Mathematics

A Beginner's Guide

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Preface

PDF Version

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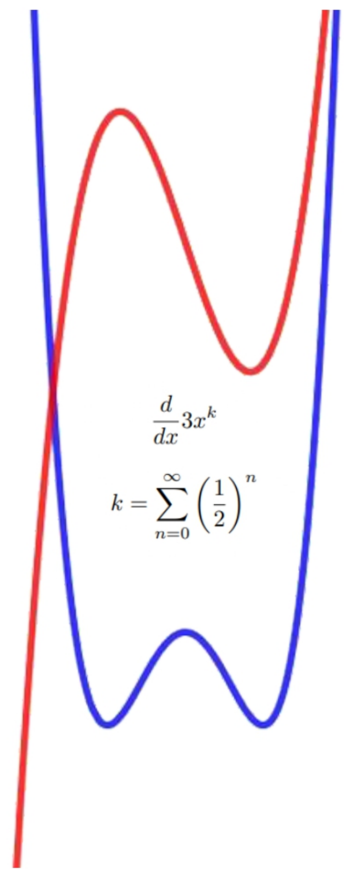

$$\frac{d}{dx}3x^k$$

$$k = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$$

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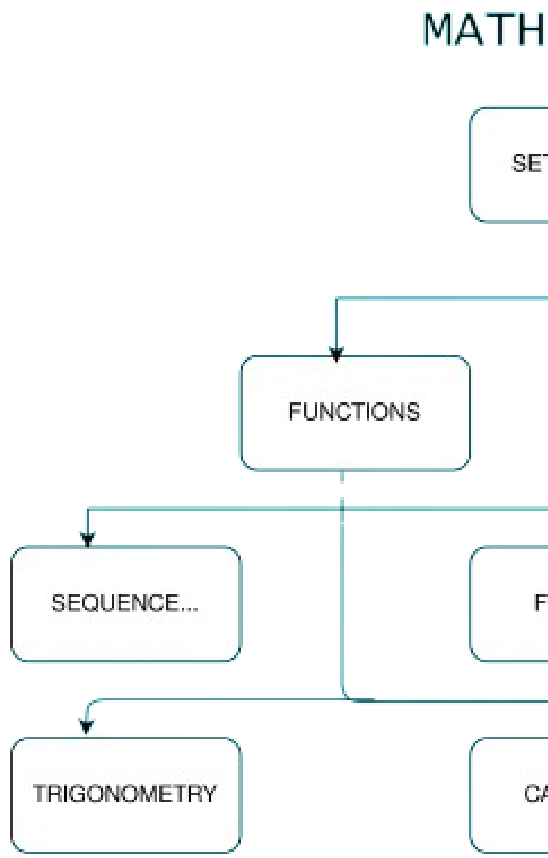
Structure



This book is structured in a way that it will cover the following topics:

in the order of seen from the table of contents. I've structured the book such that one chapter builds from the previous one. To master Probability and Functions, it would be helpful if you did Set Theory first — although not necessary(**RECOMMENDED**). The rest can be seen from the Mind-map intuitively so.

NOTE :

This is an on-going project. The resources used will be cited. Slowly but surely, step by step. **Enjoy!**

1 Set Theory: An Introduction

1.1 History

The idea of grouping things has existed for the longest of time in the history of the Modern Homo Sapiens specie. The study of grouping objects was later called set theory, where a **set** is a collection/grouping of objects. The study of modern set theory is often attributed to one of its founders, a prominent German mathematician—George Cantor along with another German mathematician Richard Dedekind, which they developed in the 1870s.

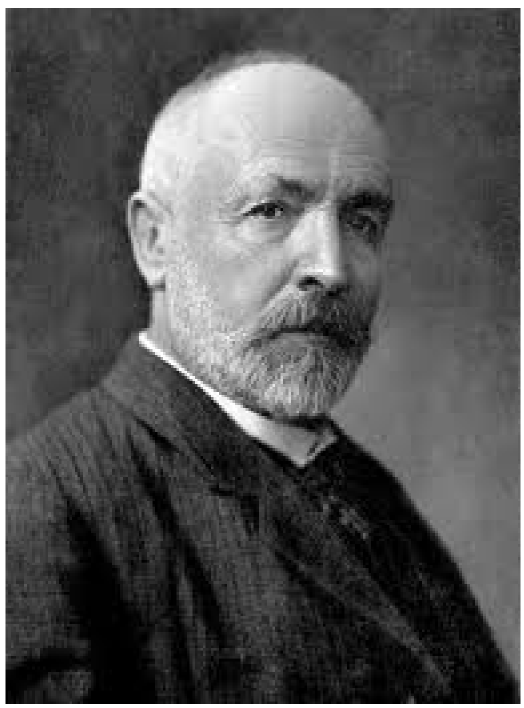


Figure 1.1: George Cantor

As the topic was relatively new and hot at the time, it received backlashes often from older mathematicians who did not embrace new idea. A famous mathematician and philosopher, Bertrand Russell also attacked some of these ideas which gave birth to a concepts like Russell's Paradoxes or fuzzy set theory.

1.2 Applications

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However, as I've outline that this isn't a new idea, you have also used sets in your daily life. Formally, a

SET is a collection of objects or elements. A set is often denoted with a capital letter & curly braces.

These objects are also called members. A classroom is a set made of students, and these students are the members of this set. In that same classroom, we can have a set which consists of only boys. Remember that the art of grouping can group things that have similarities or are entirely different. The classroom set has both girls and boys. The similarity is that they're all human. The set grouping all boys in that classroom share the similarity being that they're boys, but they're most likely different in many other ways, from names to their Blood Pressure. A set is often denoted with a capital letter & curly braces , while the elements are denoted by small letters .Let

$$V = \{a, e, i, o, u\}$$

Be an example where the set V is the set of all vowels. Where a, e, i, o, u are the members of the set V. The curly braces means "the set of". Since u and a, e, i, o, u are elements of V, we denote it as $u \epsilon V$ and

$$(a, e, i, o, u) \epsilon V \equiv a, e, i, o, u \epsilon V$$

Respectively. It oath to be noted that the *order* of elements in the set and *repetition* are not important, thus a set with elements

$$\{o, e, u, a, i\}$$

and

$$\{a, a, i, i, i, i, i, o, u, e, e\}$$

Are the same as the set V. The number of elements within a set are referred to as a **Cardinality**, denoted by the absolute bars $|X|$ where X is a set. Thus the cardinality of the set of vowels V is 5 denoted

$$|V| = 5$$

.

Another simple example would be as follows, lets call it the ***Black Box example*** :

Suppose you have a Black Box in a class, then ask each and every student to put a paper inside of the box(more like voting). The papers could be different colors and shapes, or just all plain White square papers.
The Box will act as a set while the papers of the students inside the box are regarded as the elements of the set(or elements of the box). If the box was empty, then we'd say the box is an **empty set/null set**. This is a special case of a set and it is denoted by

ϕ or empty curly braces $\{\}$. In turn, $\phi = \{\}$ with a cardinality of 0.

1.3 Example

Suppose there are three sets

$$A = \phi$$

$$B = \{\phi\}$$

$$C = \{\}$$

1. Which of these are equal?
   a. A and B
   b. A and C
   c. B and C
2. What are the cardinalities of each set?

1.3.1 Solution

1. or the first question:
   a. A and B are not equal, since A null set—without elements while B is a set that has a null set as an element. Thus, $A \neq B$.
   b. A and C are equal, since A is a null set—without elements and C is also set that has no elements in it. Thus $A = C$.
   c. B and C are not equal, since is a *set that has a null set* as an element, while C is a null set . Thus, $B \neq C$.
2. As for the cardinalities:

$$|A| = |C| = 0$$

$$|B| = 1$$

1.4 Subsets

A **subset** can be defined as a set within a set. As per the name suggests, it a portion of a set, and it is denoted by $\subseteq$. Formally, we say

SUBSET The set X is the *subset* of Y if every element in X is also an element of Y.

Suppose you have three sets $A = \{2, 4, 6, 8, 10\}$; $B = \{2, 6, 8\}$ and $C = \{2, 6\}$. C is the subset of A and B since 2 & 6 are in both the sets A and B,that is $C \subseteq A$ and $C \subseteq B$.

Set B is a subset of A but not of C. That is $B \subseteq A$ and $B \nsubseteq C$. And lastly A is not a subset of either sets.

Every set is a subset of itself.

Common basic sets which are also subsets of others include :

o ***Integers***: all set of numbers without a decimal. These are all set of integers which are either positive or negative. Represented by the symbol $\mathbb{Z}$. denoted by

$$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

. o ***Natural Numbers***: all positive integers EXCEPT 0. Represented by the symbol $\mathbb{N}$. denoted by

$$\mathbb{N} = \{1, 2, 3, ...\}$$

. o ***Whole Numbers***: all positive integers INCLUDING 0, denoted by

$$\mathbb{N}_0 = \{0, 1, 2, 3, ...\}$$

. o ***Rational Numbers***: Represented by the symbol $\mathbb{Q}$. All fractions of the form

$$\mathbb{Q} = \{\frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0\}$$

. o ***Irrational numbers***: numbers that are thought not to be rational are said to be irrational.

o ***Imaginary Numbers***: imagined numbers that may not exist where $\sqrt{-1} = i$.

o ***Real Numbers***: all numbers that can be quantified and thought not to be imaginary. These numbers are made up by both rational and irrational numbers Represented by the symbol $\mathbb{R}$.

o ***Complex Numbers***: a combination of both real and imaginary numbers. Represented by the symbol $\mathbb{C}$ and denoted as

$$\mathbb{C} = \{a + bi | a, b \in \mathbb{R}; i = \sqrt{-1}\}$$

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1.5 Example

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