Data Structures : Graph

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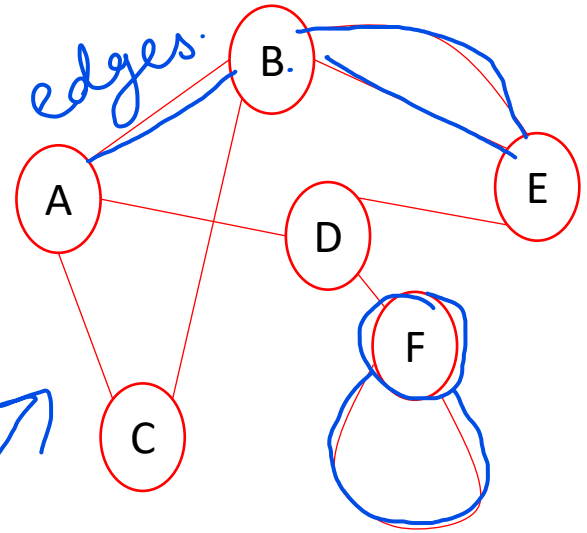
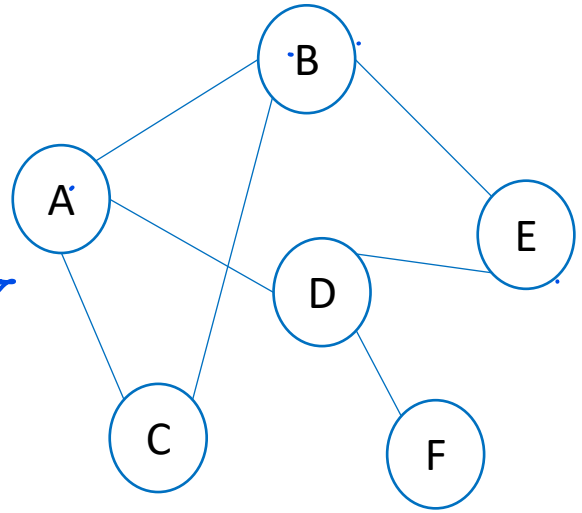
Graph

- Graph is a non-linear data structure.
- Graph is defined as set of vertices and edges. Vertices (also called as nodes) hold data, while edges connect vertices and represent relations between them.
  - G = { V, E }
- When there is an edge from vertex P to vertex Q, P is said to be adjacent to Q.
- Multi-graph
  - Contains multiple edges in adjacent vertices or loops (edge connecting a vertex to it-self).
- Simple graph
  - Doesn't contain multiple edges in adjacent vertices or loops.

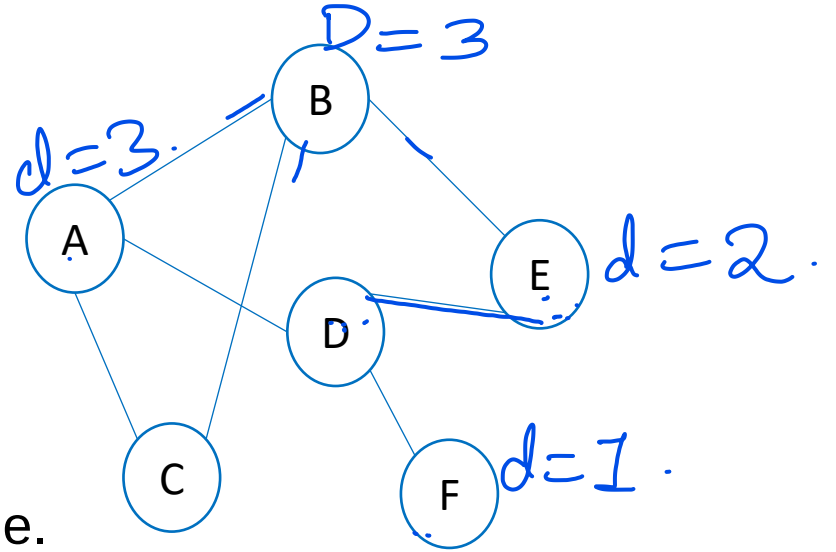




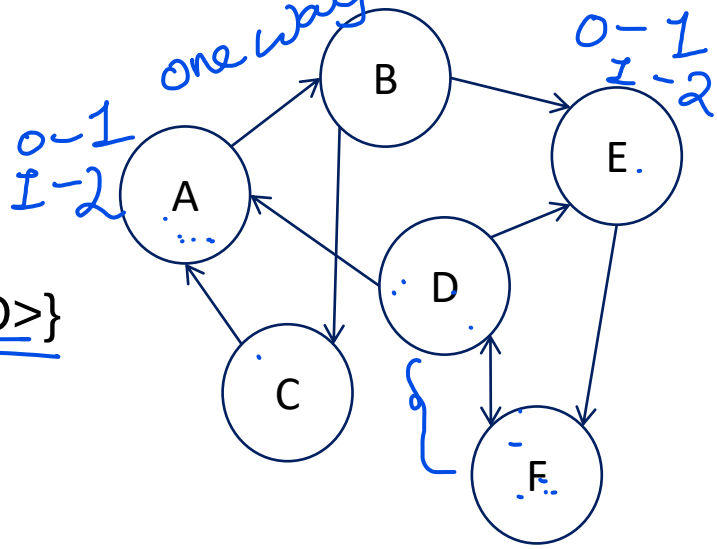
Graph

- Graph edges may or may not have directions.

- Undirected Graph: G = { V, E }
  - V = { A, B, C, D, E, F}
  - E = { (A,B), (A,C), (A,D), (B,C), (B,E), (D,E), (D,F) }
  - If P is adjacent to Q, then Q is also adjacent to P.
  - Degree of node: Number of nodes adjacent to the node.
  - Degree of graph: Maximum degree of any node in graph.

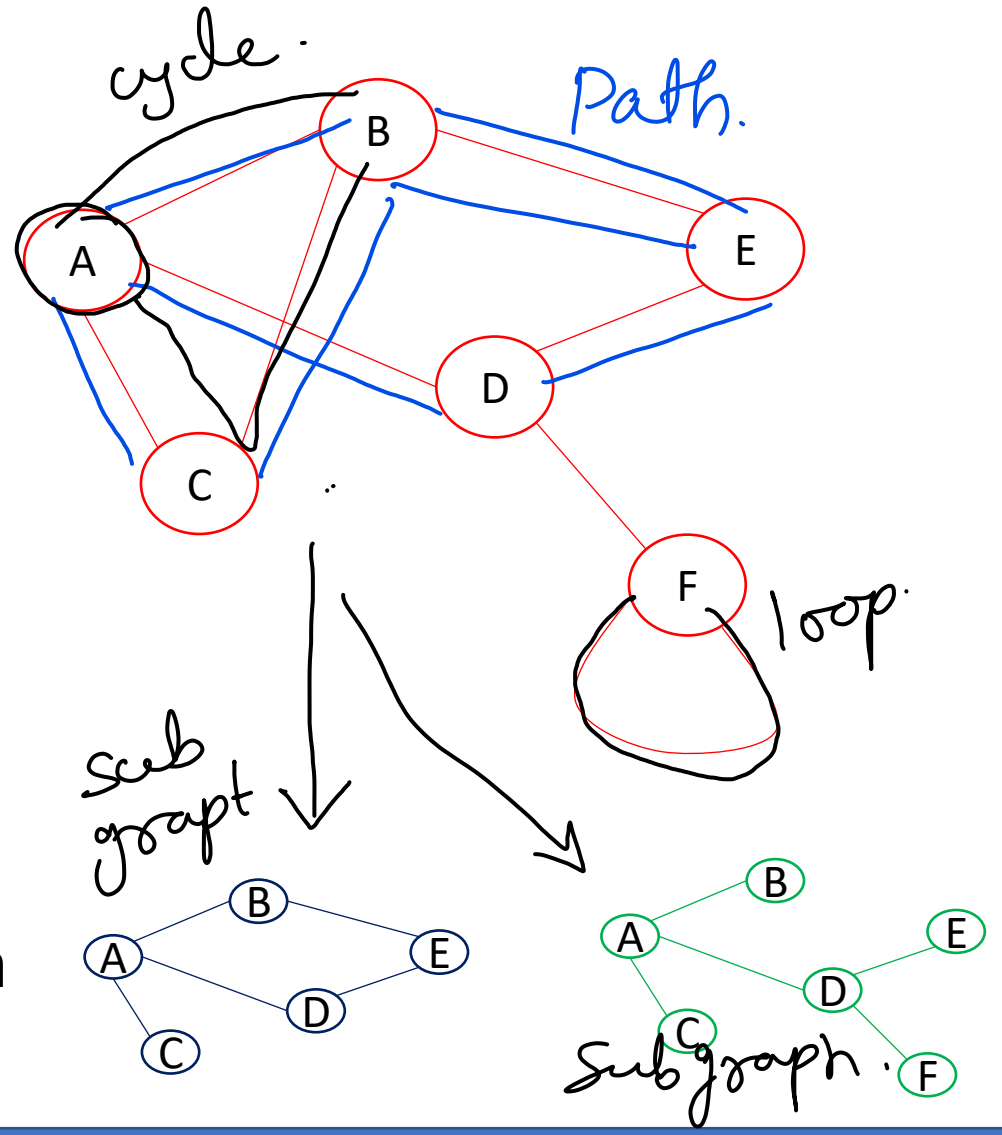


- Directed Graph: G = { V, E }
  - V = { A, B, C, D, E, F}
  - E = {<A,B>, <B,C>, <B,E>, <C,A>, <D,A>, <D,E>, <D,F>, <E,F>, <F,D>}
  - If P is adjacent to Q, then Q is may or may not be adjacent to P.
  - Out-degree: Number of edges originated from the node
  - In-degree: Number of edges terminated on the node

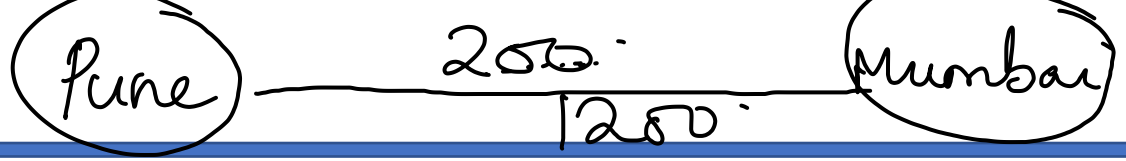


Graph

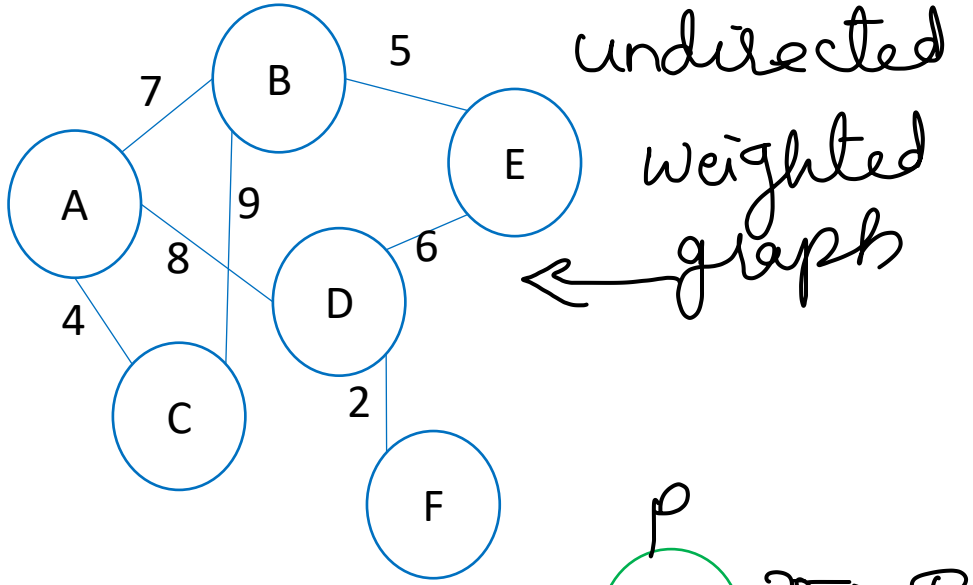
- Path: Set of edges between two vertices. There can be multiple paths between two vertices.
  - A – D – E
  - A – B – E
  - A – C – B – E
- Cycle: Path whose start and end vertex is same.
  - A – B – C – A
  - A – B – E – D – A
- Loop: Edge connecting vertex to itself. It is smallest cycle.
  - F – F
- Sub-Graph: A graph having few vertices and few edges in the given graph, is said to be sub-graph of given graph.

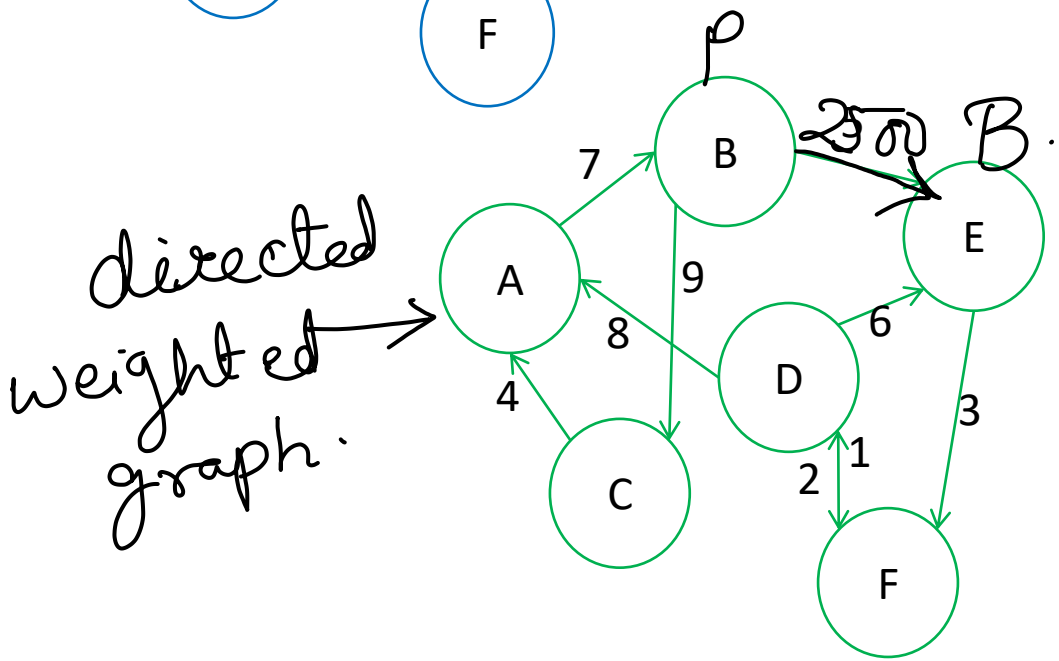


Graph



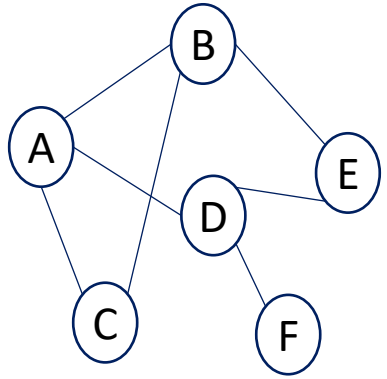
- Weighted graph
  - Graph edges have weight associated with them.
  - Weight represent some value e.g. distance, resistance.
- Directed Weighted graph (Network)
  - Graph edges have directions as well as weights.
- Applications of graph
  - Electronic circuits
  - Social media
  - Communication network
  - Road network
  - Flight/Train/Bus services
  - Bio-logical & Chemical experiments
  - Deep learning (Neural network, Tensor flow)
  - Graph databases (Neo4j)

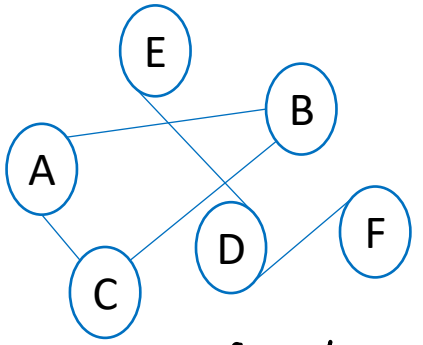




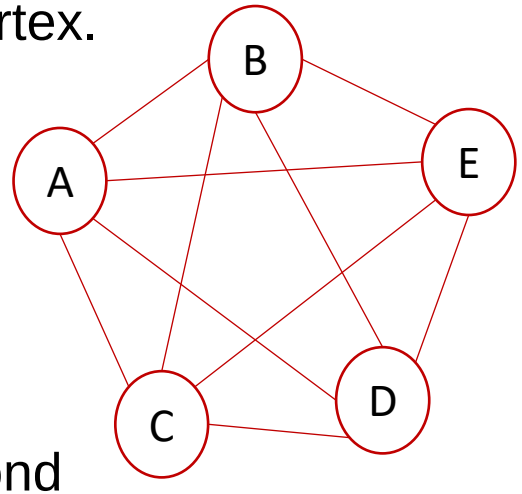
Graph

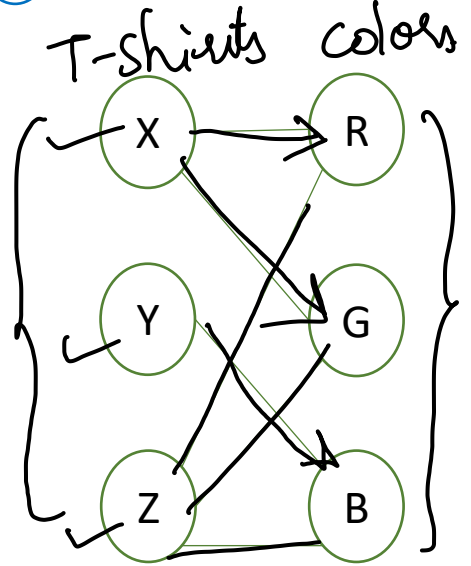
- Connected graph
  - From each vertex some path exists for every other vertex.
  - Can traverse the entire graph starting from any vertex.





- Complete graph
  - Each vertex of a graph is adjacent to every other vertex.
  - Un-directed graph: Number of edges = n (n-1) / 2
  - Directed graph: Number of edges = n (n-1)

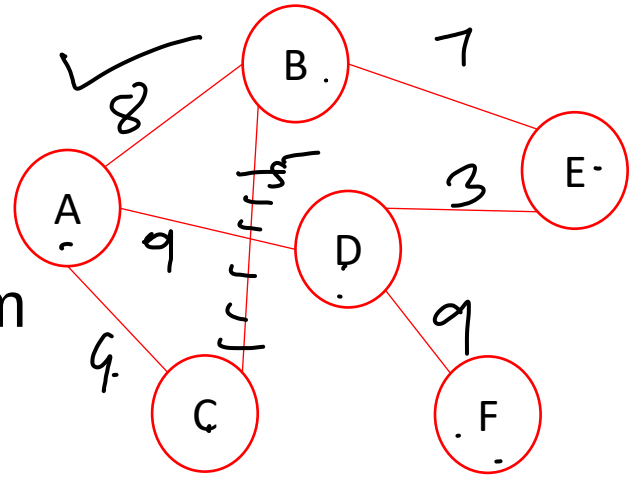


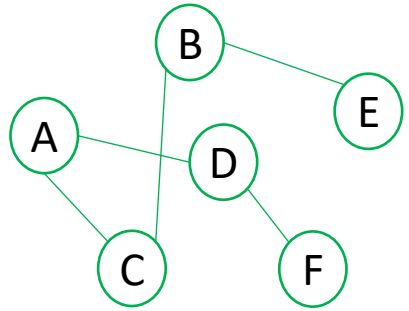


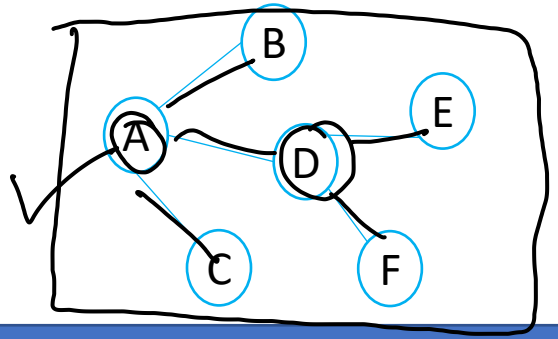
- Bi-partite graph
  - Vertices can be divided in two disjoint sets.
  - Vertices in first set are connected to vertices in second set.
  - Vertices in a set are not directly connected to each other.

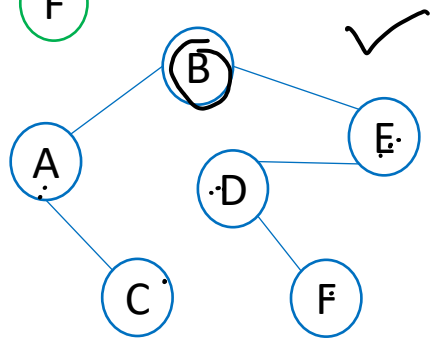
Spanning Tree

- Tree is a graph without cycles.
- Spanning tree is connected sub-graph of the given graph that contains all the vertices and sub-set of edges (V-1).
- Spanning tree can be created by removing few edges from the graph which are causing cycles to form.
- One graph can have multiple different spanning trees.
- In weighted graph, spanning tree can be made who has minimum weight (sum of weights of edges). Such spanning tree is called as Minimum Spanning Tree.
- Spanning tree can be made by various algorithms.
  - BFS Spanning tree
  - DFS Spanning tree
  - Prim's MST
  - Kruskal's MST



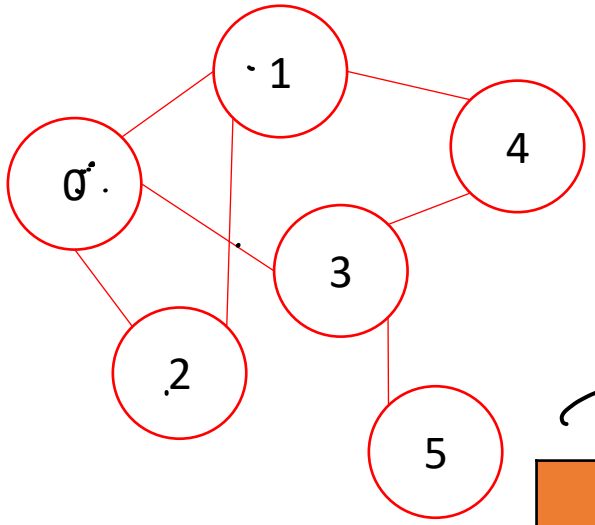






Graph Implementation – Adjacency Matrix

- If graph have V vertices, a V x V matrix can be formed to store edges of the graph.
- Each matrix element represent presence or absence of the edge between vertices.
- For *non-weighted graph*, 1 indicate edge and 0 indicate no edge.
- For un-directed graph, adjacency matrix is always symmetric across the diagonal.
- Space complexity of this implementation is $O(V^2)$.

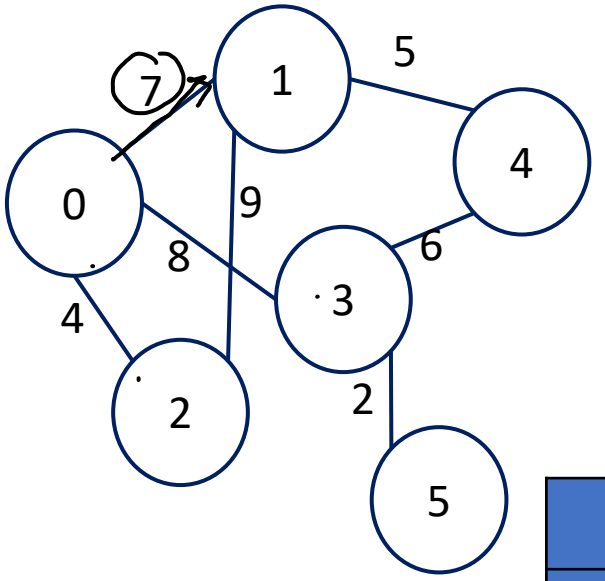


arr[6][6]

	0	1	2	3	4	5
0	0	1	1	1	0	0
1	1	0	1	0	1	0
2	1	1	0	0	0	0
3	1	0	0	0	1	1
4	0	1	0	1	0	0
5	0	0	0	1	0	0

Graph Implementation – Adjacency Matrix

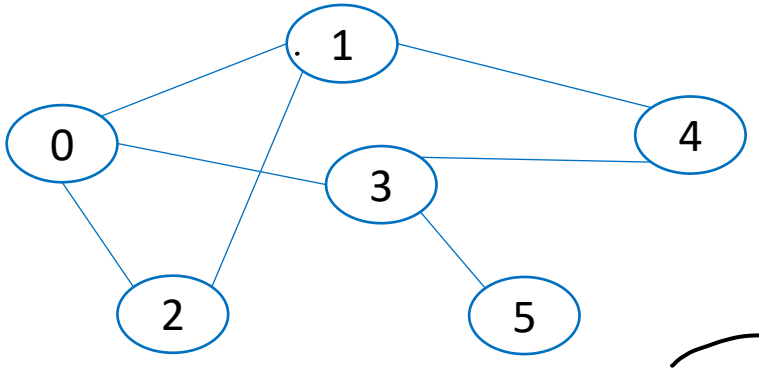
- If graph have V vertices, a V x V matrix can be formed to store edges of the graph.
- Each matrix element represent presence or absence of the edge between vertices.
- For *weighted graph*, weight value indicate the edge and infinity sign ∞ represent no edge.
- For un-directed graph, adjacency matrix is always symmetric across the diagonal.
- Space complexity of this implementation is $O(V^2)$.

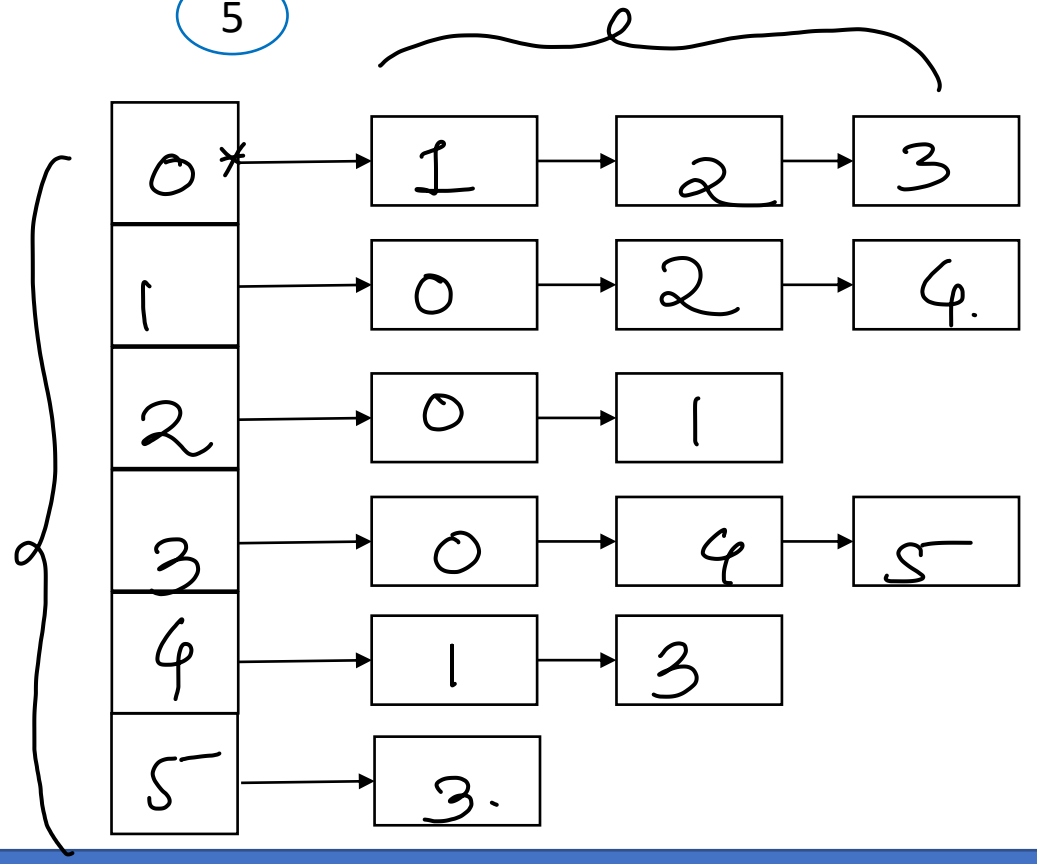


	0	1	2	3	4	5
0	∞	7	4	8	∞	∞
1	7	∞	9	∞	5	∞
2	4	9	∞	∞	∞	∞
3						
4						
5						

Graph Implementation – Adjacency List

- Each vertex holds list of its adjacent vertices.
- For non-weighted graphs only, neighbour vertices are stored.
- For weighted graph, neighbour vertices and weights of connecting edges are stored.
- Space complexity of this implementation is O(V+E).
- If graph is sparse graph (with fewer number of edges), this implementation is more efficient (as compared to adjacency matrix method).





Thank you!