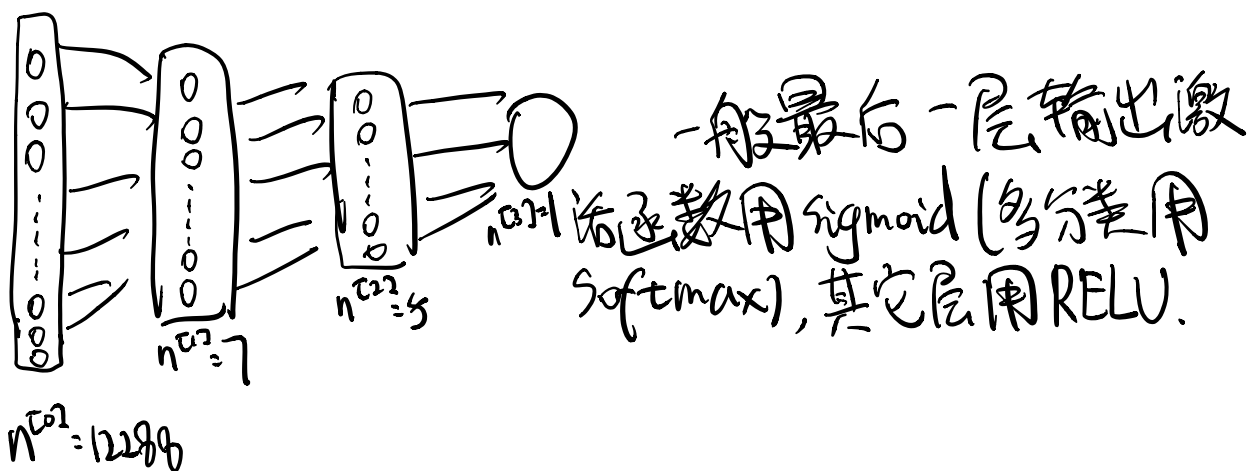


给定数据集 (如大小为 $64 \times 64 \times 3$ 的图片, m 张)
 写为向量形式:

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

X 的 shape: $(12288, m)$, 其中 $12288 = 64 \times 64 \times 3$



正向传播流程如下:

初始时, $A^{l0} = X$, w^{l1} shape: (n^{l1}, n^{l-1})

b^{l1} shape: $(n^{l1}, 1)$

$$Z^{L1} = W^{L1} A^{L0} + b^{L1} \quad (7 \times 12288) \times (12288, m) + (7 \times 1) = (7 \times m)$$

$$A^{L1} = g(Z^{L1}) \quad (7 \times m)$$

$$Z^{L2} = W^{L2} A^{L1} + b^{L2} \quad (5 \times 7) \times (7 \times m) + (5 \times 1) = (5 \times m)$$

$$A^{L2} = g(Z^{L2}) \quad (5 \times m)$$

$$Z^{L3} = W^{L3} A^{L2} + b^{L3} \quad (1 \times 5) \times (5 \times m) + (1 \times 1) = (1 \times m)$$

$$A^{L3} = g(Z^{L3}) \quad (1 \times m)$$

参数初始化:

$$W^{L1} = \text{np.random.rand}(n^{L1}, n^{L1-1}) \times \text{np.sqrt}\left(\frac{2}{n^{L1-1}}\right)$$

$$b^{L1} = \text{np.zeros}(n^{L1}, 1)$$

np.random.rand 产生一个或组服从正态分布的随机取样值。

正向模板:

$$Z^{L1} = W^{L1T} A^{L1-1} + b^{L1} \quad (A^{L0} = X)$$

$$A^{L1} = g(Z^{L1})$$

反向传播流程如下:

$$L = \frac{1}{m} \sum_{i=1}^m \ell(A^{[3]}(i), y^{(i)})$$

$$= -\frac{1}{m} \left(\sum_{i=1}^m y^{(i)} \ln A^{[3]}(i) + (1 - y^{(i)}) \ln (1 - A^{[3]}(i)) \right)$$

$$dA^{[3]} = -\frac{1}{m} \left(\frac{Y}{A^{[3]}} - \frac{1-Y}{1-A^{[3]}} \right) \quad (1 \times m)$$

$$dz^{[3]} = dA^{[3]} \cdot g'(z^{[3]}) \quad \text{np.multiply}(1 \times m, 1 \times m) = (1 \times m)$$

$$dw^{[3]} = dz^{[3]} \cdot A^{[2]} \quad (1 \times m) \times (5 \times m)^T = (1 \times 5)$$

$$db^{[3]} = dz^{[3]} \quad \text{np.sum}(1 \times m, \text{axis}=1) = (1 \times 1)$$

$$dA^{[2]} = dz^{[3]} \cdot w^{[3]} \quad (1 \times 5)^T \times (1 \times m) = (5 \times m)$$

$$dz^{[2]} = dA^{[2]} \cdot g'(z^{[2]}) \quad \text{np.multiply}(5 \times m, 5 \times m) = (5 \times m)$$

$$dw^{[2]} = dz^{[2]} \cdot A^{[1]} \quad (5 \times m) \times (7 \times m)^T = (5 \times 7)$$

$$db^{[2]} = dz^{[2]} \quad \text{np.sum}(5 \times m, \text{axis}=1) = (5 \times 1)$$

$$dA^{[1]} = dz^{[2]} \cdot w^{[2]} \quad (5 \times 7)^T \times (5 \times m) = (7 \times m)$$

$$dz^{[1]} = dA^{[1]} \cdot g'(z^{[1]}) \quad \text{np.multiply}(7 \times m, 7 \times m) = (7 \times m)$$

$$dw^{[1]} = dz^{[1]} \cdot A^{[0]} \quad (7 \times m) \times (12288, m)^T = (7 \times 12288)$$

$$db^{[1]} = dz^{[1]} \quad \text{np.sum}(7 \times m, \text{axis}=1) = (7 \times 1)$$

因为反向传播时会用到 $A^{[2]}$, $z^{[2]}$, 所以应在正向传播中将它们存起来。

反向传播:

$$dz^{l+1} = \text{np.multiply}(dA^{l+1}, g'(z^{l+1}))$$

$$dw^{l+1} = dz^{l+1} \cdot A^{[l+1]T}$$

$$db^{l+1} = \text{np.sum}(dz^{l+1}, \text{axis}=1)$$

$$dA^{l+1} = W^{[l+1]T} \cdot dz^{[l+1]} \text{ (第一层除外)}$$