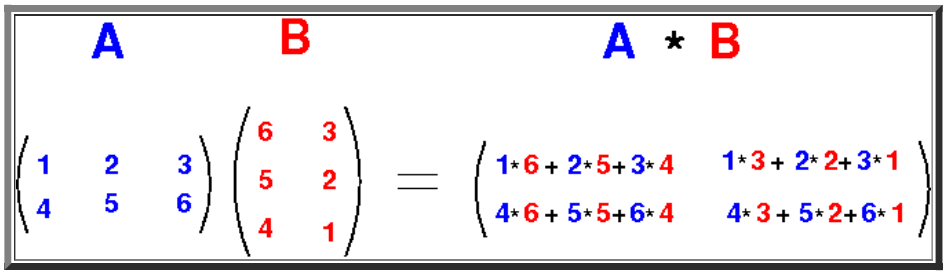
MapReduce Algorithm for Matrix Multiplication

- **Matrix Multiplication**
  - From **high school calculus**:

$$A \times B = C$$

$$c_{ij} = \sum_{k=1,2,\ldots,n} a_{ik} \times c_{kj}$$

  - **Example:**



- **The reduce( ) step in the MapReduce Algorithm for matrix multiplication**
  - **Facts:**

1. The ***final*** **step** in the **MapReduce** algorithm is to produce the **matrix A × B**

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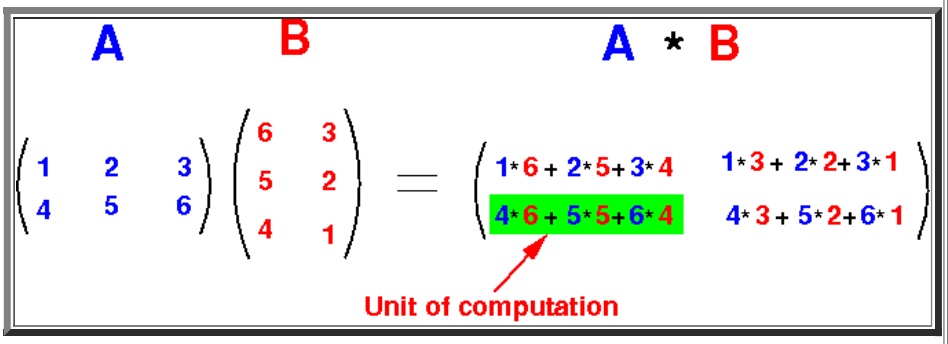
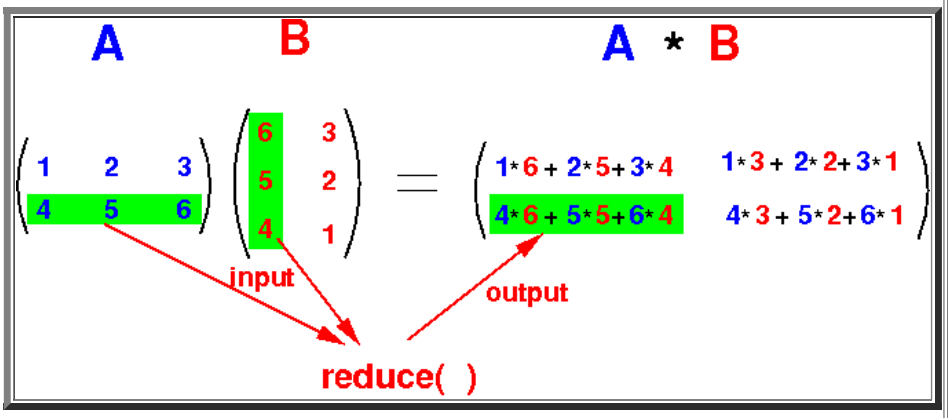
2. The **unit** of **computation** of of **matrix A × B** is ***one*** **element** in the **matrix**:



  - **Conclusion:**

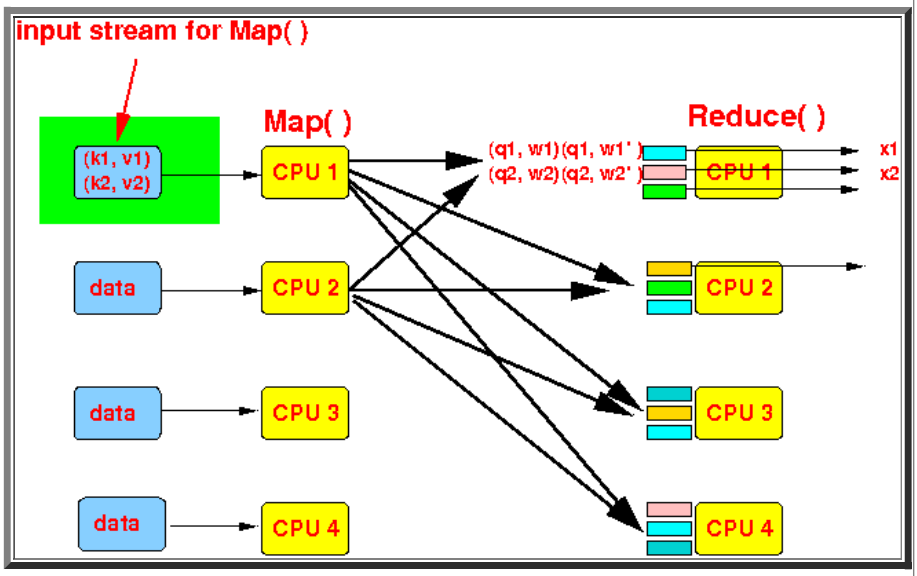
- The ***input*** **information** of the **reduce( )** step (function) of the **MapReduce algorithm** are:



1. **One row vector** from **matrix A**
2. **One column vector** from **matrix B**

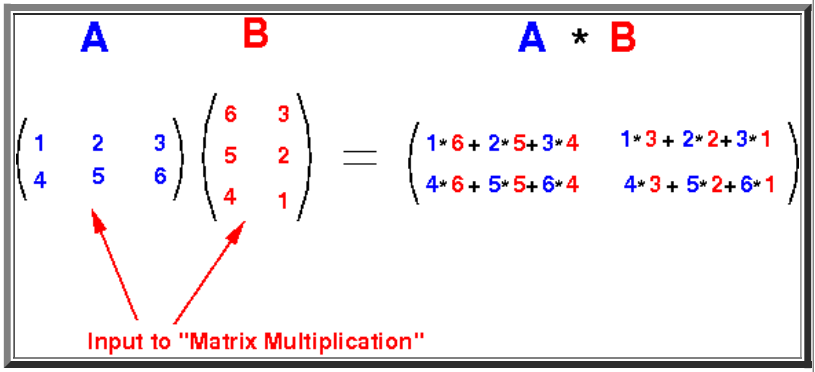
- The `reduce( )` function will **compute**:
  - The ***inner* product** of the
    - **One row vector** from **matrix A**
    - **One column vector** from **matrix B**

- **Preprocessing for the `map( )` function**
  - **Fact:**
    - The `map( )` function (really) ***only*** has ***one* input stream**:



of the **format** (key_i , $value_i$)

  - The **inputs** of the **matrix multiplication** are:
    - ***Tow* (2)** input **matrices**:



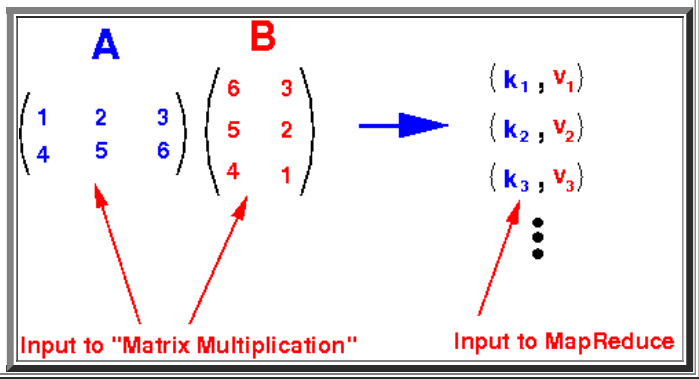
  - **Therefore:**

- We must **insert** a ***pre-processing*** **step** to:

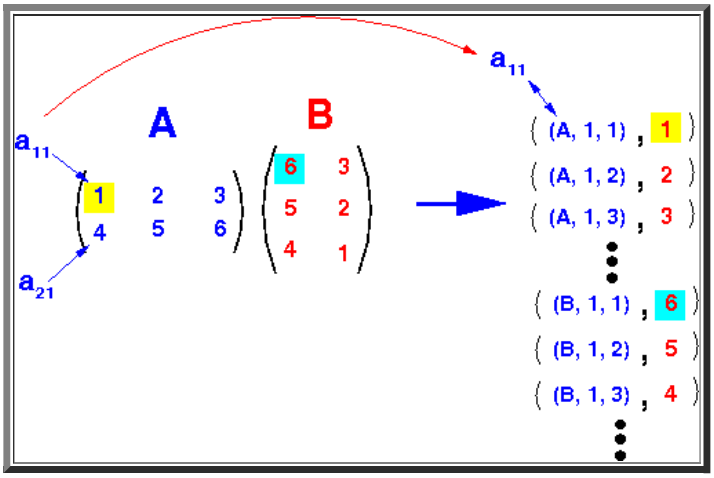
  - **Convert** the **input matrices** to the **form**:

    ```
    ( key1 , value1 )
    ( key2 , value2 )
    ( key3 , value3 )
    ...
    ```

  Graphically:



- **Pre-processing** **used** for **matrix multiplication**:



- **Overview of the MapReduce Algorithm for Matrix Multiplication**
  - **So far**, we have **discovered**:

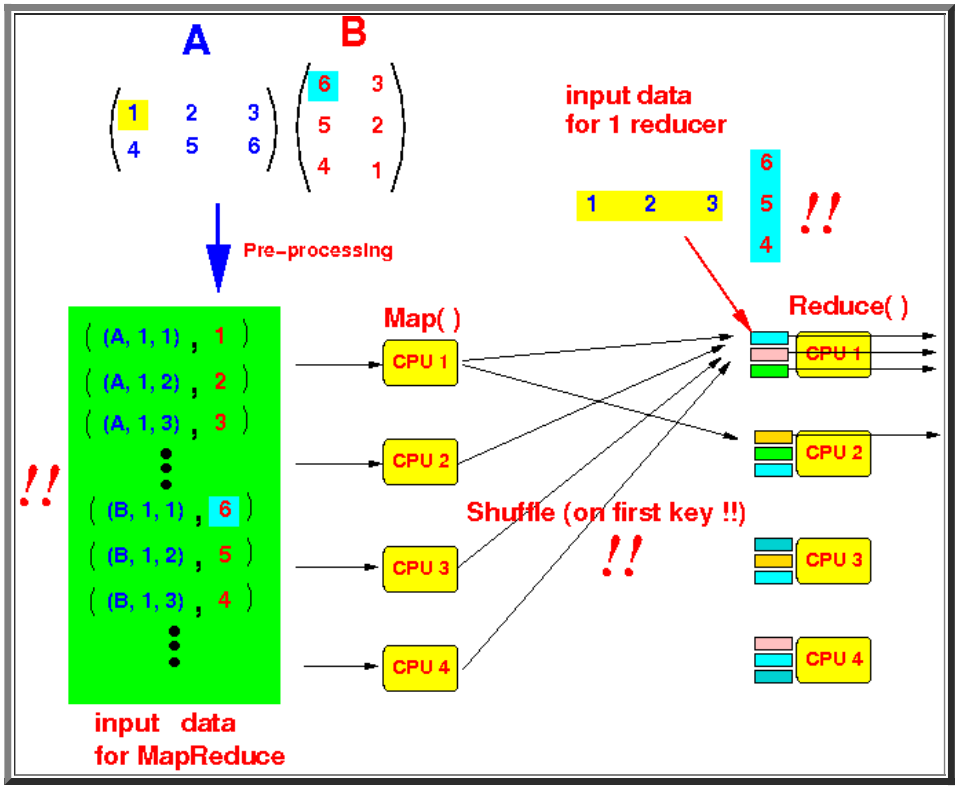
    - The **input** to the **Map( )** is as **follows**:

      ```
      ( (A, 1, 1) , a11 )
      ( (A, 1, 2) , a12 )
      ( (A, 1, 3) , a13 )
      ...
      ( (B, 1, 1) , b11 )
      ( (B, 1, 2) , b12 )
      ( (B, 1, 3) , b13 )
      ...
      ```

    - The **input** to ***one*** **reduce( )** function is as **follows**:

      - A **row vector** from **matrix A**
      - A **column vector** from **matrix B**

- **Graphical summary:**



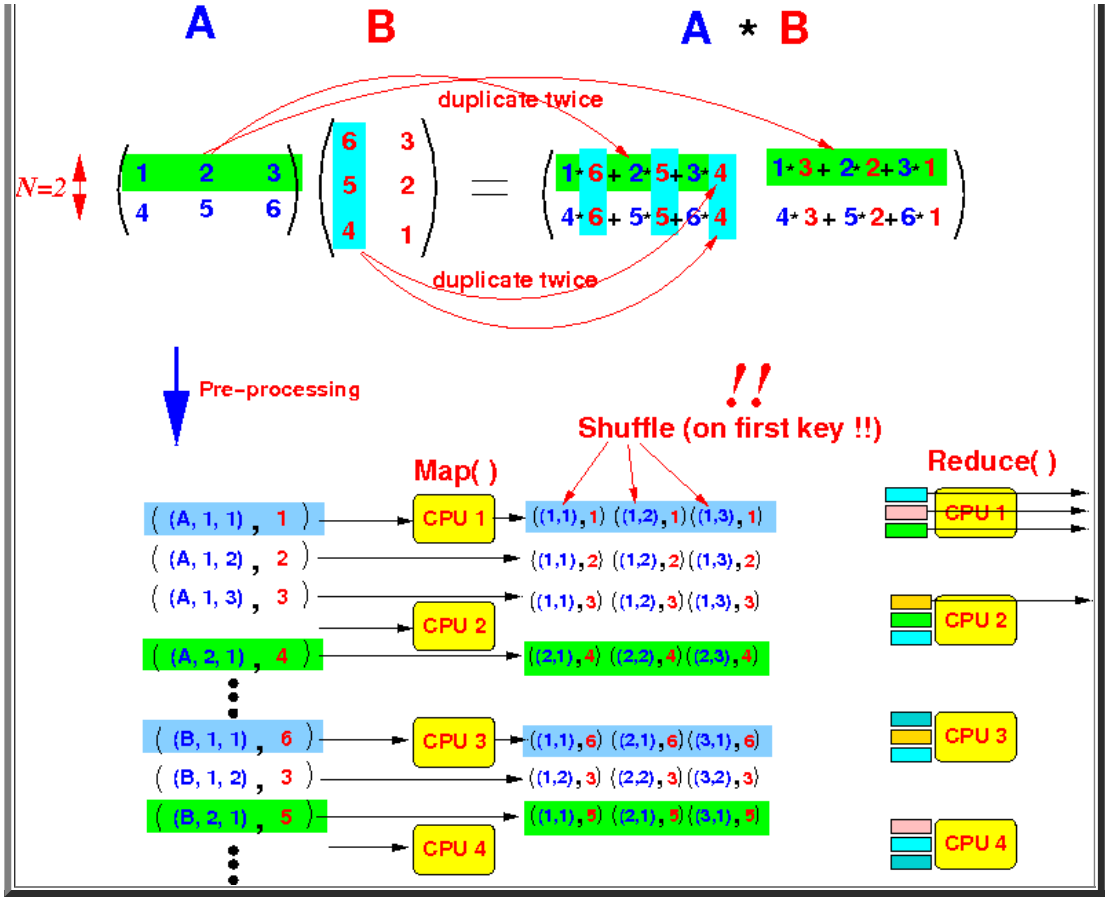
- **The MapReduce Algorithm for Matrix Multiplication**
  - The `map( )` function:

    - The `map( )` will **duplicate** ***N*** **times** as **follows**:

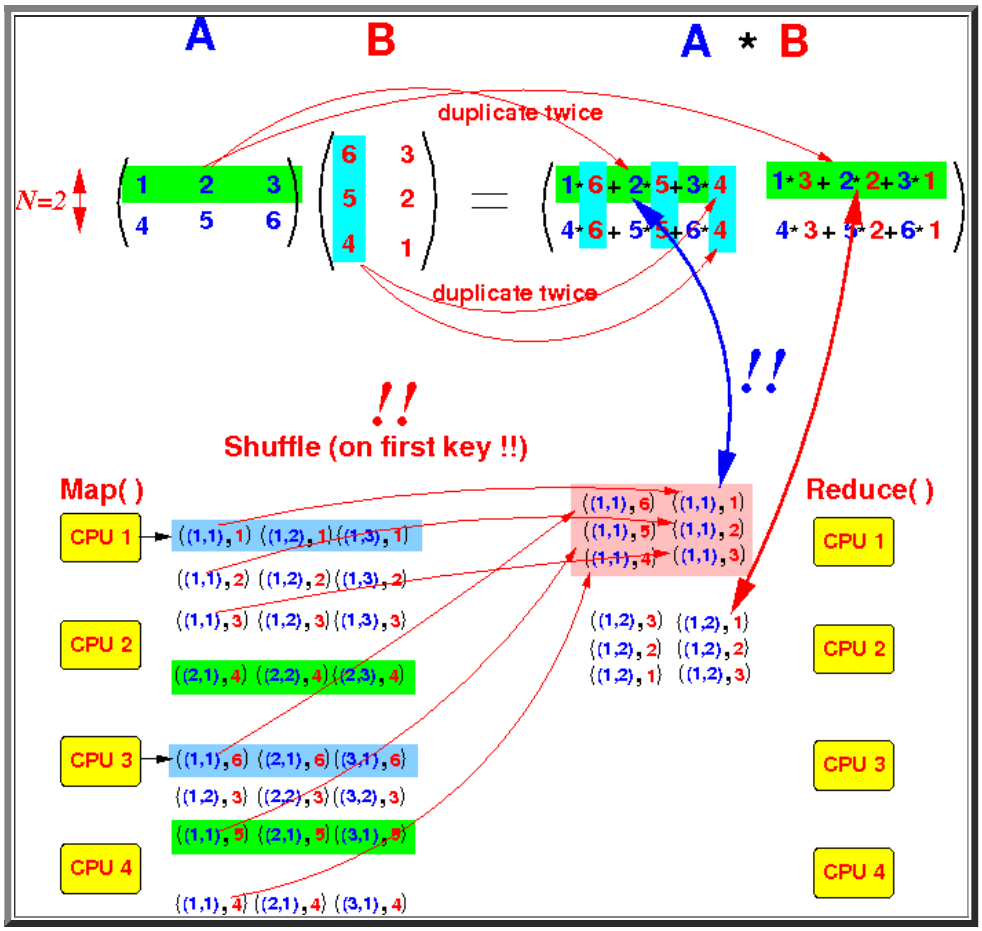
      ```
      (A, i, j), x) --->  ( (i,1), x )  ( (i,2), x ) ....  ( (i,N), x )
      (B, i, j), x) --->  ( (1,j), x )  ( (2,j), x ) ....  ( (N,j), x )
      ```

      wher ***N*** = # **rows** in **matrix A** (= # **columnss** in **matrix B**)

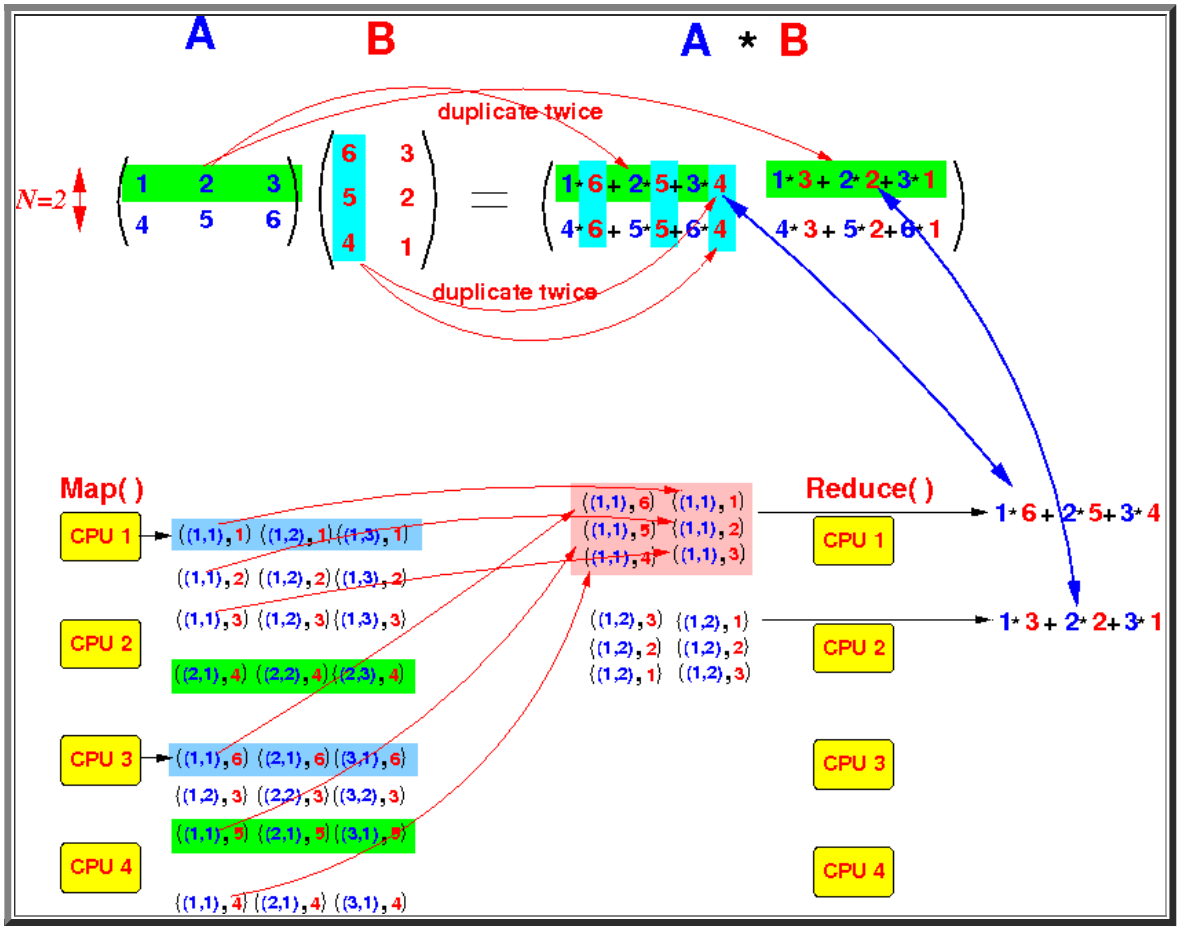
    Example:



- The ***shuffle*** **mechanism** of **MapReduce** will ***re-organize*** **(group)** the `map( )` **output** as **follows**:



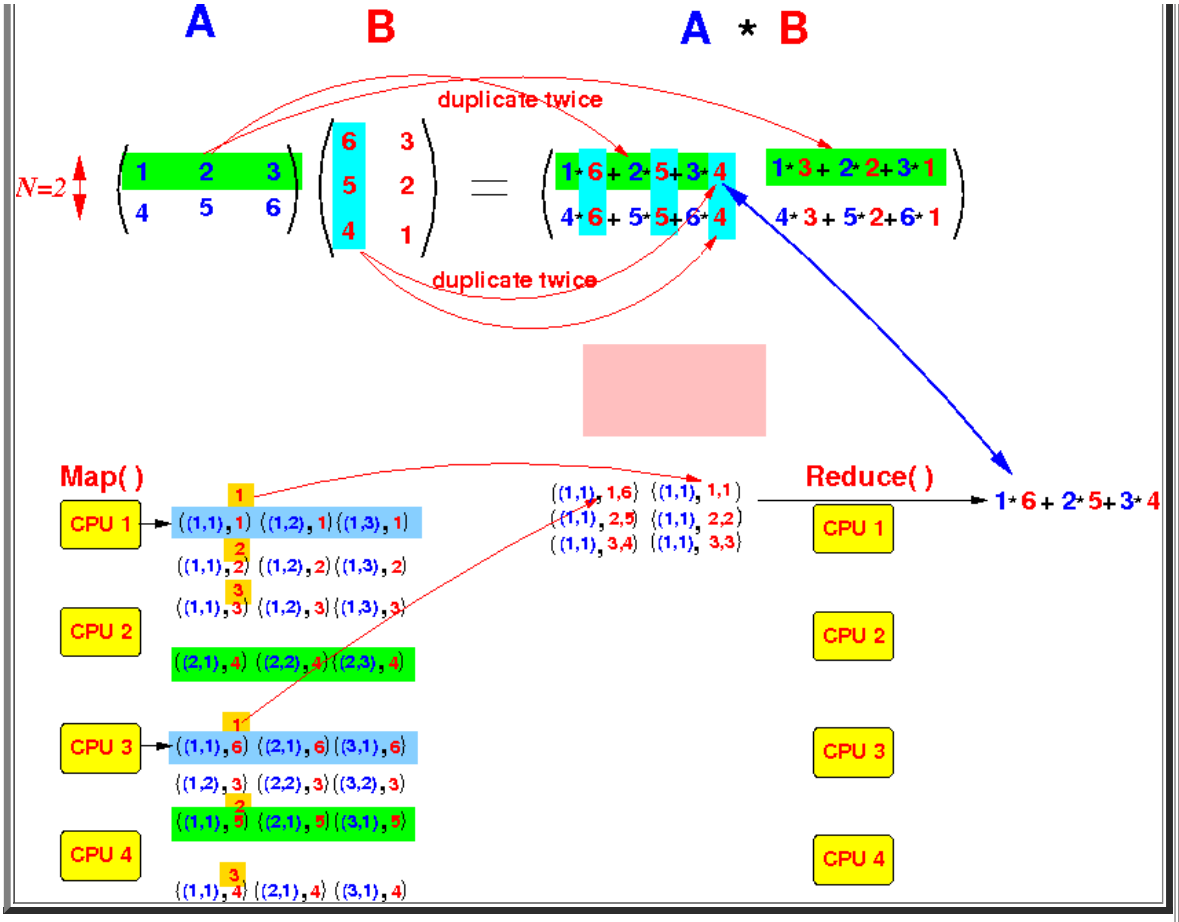
- The `reduce( )` **function** will **compute** the ***inner product*** of the **input vectors**



- **Postscript:**

  - We need to **tag** the **map( ) function output** with the **position** so the **reduce( ) function** can **identify** the ***components*** in the ***different vectors***

    Example:



(This **detail** was **omitted** for ***brevity*** --- **figure** is kinda ***full***)

- The `reduce( )` function is as **follows**:

```
sum = 0;

for ( pos = 1 to N ) do
{
   x = first  value at position pos
   y = second value at position pos

   sum = sum + x*y;
}
```