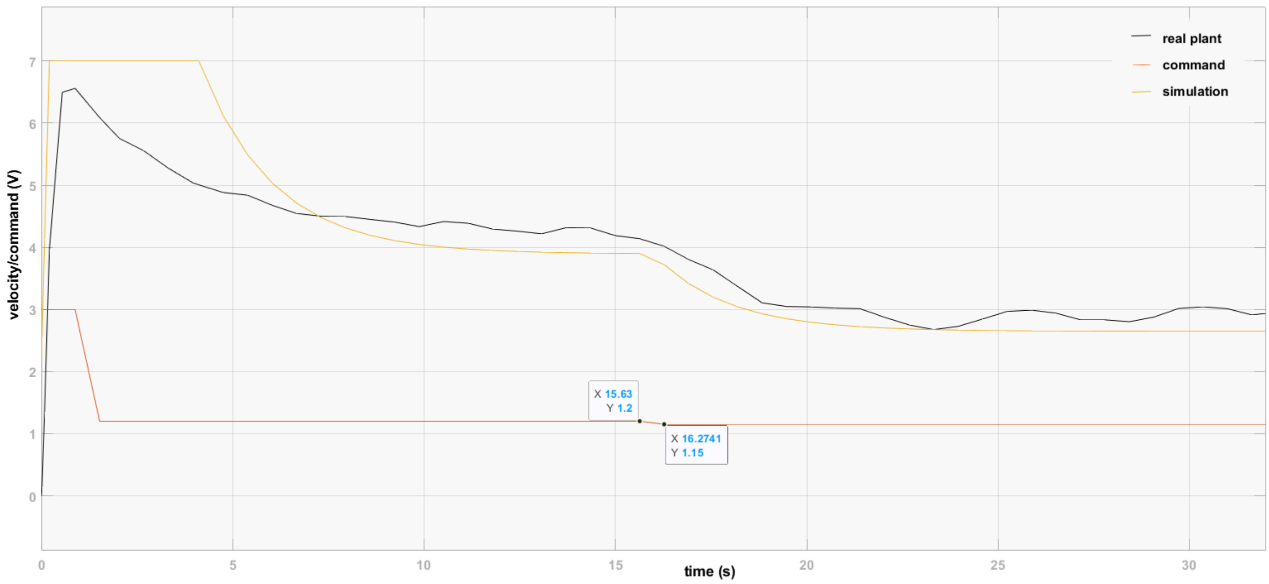

Figure 1.4: Validation of the model $G_{1+}(s)$

It appears quite clearly that the simulated response is not corresponding to the real one. Note that the first part of the response (until $\sim 10s$) is not interesting as the model don't have to take the velocity saturation into account. Based on the observation that the asymptotic value of the velocity was not corresponding between the real plant and the simulation, we tried changing A_0 to get a better matching. With $A_0 = 30$, the model was close to the reality as shown in figure 1.5, which leads to the transfer function:

$$\tilde{G}_{1+}(s) = \frac{30}{1.915s + 1} \tag{1.4}$$

The conclusion that can be drawn from this whole section is that depending on the operating point, the transfer function needs to be modified *by a multiplicative factor* but the position of the pole don't change. This means that $G_{1+}(s)$ can be used if we keep in mind that the numerator can vary a little.

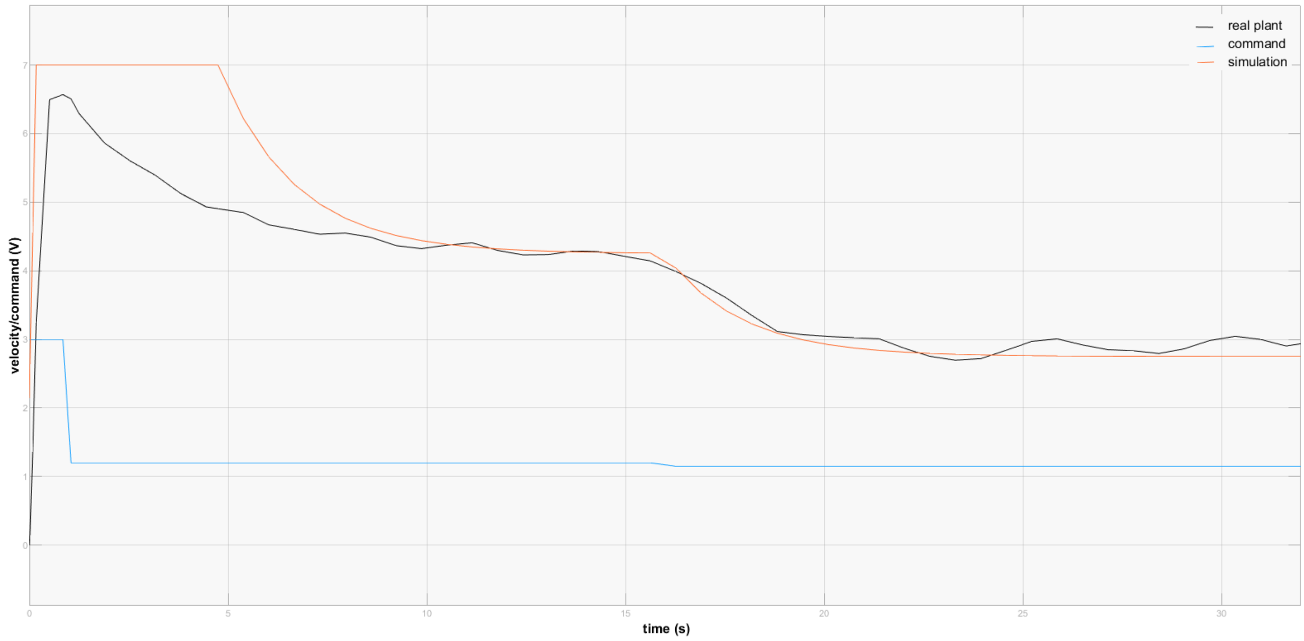

Figure 1.5: Validation of the modified model $\tilde{G}_{1+}(s)$

1.3 Other transfer functions

Based on the same experiment, we determined the other transfer functions[2]:

$$G_{1-}(s) = \frac{A_0}{\tau s + 1} \qquad OP_{1-} = \begin{bmatrix} \text{command} & = & \alpha V \\ \text{velocity} & = & \beta V \end{bmatrix} \tag{1.5}$$

$$G_{2+}(s) = \frac{19.51}{1.7s + 1} \qquad OP_{2+} = \begin{bmatrix} \text{command} & = & 1.1V \\ \text{velocity} & = & 2.4V \end{bmatrix} \tag{1.6}$$

$$G_{2-}(s) = \frac{A_0}{\tau s + 1} \qquad OP_{2-} = \begin{bmatrix} \text{command} & = & \alpha V \\ \text{velocity} & = & \beta V \end{bmatrix} \tag{1.7}$$

[2]With the naming convention mentioned before

Conclusion