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Факултет по математика и информатика
Специалност: Софтуерно инженерство
Курс: I курс

# Курсова работа по ЛААГ – Част 1

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## Зад. 1.

$$A=\begin{pmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad B=\begin{pmatrix} 4 & -1 & 1 \\ -3 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

а) $\det A = ?$

$$\det A = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}\begin{matrix} 2 & 3 \\ 1 & -1 \\ 1 & 0 \end{matrix} = -2+3-(-1+3) = 1-2 = -1$$

отг. $\det A = -1$

б) $A^{-1} = ?$

$\det A = -1 \neq 0 \quad \exists A^{-1}$

$A_{11} = \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = -1 \qquad A_{21} = -\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} = -3 \qquad A_{31} = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 4$

$A_{12} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \qquad A_{22} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \qquad A_{32} = -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1 \qquad A_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 3 \qquad A_{33} = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -5$

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \frac{1}{-1}\begin{pmatrix} -1 & -3 & 4 \\ 0 & 1 & -1 \\ 1 & 3 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 3 & -4 \\ 0 & -1 & 1 \\ -1 & -3 & 5 \end{pmatrix}$$

отг. $A^{-1} = \begin{pmatrix} 1 & 3 & -4 \\ 0 & -1 & 1 \\ -1 & -3 & 5 \end{pmatrix}$

в) $A^{-1}B$ и $BA^{-1} = ?$

$$A^{-1}\begin{pmatrix} 1 & 3 & -4 \\ 0 & -1 & 1 \\ -1 & -3 & 5 \end{pmatrix} \qquad B\begin{pmatrix} 4 & -1 & 1 \\ -3 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

$$A^{-1}B=\underset{3\times 3}{\begin{pmatrix}1 & 3 & -4\\ 0 & -1 & 1\\ -1 & -3 & 5\end{pmatrix}}\underset{3\times 3}{\begin{pmatrix}4 & -1 & 1\\ -3 & 0 & 1\\ 0 & 2 & -1\end{pmatrix}}=\underset{3\times 3}{\begin{pmatrix}-5 & -9 & 8\\ 3 & 2 & -2\\ 5 & 11 & -9\end{pmatrix}}$$

$4-9=-5$ $\qquad$ $-1-8=-9$ $\qquad$ $1+3+4=8$

$3$ $\qquad$ $2$ $\qquad$ $-1-1=-2$

$-4+9=5$ $\qquad$ $1+10=11$ $\qquad$ $-1-3-5=-9$

$$BA^{-1}=\underset{3\times 3}{\begin{pmatrix}4 & -1 & 1\\ -3 & 0 & 1\\ 0 & 2 & -1\end{pmatrix}}\underset{3\times 3}{\begin{pmatrix}1 & 3 & -4\\ 0 & -1 & 1\\ -1 & -3 & 5\end{pmatrix}}=\underset{3\times 3}{\begin{pmatrix}3 & 10 & -12\\ -4 & -12 & 17\\ 1 & 1 & -3\end{pmatrix}}$$

$4-1=3$ $\qquad$ $12+1-3=10$ $\qquad$ $-16-1+5=-12$

$-3-1=-4$ $\qquad$ $-9-3=-12$ $\qquad$ $12+5=17$

$1$ $\qquad$ $-2+3=1$ $\qquad$ $2-5=-3$

<u>отг.</u> $A^{-1}B=\begin{pmatrix}-5 & -9 & 8\\ 3 & 2 & -2\\ 5 & 11 & -9\end{pmatrix}$, $BA^{-1}=\begin{pmatrix}3 & 10 & -12\\ -4 & -12 & 17\\ 1 & 1 & -3\end{pmatrix}$

<u>Зад. 2.</u>

а) $A\begin{vmatrix}2 & 1 & -3 & 0\\ 4 & 1 & 1 & 2\\ -1 & 2 & 1 & -1\\ 1 & 5 & 2 & 0\end{vmatrix}\begin{matrix}\\ \leftarrow + \\ \cdot 2\\ \\ \end{matrix}\sim\begin{vmatrix}2 & 1 & -3 & 0\\ 2 & 5 & 3 & 0\\ -1 & 2 & 1 & -1\\ 1 & 5 & 2 & 0\end{vmatrix}$ $\qquad$ $\det A=?$

$\det A=-1A_{34}=-1\cdot 26=-26$

$$A_{34}=-\begin{vmatrix}2 & 1 & -3\\ 2 & 5 & 3\\ 1 & 5 & 2\end{vmatrix}\begin{matrix}2 & 1\\ 2 & 5\\ 1 & 5\end{matrix}=-(20+3-30-(-15+30+4))=-(-7-19)=-(-26)=26$$

отг. $\det A = -26$

б) $B \begin{vmatrix} 1 & -1 & 2 & 3 \\ 2 & 0 & 1 & 1 \\ 3 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{vmatrix} \begin{matrix} \\ \\ \\ 1\cdot(-1) \quad 1\cdot(-3) \end{matrix} \sim \begin{vmatrix} 1 & -4 & -4 & 0 \\ 2 & -1 & -1 & 0 \\ 3 & 1 & -1 & 0 \\ 0 & 1 & 2 & 1 \end{vmatrix}$ $\quad \det B = ?$

$\det B = 1\, B_{44} = 1(-14) = -14$

$B_{44} = \begin{vmatrix} 1 & -4 & -4 \\ 2 & -1 & -1 \\ 3 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & -4 \\ 2 & -1 \\ 3 & 1 \end{matrix} = 1 + 12 - 8 - (12 - 1 + 8) = 5 - 19 = -14$

отг. $\det B = -14$

в) $C \begin{vmatrix} 2 & 2 & 0 & -2 \\ 1 & 0 & 3 & 4 \\ -1 & 3 & -2 & 1 \\ 2 & 1 & 2 & -1 \end{vmatrix} \begin{matrix} \\ 1\cdot 1 \quad 1\cdot(-2) \\ \\ \\ \end{matrix} \sim \begin{vmatrix} 0 & 2 & -6 & -10 \\ 1 & 0 & 3 & 4 \\ 0 & 3 & 1 & 5 \\ 0 & 1 & -4 & -9 \end{vmatrix}$ $\quad \det C = ?$

$\det C = 1\, C_{21} = 1 \cdot 40 = 40$

$C_{21} = -\begin{pmatrix} 2 & -6 & -10 \\ 3 & 1 & 5 \\ 1 & -4 & -9 \end{pmatrix} \begin{matrix} 2 & -6 \\ 3 & 1 \\ 1 & -4 \end{matrix} = -(-18 - 30 + 120) - (-10 - 40 + 162)) =$

$= -(72 - 112) = 40$

отг. $\det C = 40$

Зад. 3.

а) $A = \begin{vmatrix} 1 & 2 & 1 & -1 \\ 2 & 0 & 1 & 1 \\ -1 & 1 & 2 & 1 \\ -2 & 1 & 0 & 3 \end{vmatrix} \begin{matrix} 1\cdot(-2) \quad 1\cdot 1 \quad 1\cdot 2 \\ \\ \\ \\ \end{matrix} \sim \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & -4 & -1 & 3 \\ 0 & 3 & 3 & 0 \\ 0 & 5 & 2 & 1 \end{vmatrix} \begin{matrix} \\ \\ 1:3 \\ \\ \end{matrix} \sim$ $\quad \operatorname{rang} A = ?$

$\sim \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & -1 & 3 \\ 0 & 5 & 2 & 1 \end{vmatrix} \begin{matrix} \\ 1\cdot 4 \quad 1\cdot(-5) \\ \\ \\ \end{matrix} \sim \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & -3 & 1 \end{vmatrix} \begin{matrix} \\ \\ 1\cdot 1 \\ \\ \end{matrix} \sim \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix} \Rightarrow \operatorname{rang} A = 4$

отг. $\operatorname{rang} A = 4$

б) $B = \begin{vmatrix} 2 & 3 & 1 & -1 \\ 2 & 4 & -2 & 2 \\ 1 & -2 & 2 & 1 \\ 5 & 5 & 1 & 2 \end{vmatrix} \sim \begin{vmatrix} 1 & -2 & 2 & 1 \\ 2 & 4 & -2 & 2 \\ 2 & 3 & 1 & -1 \\ 5 & 5 & 1 & 2 \end{vmatrix} \begin{matrix} /(-2) & /(-5) \\ + & \\ & + \\ & \end{matrix} \sim$ rang $B = ?$

$\sim \begin{vmatrix} 1 & -2 & 2 & 1 \\ 0 & 8 & -6 & 0 \\ 0 & 7 & -3 & -3 \\ 0 & 15 & -9 & -3 \end{vmatrix} /:3 \sim \begin{vmatrix} 1 & -2 & 2 & 1 \\ 0 & 5 & -3 & -1 \\ 0 & 7 & -3 & -3 \\ 0 & 8 & -6 & 0 \end{vmatrix} \begin{matrix} /(-\frac{7}{5}) & /(-\frac{8}{5}) \\ + & \\ & + \end{matrix} \sim \begin{vmatrix} 1 & -2 & 2 & 1 \\ 0 & 5 & -3 & -1 \\ 0 & 0 & 6/5 & -8/5 \\ 0 & 0 & -6/5 & 8/5 \end{vmatrix} \begin{matrix} /.1 \\ + \end{matrix} \sim$

$\sim \begin{vmatrix} 1 & -2 & 2 & 1 \\ 0 & 5 & -3 & -1 \\ 0 & 0 & 6/5 & -8/5 \\ 0 & 0 & 0 & 0 \end{vmatrix} \Rightarrow$ rang $B = 3$

отг. rang $B = 3$

в) $C = \begin{vmatrix} 3 & 1 & 0 & 1 \\ -2 & 2 & 1 & 4 \\ -1 & 5 & 2 & 9 \\ 1 & 3 & 1 & 5 \end{vmatrix} \sim \begin{vmatrix} 1 & 3 & 1 & 5 \\ -2 & 2 & 1 & 4 \\ -1 & 5 & 2 & 9 \\ 3 & 1 & 0 & 1 \end{vmatrix} /2 \; /1 \; /(-3) \sim$ rang $C = ?$

$\sim \begin{vmatrix} 1 & 3 & 1 & 5 \\ 0 & 8 & 3 & 14 \\ 0 & 8 & 3 & 14 \\ 0 & -8 & -3 & -14 \end{vmatrix} /(-1) \; /.1 \sim \begin{vmatrix} 1 & 3 & 1 & 5 \\ 0 & 8 & 3 & 14 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \Rightarrow$ rang $C = 2$

отг. rang $C = 2$

Зад. 4.

а) $\begin{cases} x_1 + x_2 + x_3 = 2 \\ x_1 - x_2 - x_3 = 0 \\ 2x_1 - x_2 + x_3 = 5 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & -1 & 0 \\ 2 & -1 & 1 & 5 \end{array}\right) /(-1) \; /(-2) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -2 & -2 \\ 0 & -3 & -1 & 1 \end{array}\right) /:(-2) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -3 & -1 & 1 \end{array}\right) /.3 \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 4 \end{array}\right) /:2 \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{matrix} \Rightarrow x_1 = 2 - 2 + 1 = 1 \\ \Rightarrow x_2 = 1 - 2 = -1 \\ \Rightarrow x_3 = 2 \end{matrix}$

отг. $(x_1, x_2, x_3) = (1, -1, 2)$

б) $\begin{cases} x_1 + x_2 - x_4 = 1 \\ x_1 - x_2 + 2x_3 - x_4 = -3 \\ 2x_1 - x_2 - x_3 = 0 \\ 3x_1 + x_2 - 2x_3 - x_4 = 3 \end{cases}$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 1 & -1 & 2 & -1 & -3 \\ 2 & -1 & -1 & 0 & 0 \\ 3 & 1 & -2 & -1 & 3 \end{array}\right) \begin{array}{l} /(-1)\ /(-2)\ /(-3) \\ \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 0 & -2 & 2 & 0 & -4 \\ 0 & -3 & -1 & 2 & -2 \\ 0 & -2 & -2 & 2 & 0 \end{array}\right) \begin{array}{l} \\ /:(-2) \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & -3 & -1 & 2 & -2 \\ 0 & -2 & -2 & 2 & 0 \end{array}\right) \begin{array}{l} \\ /.3\ /.2 \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & -4 & 2 & 4 \\ 0 & 0 & -4 & 2 & 4 \end{array}\right) \begin{array}{l} \\ \\ /(-1) \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & -4 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \\ \\ /:(-2) \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 2 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \Rightarrow x_1 = 1 - \frac{p}{2} - 1 + p = \frac{p}{2} \\ \Rightarrow x_2 = 2 + \frac{p}{2} - 1 = \frac{p}{2} + 1 \\ \Rightarrow 2x_3 = p - 2 /:2 \Rightarrow x_3 = \frac{p}{2} - 1 \\ \\ \end{array}$$

$x_4 = p,\ p \in R$

<u>отг.</u> $\{(x_1, x_2, x_3, x_4) = (\frac{p}{2}, \frac{p}{2} + 1, \frac{p}{2} - 1, p),\ p \in R\}$

в) $\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 0 \\ 2x_1 + 3x_2 - x_3 - x_4 = 0 \\ 2x_1 + x_2 - 2x_3 - 3x_4 = 0 \\ x_1 + 4x_2 - 2x_3 + 3x_4 = 0 \end{cases}$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 0 \\ 2 & 3 & -1 & -1 & 0 \\ 2 & 1 & -2 & -3 & 0 \\ 1 & 4 & -2 & 3 & 0 \end{array}\right) \begin{array}{l} /(-2)\ /(-1) \\ \\ \\ \\ \end{array} \sim \left|\begin{array}{cccc} 1 & 2 & -3 & 1 \\ 0 & -1 & 5 & -3 \\ 0 & -3 & 4 & -5 \\ 0 & 2 & 1 & 2 \end{array}\right| \begin{array}{l} \\ /(-3)\ /.2 \\ \\ \\ \end{array} \sim$$

$$\sim \left|\begin{array}{cccc} 1 & 2 & -3 & 1 \\ 0 & -1 & 5 & -3 \\ 0 & 0 & -11 & 4 \\ 0 & 0 & 11 & -4 \end{array}\right| \begin{array}{l} \\ \\ /(-1) \\ \\ \end{array} \sim \left|\begin{array}{cccc} 1 & 2 & -3 & 1 \\ 0 & -1 & 5 & -3 \\ 0 & 0 & -11 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right| \begin{array}{l} \Rightarrow x_1 = \frac{26p}{11} + \frac{12p}{11} - p = \frac{27p}{11} \\ \Rightarrow -x_2 = -\frac{20p}{11} + 3p = \frac{13p}{11} /.(-1) \Rightarrow x_2 = -\frac{13p}{11} \\ \Rightarrow -11x_3 = -4p /:(-11) \Rightarrow x_3 = \frac{4p}{11} \\ \\ \end{array}$$

$x_4 = p,\ p \in R$

<u>отг.</u> $\{(x_1, x_2, x_3, x_4) = (\frac{27p}{11}, -\frac{13p}{11}, \frac{4p}{11}, p),\ p \in R\}$

Зад. 5.

$$v_1=(1,0,1,-1) \quad v_2=(2,1,-1,0) \quad v_3=(2,-1,1,-1) \quad v_4=(1,1,1,-1) \Rightarrow \begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 0 & -1 & -1 \end{vmatrix} \begin{matrix} /(-1) & /(1) \end{matrix} \sim \begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & -3 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{vmatrix} = A$$

$$\det A = 1A_{11} = 1(-1) = -1$$

$$A_{11} = \begin{vmatrix} 1 & -1 & 1 \\ -3 & -1 & 0 \\ 2 & 1 & 0 \end{vmatrix} \begin{matrix} 1 & -1 \\ -3 & -1 \\ 2 & 1 \end{matrix} = -3-(-2) = -1$$

$\det A = -1 \neq 0 \Rightarrow$ ЛНЗ $\Rightarrow$ образува база на $\mathbb{R}^4$

$$u=(1,2,-3,0)$$

$$u = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 + \lambda_4 v_4$$

$$\Downarrow$$

$$\begin{cases} \lambda_1 + 2\lambda_2 + 2\lambda_3 + \lambda_4 = 1 \\ \lambda_2 - \lambda_3 + \lambda_4 = 2 \\ \lambda_1 - \lambda_2 + \lambda_3 + \lambda_4 = -3 \\ -\lambda_1 - \lambda_3 - \lambda_4 = 0 \end{cases} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 1 & -1 & 1 & 1 & -3 \\ -1 & 0 & -1 & -1 & 0 \end{array}\right) \begin{matrix} /(-1) & /1 \end{matrix} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & -3 & -1 & 0 & -4 \\ 0 & 2 & 1 & 0 & 1 \end{array}\right) \begin{matrix} /.3 & /(-2) \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & -4 & 3 & 2 \\ 0 & 0 & 3 & -2 & -3 \end{array}\right) /\tfrac{3}{4} \sim \left(\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & -4 & 3 & 2 \\ 0 & 0 & 0 & 1/4 & -\tfrac{3}{2} \end{array}\right) /.4 \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & -4 & 3 & 2 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right) \begin{array}{l} \Rightarrow \lambda_1 = 1-6+10+6 \Rightarrow \lambda_1 = 11 \\ \Rightarrow \lambda_2 = 2-5+6 \Rightarrow \lambda_2 = 3 \\ \Rightarrow -4\lambda_3 = 2+18 \Rightarrow -4\lambda_3 = 20 /:-4 \Rightarrow \lambda_3 = -5 \\ \Rightarrow \lambda_4 = -6 \end{array}$$

отг. $[u]_{\{v_1,v_2,v_3,v_4\}} = (11, 3, -5, -6)$

## Зад. 6.

а) $e_1' = e_1 + e_2 + 2e_3 \to e_1'(1, 1, 2)$

$e_2' = e_1 + e_2 + e_3 \to e_2'(1, 1, 1)$

$e_3' = 3e_1 + 2e_2 + e_3 \to e_3'(3, 2, 1)$

$$T = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$$ матр. на прехода

$$\det T = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 1 \\ 2 & 1 \end{matrix} = 1 + 4 + 3 - (6 + 2 + 1) = 8 - 9 = -1$$

$\det T = -1 \neq 0 \Rightarrow$ ЛНЗ $\Rightarrow$ образува база на $V$.

б) $v(1, -2, 3)$

$$x' = T^{-1} x$$

$T_{11} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$ $\quad T_{21} = -\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -(1-3) = 2$ $\quad T_{31} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = 2 - 3 = -1$

$T_{12} = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -(1-4) = 3$ $\quad T_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 - 6 = -5$ $\quad T_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -(2-3) = 1$

$T_{13} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$ $\quad T_{23} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$ $\quad T_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$$T^{-1} = \frac{1}{-1} \begin{vmatrix} -1 & 2 & -1 \\ 3 & -5 & 1 \\ -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 \\ -3 & 5 & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

$$x' = T^{-1} x = \underbrace{\begin{vmatrix} 1 & -2 & 1 \\ -3 & 5 & -1 \\ 1 & -1 & 0 \end{vmatrix}}_{3\times 3} \cdot \underbrace{\begin{vmatrix} 1 \\ -2 \\ 3 \end{vmatrix}}_{3\times 1} = \begin{pmatrix} 8 \\ -16 \\ 3 \end{pmatrix}$$

$1 + 4 + 3 = 8$

$-3 - 10 - 3 = -16$

$1 + 2 = 3$

отг. $\vec{v}(8, -16, 3)$ относно $e'$