Class 1

Chapter 3

3)

a) iii. is correct because even though in the baseline case females earn more, the interaction effect is negative, and if the GPA is sufficiently high (at least 3.5), men earn more.

b)

```
beta_0 <- 50
beta_1 <- 20
beta_2 <- 0.07
beta_3 <- 35
beta_4 <- 0.01
beta_5 <- -10
IQ <- 110
GPA <- 4
GEN <- 1

beta_0 + GPA * beta_1 + IQ * beta_2 + GEN * beta_3 + GPA * IQ * beta_4 + GPA * GEN * beta_
```

```
[1] 137.1
```

c) FALSE

In the interaction effect we multiply IQ with GPA. The resulting number is thus big, so a small coefficient is okay. Whether it is statistically significant depends on the standard deviation of the variable and the sample size. (t test)

6) In the simple regression case we have

$$\hat{y_i} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

If we plug in $x_i = \bar{x}$ and use 3.4, we get:

$$\hat{y_i} = \bar{y} - \hat{\beta}_1\bar{x} + \hat{\beta}_1\bar{x} = \bar{y}$$

Thus the statement is proven.

11)

a)

```
set.seed(1)
x = rnorm(100)
y = 2*x+rnorm(100)

reg_a <- lm(y~x+0)
summary(reg_a)
```

```
Call:
lm(formula = y ~ x + 0)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.9154 -0.6472 -0.1771  0.5056  2.3109 

Coefficients:
  Estimate Std. Error t value Pr(>|t|)    
x   1.9939     0.1065   18.73   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9586 on 99 degrees of freedom
Multiple R-squared:  0.7798,    Adjusted R-squared:  0.7776 
F-statistic: 350.7 on 1 and 99 DF,  p-value: < 2.2e-16
```

The coefficient is 1.9939, and it is significant, because the p value is very small.

b)

```
reg_b <- lm(x~y+0)
summary(reg_b)
```

```
Call:
lm(formula = x ~ y + 0)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.8699 -0.2368  0.1030  0.2858  0.8938 

Coefficients:
  Estimate Std. Error t value Pr(>|t|)    
y  0.39111    0.02089   18.73   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4246 on 99 degrees of freedom
Multiple R-squared:  0.7798,    Adjusted R-squared:  0.7776 
F-statistic: 350.7 on 1 and 99 DF,  p-value: < 2.2e-16
```

The coefficient is 0.39, and it is significant, with t t and p value being the same as in the last case.

c) t the test and the p value are the same

d) ?

e) ?

f)

```r
reg_f_1 <- lm(x~y)
reg_f_2 <- lm(y~x)
summary(reg_f_1)
```

```
Call:
lm(formula = x ~ y)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.90848 -0.28101  0.06274  0.24570  0.85736 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
```

```
(Intercept)  0.03880    0.04266    0.91    0.365
y            0.38942    0.02099   18.56   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4249 on 98 degrees of freedom
Multiple R-squared:  0.7784,    Adjusted R-squared:  0.7762
F-statistic: 344.3 on 1 and 98 DF,  p-value: < 2.2e-16
```

```
summary(reg_f_2)
```

```
Call:
lm(formula = y ~ x)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8768 -0.6138 -0.1395  0.5394  2.3462

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.03769    0.09699  -0.389    0.698
x            1.99894    0.10773  18.556   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9628 on 98 degrees of freedom
Multiple R-squared:  0.7784,    Adjusted R-squared:  0.7762
F-statistic: 344.3 on 1 and 98 DF,  p-value: < 2.2e-16
```

15)

a)

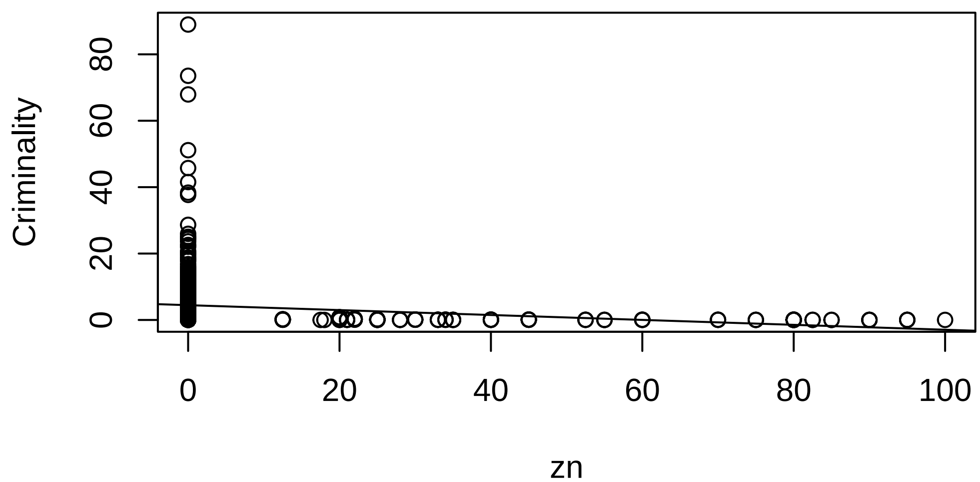
```
library(ISLR2)
df <- Boston
regs <- list() #create a list
for (i in colnames(df[-1])){ #loop over all the variables
  tmp_y <- df[["crim"]]
  tmp_x <- df[[i]]
  tmp <- lm(tmp_y ~ tmp_x)
```

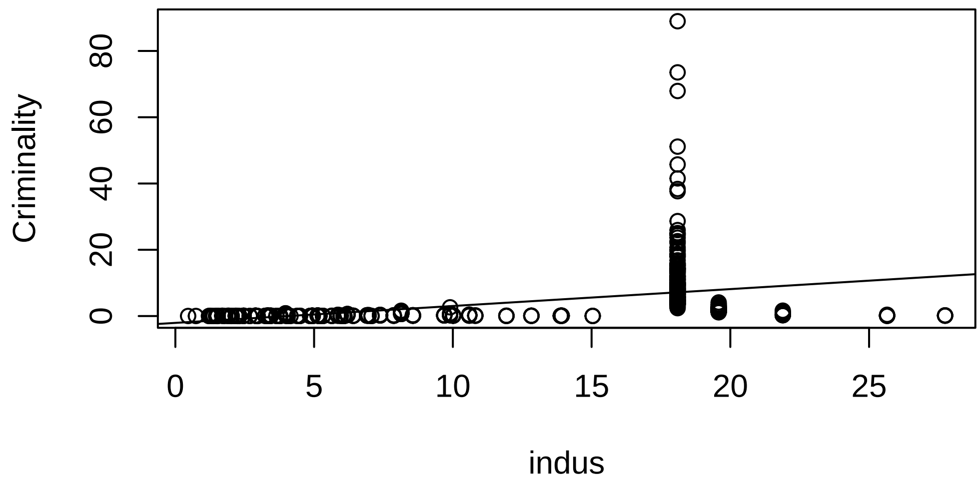
```r
    regs[[i]] <- tmp
  }
  reg_sig <- list()
  sig_num <- c()
  for (r in names(regs)) { #find significant results
    if (summary(regs[[r]])$coefficients[2,4] <= 0.05){
      tmp <- regs[[r]]
      reg_sig[[r]] <- tmp
    }
  }
  names(reg_sig)
```

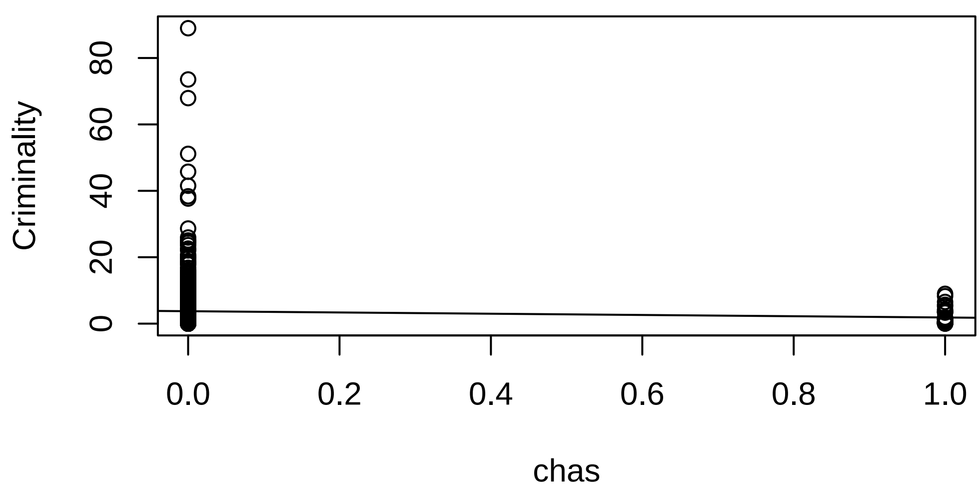
```
 [1] "zn"      "indus"   "nox"     "rm"      "age"     "dis"     "rad"
 [8] "tax"     "ptratio" "lstat"   "medv"
```

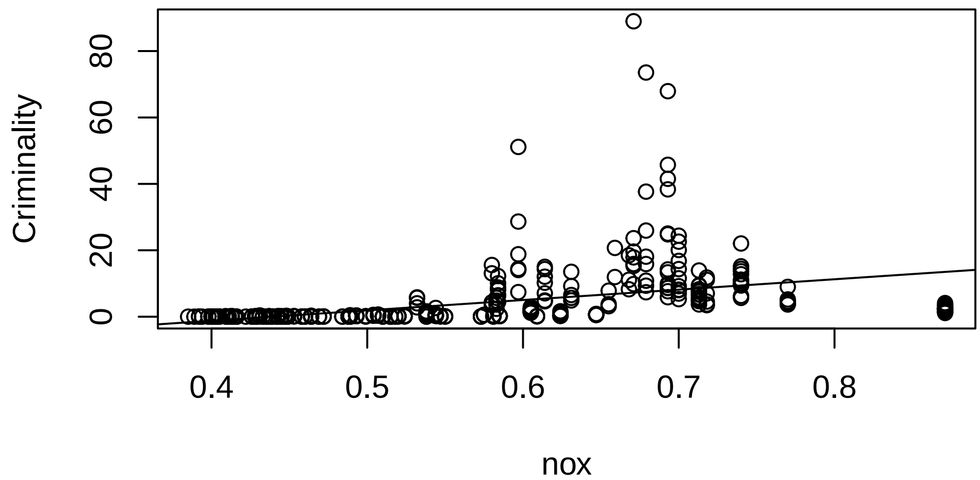
11 variables are significant with a p value less than 0.05. The exception is chas.

```r
  for (r in names(regs)) { #plot the results
    plot(df[[r]],df$crim, ylab = "Criminality", xlab = r)
    abline(regs[[r]])
  }
```

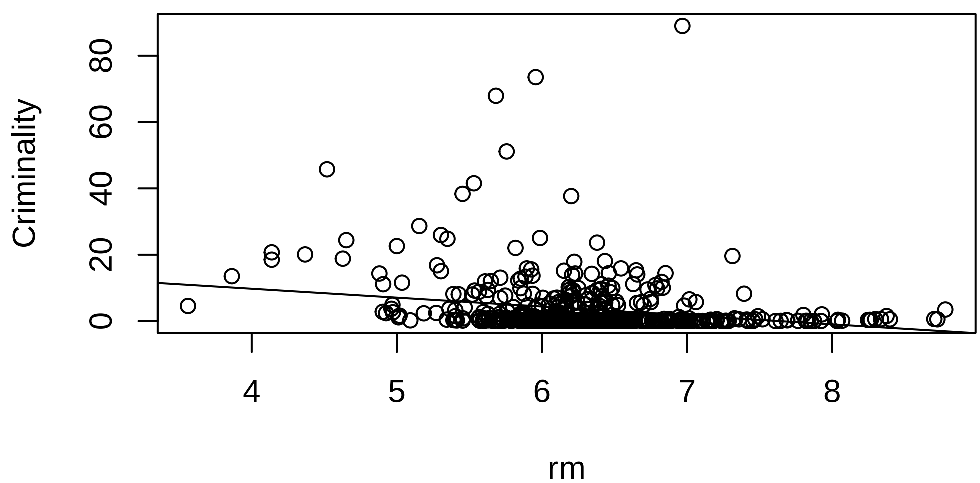



Criminality
0
20
40
60
80
0
5
10
15
20
25
indus

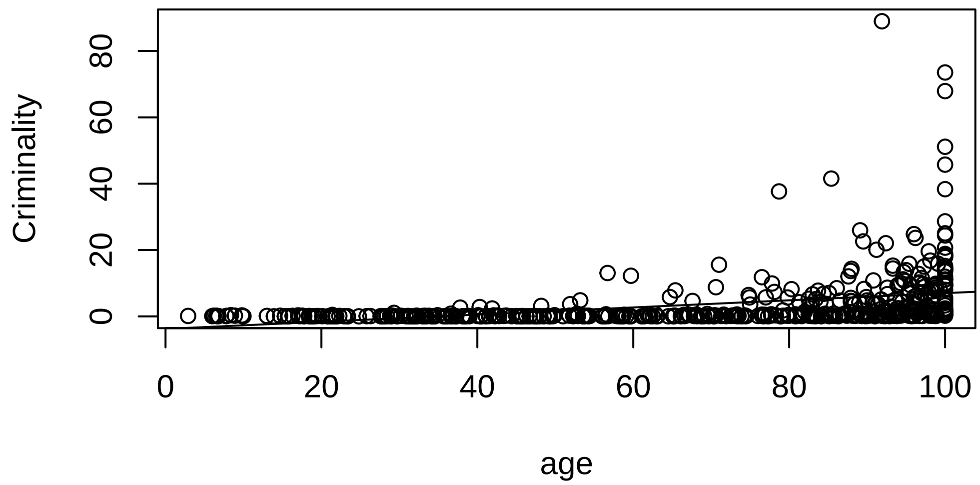

Criminality
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
1.0
chas

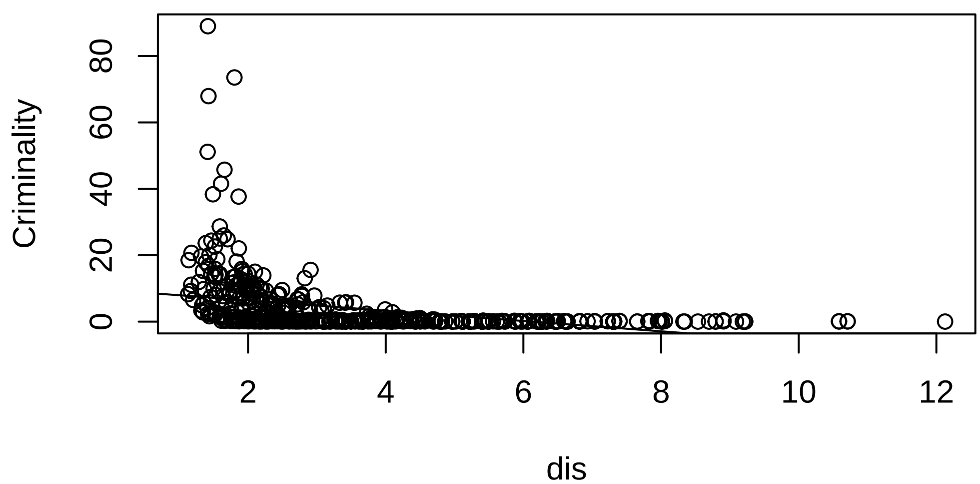


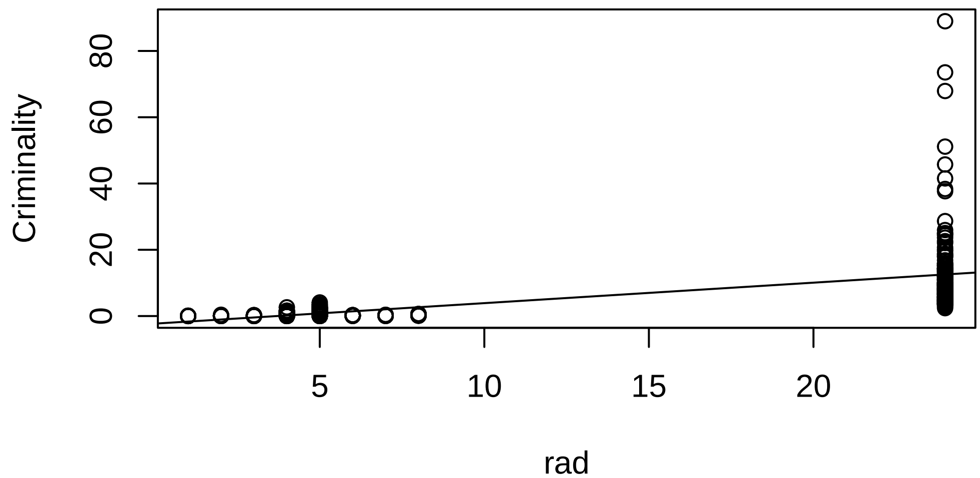
Criminality
0
20
40
60
80
0.4
0.5
0.6
0.7
0.8
nox

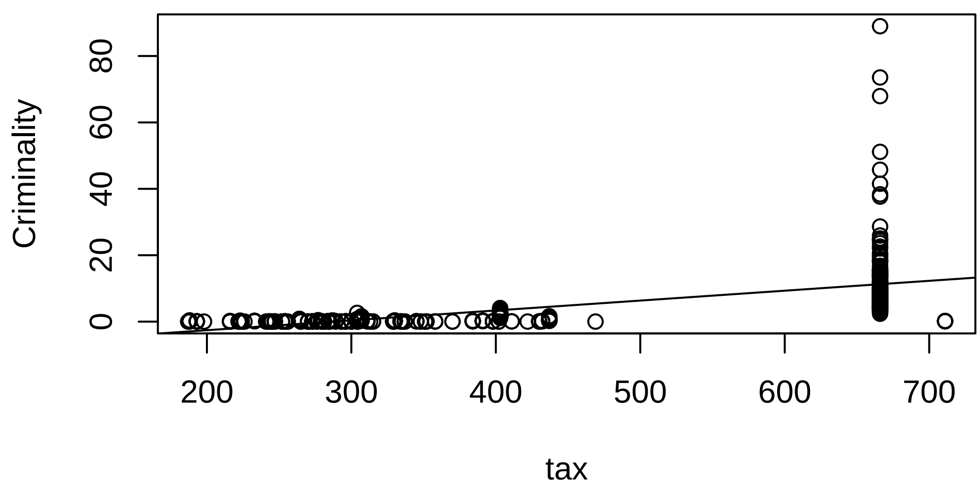


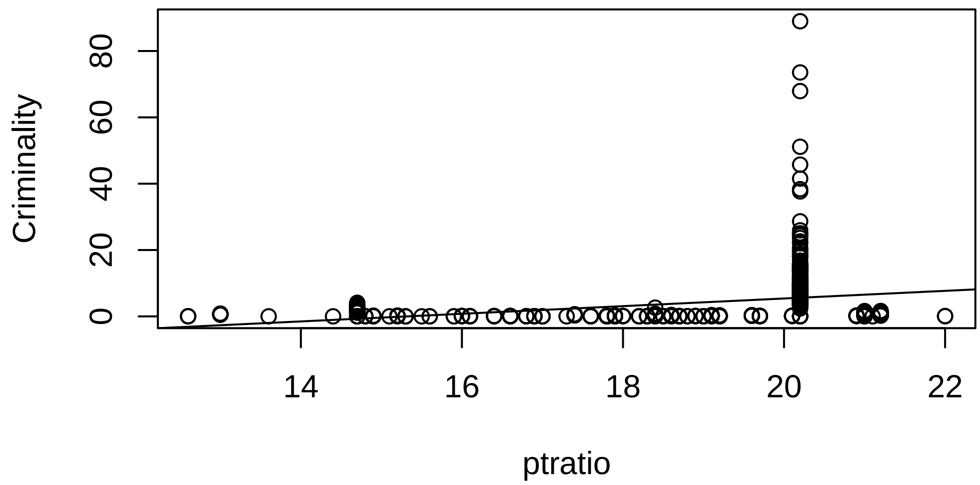
Criminality
0
20
40
60
80
4
5
6
7
8
rm

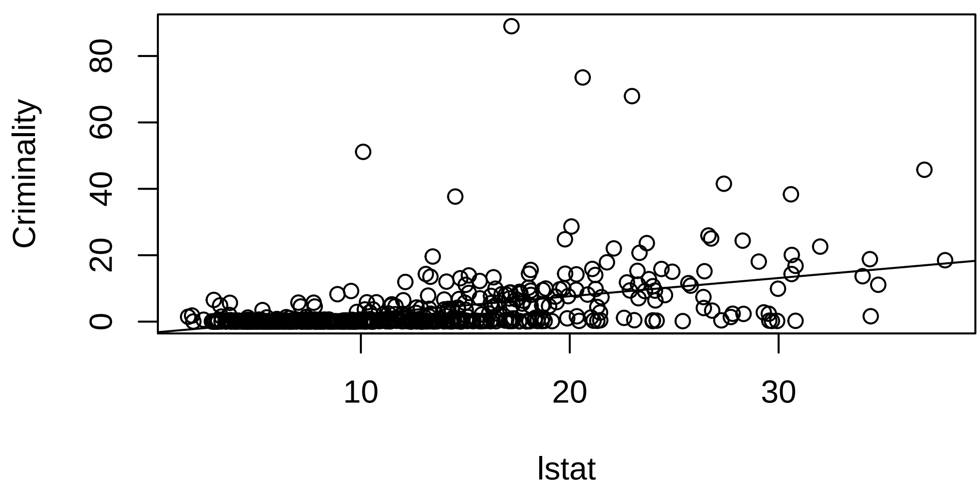

Criminality
0
20
40
60
80
0
20
40
60
80
100
age

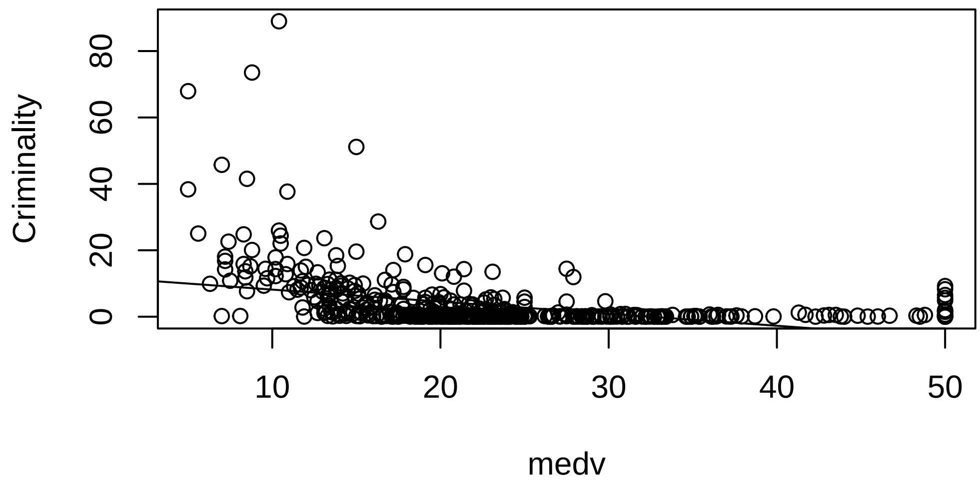

Criminality
0
20
40
60
80
2
4
6
8
10
12
dis


Criminality
0
20
40
60
80
5
10
15
20
rad


Criminality
0
20
40
60
80
200
300
400
500
600
700
tax


Criminality
0
20
40
60
80
14
16
18
20
22
ptratio


Criminality
0
20
40
60
80
10
20
30
lstat



b)

```
reg_multi <- lm(crim ~ ., df)
summary(reg_multi)
```

```
Call:
lm(formula = crim ~ ., data = df)

Residuals:
   Min     1Q Median     3Q    Max
-8.534 -2.248 -0.348  1.087 73.923

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 13.7783938  7.0818258   1.946 0.052271 .
zn           0.0457100  0.0187903   2.433 0.015344 *
indus       -0.0583501  0.0836351  -0.698 0.485709
chas        -0.8253776  1.1833963  -0.697 0.485841
nox         -9.9575865  5.2898242  -1.882 0.060370 .
rm           0.6289107  0.6070924   1.036 0.300738
age         -0.0008483  0.0179482  -0.047 0.962323
```

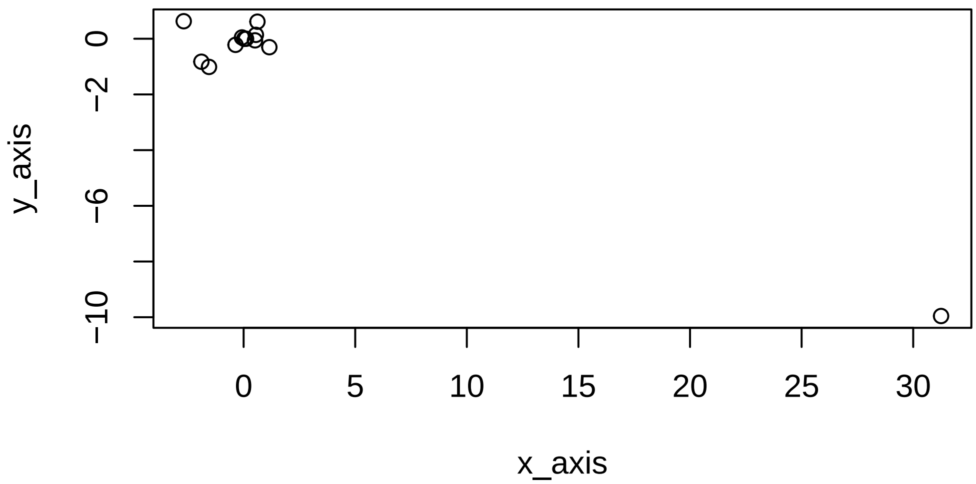
```
dis         -1.0122467  0.2824676  -3.584 0.000373 ***
rad          0.6124653  0.0875358   6.997 8.59e-12 ***
tax         -0.0037756  0.0051723  -0.730 0.465757
ptratio     -0.3040728  0.1863598  -1.632 0.103393
lstat        0.1388006  0.0757213   1.833 0.067398 .
medv        -0.2200564  0.0598240  -3.678 0.000261 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.46 on 493 degrees of freedom
Multiple R-squared:  0.4493,    Adjusted R-squared:  0.4359
F-statistic: 33.52 on 12 and 493 DF,  p-value: < 2.2e-16
```

Here in much less variables is the result significant. Only for the following (with a alpha of 0.05):

indus, dis, rad, medv

c)

```r
x_axis <-c()
y_axis <- c()
for (r in names(regs)) {
  x_axis[r] <- summary(regs[[r]])$coefficients[2,1]
}
y_axis <- summary(reg_multi)$coefficients[-1,1]
plot(x_axis, y_axis)
```



d)

```
regs_nonlinear <- list()
for (i in colnames(df[-1])){
  tmp_y <- df[["crim"]]
  tmp_x <- df[[i]]
  tmp_x_2 <- tmp_x * tmp_x
  tmp_x_3 <- tmp_x * tmp_x * tmp_x
  tmp <- lm(tmp_y ~ tmp_x + tmp_x_2 + tmp_x_3)
  regs_nonlinear[[i]] <- tmp
}
```

Chapter 4

1)

Starting from 4.2:

$$p(X) + p(X)e^{\beta_0+\beta_1 X} = e^{\beta_0+\beta_1 X}$$

$$\frac{p(X)}{1-p(X)} + \frac{p(X)e^{\beta_0+\beta_1 X}}{1-p(X)} = \frac{e^{\beta_0+\beta_1 X}}{1-p(x)}$$

$$\frac{p(X)}{1-p(X)} = \frac{e^{\beta_0+\beta_1 X}}{1-p(x)} - \frac{p(X)e^{\beta_0+\beta_1 X}}{1-p(X)}$$

$$\frac{p(X)}{1-p(X)} = \frac{e^{\beta_0+\beta_1 X} - p(X)e^{\beta_0+\beta_1 X}}{1-p(X)}$$

$$\frac{p(X)}{1-p(X)} = \frac{e^{\beta_0+\beta_1 X}(1-p(X))}{1-p(X)}$$

$$\frac{p(X)}{1-p(X)} = e^{\beta_0+\beta_1 X}$$

2) ?

4)

a) 10%

b)1%

c) 10^{100}%

d) As we can see, the higher the dimensionality goes, the less probable it is that we already have an observation which is similar in each dimension. Because of this, our averages will be based on a low sample size, making our prediction weak.

e)

p=1: the "side" is 1/10 length

p=2: 0,316227766 ($\sqrt{0.1}$)

p=100: 0.97237221 ($\sqrt[100]{0.1}$)

16) ?