Mathematics

A Beginner's Guide

Katleho Nyoni

Table of contents

Preface

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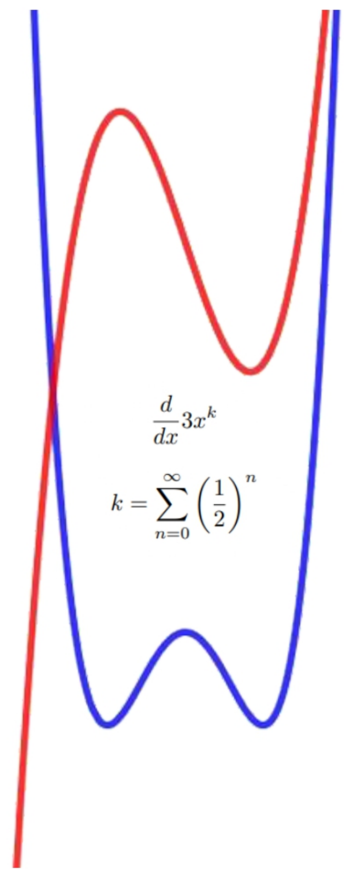
Mathematics

A Beginner's Guide



$$\frac{d}{dx}3x^k$$

$$k = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$$

KATIEUM

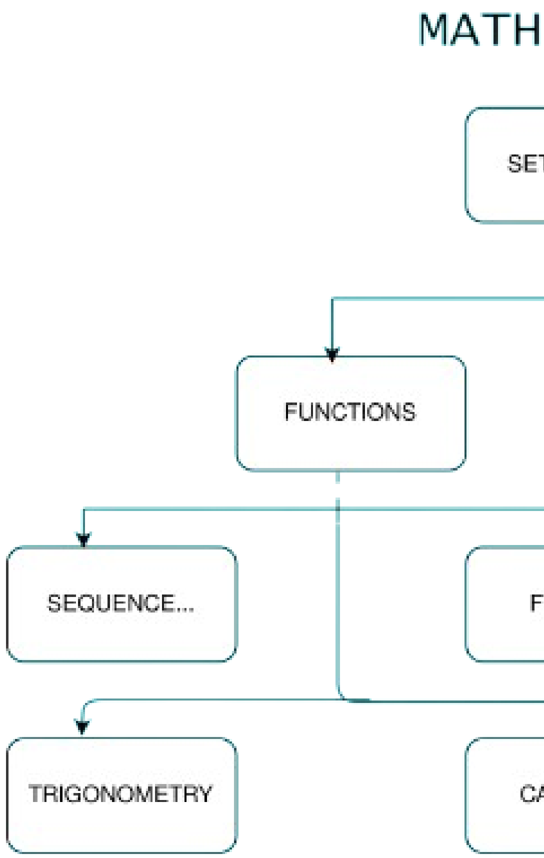
In Mathematics we Trust

Katleho Nyoni

Welcome !

Katieum uses animation to teach Mathematics and Physical Sciences to Grade 10 — 12 learners of South Africa. Click to check the demo.

Structure



This book is structured in a way that it will cover the following topics:

in the order of seen from the table of contents. I've structured the book such that one chapter builds from the previous one. To master Probability and Functions, it would be helpful if you did Set Theory first — although not necessary(**RECOMMENDED**). The rest can be seen from the Mind-map intuitively so.

NOTE :

This is an on-going project. The resources used will be cited. Slowly but surely, step by step.
Enjoy!

1 Set Theory: An Introduction

1.1 History

The idea of grouping things has existed for the longest of time in the history of the Modern Homo Sapiens specie. The study of grouping objects was later called set theory, where a **set** is a collection/grouping of objects. The study of modern set theory is often attributed to one of its founders, a prominent German mathematician—George Cantor along with another German mathematician Richard Dedekind, which they developed in the 1870s.

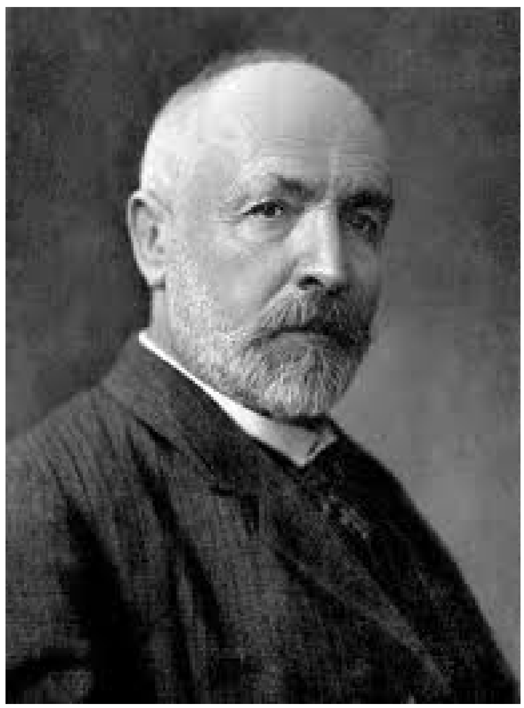


Figure 1.1: George Cantor

As the topic was relatively new and hot at the time, it received backlashes often from older mathematicians who did not embrace new idea. A famous mathematician and philosopher, Bertrand Russell also attacked some of these ideas which gave birth to a concepts like Russell's Paradoxes or fuzzy set theory.

1.2 Applications

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However, as I've outline that this isn't a new idea, you have also used sets in your daily life. Formally, a

SET is a collection of objects or elements. A set is often denoted with a capital letter & curly braces.

These objects are also called members. A classroom is a set made of students, and these students are the members of this set. In that same classroom, we can have a set which consists of only boys. Remember that the art of grouping can group things that have similarities or are entirely different. The classroom set has both girls and boys. The similarity is that they're all human. The set grouping all boys in that classroom share the similarity being that they're boys, but they're most likely different in many other ways, from names to their Blood Pressure. A set is often denoted with a capital letter & curly braces , while the elements are denoted by small letters .Let

$$V = \{a, e, i, o, u\}$$

Be an example where the set V is the set of all vowels. Where a, e, i, o, u are the members of the set V. The curly braces means "the set of". Since u and a, e, i, o, u are elements of V, we denote it as $u \epsilon V$ and

$$(a, e, i, o, u) \epsilon V \equiv a, e, i, o, u \epsilon V$$

Respectively. It oath to be noted that the *order* of elements in the set and *repetition* are not important, thus a set with elements

$$\{o, e, u, a, i\}$$

and

$$\{a, a, i, i, i, i, i, o, u, e, e\}$$

Are the same as the set V. The number of elements within a set are referred to as a **Cardinality**, denoted by the absolute bars $|X|$ where X is a set. Thus the cardinality of the set of vowels V is 5 denoted

$$|V| = 5$$

.

Another simple example would be as follows, lets call it the ***Black Box example*** :

Suppose you have a Black Box in a class, then ask each and every student to put a paper inside of the box(more like voting). The papers could be different colors and shapes, or just all plain White square papers.
The Box will act as a set while the papers of the students inside the box are regarded as the elements of the set(or elements of the box). If the box was empty, then we'd say the box is an **empty set/null set**. This is a special case of a set and it is denoted by

ϕ or empty curly braces $\{\}$. In turn, $\phi = \{\}$ with a cardinality of 0.

1.3 Example

Suppose there are three sets

$$A = \phi$$

$$B = \{\phi\}$$

$$C = \{\}$$

1. Which of these are equal?

   a. and
   b. and
   c. and

2. What are the cardinalities of each set?

1.3.1 Solution

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2 Probability

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3 Combinorics : Counting Principles

4 Functions

Polynomials

Trigonometry

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5 Sequence & Series

5.1 History

The idea of sequences and series' go a long way back into the past. One of the earliest encounters could be seen around 5th BCE where the famous Greek philosopher *Zeno of Elea* introduced a number of problems where only Nine survived, which came to be known as Zeno's Paradoxes, these were his proposals to the questions concerning space and time at their time.

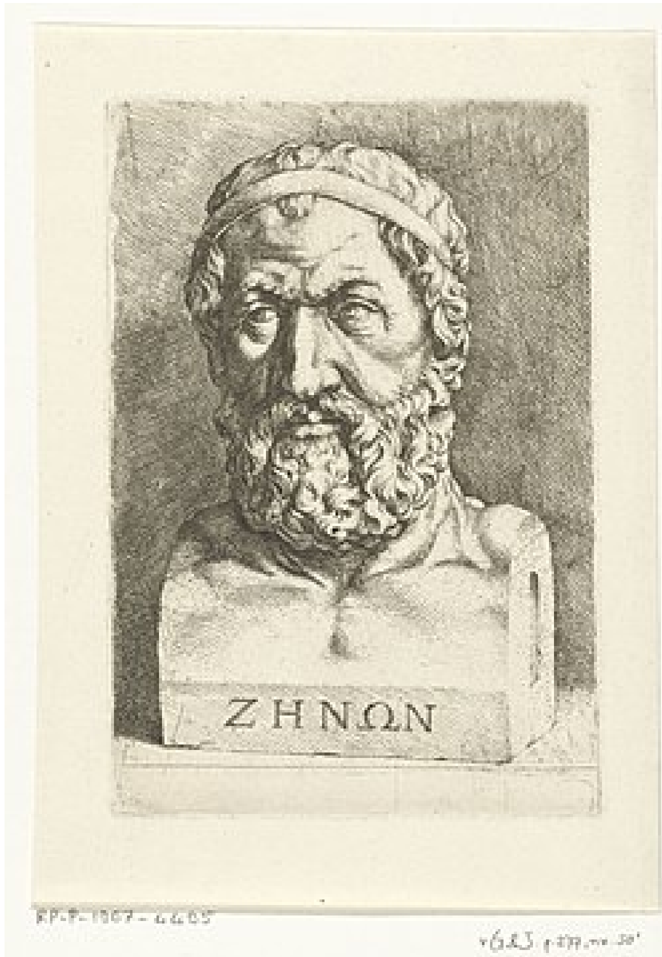

Figure 5.1: Zeno of Elea

On one of them, Zeno argued that a man who wanted to walk across a room, had to walk half the distance of the room first, but before travelling that distance they had to travel halfway of that half, then half of that, and repeatedly infinitely many times producing the sequence

$$..., \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1$$

Which he claimed was impossible since it requires a number of infinite tasks and wouldn't have the first distance to travel, meaning the journey wouldn't even start. Traveling across the same room would give the sequence

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ..., \frac{1}{2^n}$$

5.2 Applications

This sequence is popularly known as the Geometric sequence. More about that later. In present day, sequence and series has become increasingly important as it is widely used in almost every field. In finance it is used to calculate the return of loans and investments, in physics it is used to study waves and their properties. In Chemistry it is used to see how chemical reactions end, in Statistics it is used to study trends, in ecology and epidemiology it is used to model populations of the same and different species. In astronomy and cosmology it is used to study the emission of radiation/energy of celestial bodies; and it is used by TikTok and Netflix to recommend you videos and movies you may like.
First and foremost, what's the difference between a sequence and a series?

5.3 Introduction

SEQUENCE a list of ordered numbers separated by a comma

e.g.

$$1, 2, 3, 4, ..., n$$

where 1 is the 1st term denoted by T_1

$$2, 4, 6, 8, ..., 2n$$

where $2n$ is the formula for the sequence denoted T_n

$$3, 6, 9, 12, ..., 2n + 1$$

Thus, a sequence is denoted by

$$1, T_2, T_3, ..., T_n$$

SERIES the sum of individual terms that form a sequence, separated by an operator $\pm$

e.g.

$$1 + 2 + 3 + 4 + \cdots$$

$$2 + 4 + 6 + 8 + \cdots$$

$$3 + 6 + 9 + 12 + \cdots$$

Thus, a series is denoted by

$$S_n = T_1 + T_2 + T_3 + \cdots + T_n = \sum_{i=1}^{n} T_i$$

Where

$$S_1 = T_1$$

$$S_2 = T_1 + T_2 = \sum_{i=1}^{2} T_i$$

$$S_3 = T_1 + T_2 + T_3 = \sum_{i=1}^{3} T_i$$

There are recursive and non-recursive sequences.

RECURSIVE these are sequences which whose terms are dependent on the previous term. They are a 'regress'.

A very famous one is the Fibonacci sequence that appears a lot in nature, especially in the rows of corn or becomes the sequence of the petals on a flower. *Fibonacci sequence* :

$$0, 1, 1, 2, 3, 5, 8, 13, ...$$

Another interesting example is the logistic equation, which is often a realistic approach used to model the population of species.

NON-RECURSIVE sequences that are not recursive. These are the 'normal' ones.

We'll only be dealing with the Arithmetic(linear), Quadratic and Geometric sequences and series'.

Arithmetic

5.3.1 Arithmetic Sequence

An Arithmetic sequence can be seen as a linear function where the c in

$$y = mx + c$$

can either be negative or positive. This is a constant(fixed) number that is added to each previous term to find the next. For the sake of formality, in this chapter we use the notation

$$T_n = \phi n + \gamma$$

But more importantly, to obtain the equation in 2.2 , we use the equation

$$T_n = a + (n - 1)d$$

Where $\phi = d$ and \$ $= -$ \$. Here T_n denotes the general term, the first term of the sequence and is the common difference. Since a sequence is defined by

$$T_1, T_2, T_3, ..., T_n$$

5.3.2 Arithmetic Series

Geometric

5.3.1 Geometric Sequence

5.3.2 Geometric Series

5.3.2.1 Infinite Geometric Series

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6 Financancial Mathematics

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7 Calculus

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8 Statistics

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9 Geometry

Analytical Geometry

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Euclidean Geometry