

1.

$x$	$\cos(x)$
0.698	0.9661
0.733	0.7432
0.768	0.7193
0.803	0.6946

$$|P_1^{cc} P_1(x)| = \frac{x-x_2}{x_1-x_2} y_1 + \frac{x-x_1}{x_2-x_1} y_2$$

$$P_1(0.75) (x_1=0.733, x_2=0.768)$$

$$= \frac{0.75-0.768}{0.733-0.768} \cdot 0.7432 + \frac{0.75-0.733}{0.768-0.733} \cdot 0.7193$$

$$= 0.73159$$

$$\text{error bound: } |E_1(x)| \leq \frac{\max |f^{(n+1)}(x)|}{(n+1)!} \left| \prod_{i=0}^n (x-x_i) \right|$$

$$|E_1(x)| \leq \frac{|f^{(n+1)}(x)|}{2!} |(0.75-0.768)(0.75-0.733)| = 1.1391 \times 10^{-4}$$

$$\begin{aligned} 2. P_2^{cc} P_2(x) &= \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3 \\ &= \sum_{i=1}^3 L_i(x) y_i \end{aligned}$$

$$P_2(0.75) (x_1=0.698, x_2=0.733, x_3=0.768)$$

$$= \frac{0.017 \cdot -0.018}{-0.035 \cdot -0.070} \cdot 0.7432 + \frac{0.052 \cdot -0.018}{0.035 \cdot -0.035} \cdot 0.7193 + \frac{0.052 \cdot 0.017}{0.070 \cdot 0.035} \cdot 0.6946$$

$$= 0.731716$$

$$\begin{aligned} \text{error bound } |E_2(x)| &\leq \frac{|f^{(n+1)}(x)|}{3!} |(0.75-0.698)(0.75-0.733)(0.75-0.768)| \\ &= 1.84234 \times 10^{-6} \end{aligned}$$

$$\text{三階 } L_i = \sum_{\substack{j=1 \\ j \neq i}}^4 \frac{(x-x_j)}{(x_i-x_j)} \Rightarrow L_1 = -\frac{2703}{42875}$$

$$L_2 = 0.5785189504$$

$$L_3 = 0.5463790087$$

$$L_4 = -\frac{2652}{42875}$$

$$\Rightarrow P_3(0.75) = \sum_{i=1}^4 L_i y_i = 0.7317039$$

$$\text{error bound: } E_4(x) = \frac{0.7661}{4!} |(0.75-0.698)(0.75-0.733)(0.75-0.768)(0.75-0.803)| \\ = 2.692 \times 10^{-8}$$

四階：無第五個參考點  $\Rightarrow$  無法求

2.

$x$	$e^{-x}$
0.3	0.740818
0.4	0.670320
0.5	0.606531
0.6	0.548812

$$x - e^{-x} = 0 \Rightarrow x = e^{-x} = y$$

$$L_0 = \frac{(y-0.670320)(y-0.606531)(y-0.548812)}{(0.740818-0.670320)(0.740818-0.606531)(0.740818-0.548812)}$$

$$L_1 = \frac{(y-0.740818)(y-0.606531)(y-0.548812)}{(0.670320-0.670320)(0.670320-0.606531)(0.670320-0.548812)}$$

$$L_2 = \frac{(y-0.740818)(y-0.670320)(y-0.548812)}{(0.606531-0.740818)(0.606531-0.670320)(0.606531-0.548812)}$$

$$L_3 = \frac{(y-0.740818)(y-0.670320)(y-0.606531)}{(0.548812-0.740818)(0.548812-0.670320)(0.548812-0.606531)}$$

使用 wolfram alpha:  $x = 0.567185$ , 使用計算機解方程式:  $x = 0.5671432904$

3. T	0	3	5	8	13
D	0	200	375	620	990
V	75	77	80	74	72

t D(t)

0 0

0 0 75

3 200  $\frac{200}{3} - \frac{25}{9}$

3 200 77  $\frac{31}{9} \frac{56}{27}$

t D(t)

8 620

8 620 74

13 990  $\frac{990-620}{5} = 74 \frac{74 \cdot 74}{5} =$

13 990 72  $\frac{72 \cdot 74}{5} = \frac{2}{5} \frac{-\frac{2}{5} - 0}{5} = -\frac{2}{25}$

$$H_3(x) = 620 + 74(x-8) + 0(x-8)^2 + -\frac{2}{25}(x-8)^2(x-13)$$

$$H_3'(x) = 74 + -\frac{2}{25}[2(x-8)(x-13) + (x-8)^2] = -\frac{6}{25}x^2 + \frac{116}{25}x + 52.24$$

$$H_3''(x) = -\frac{12}{25}x + \frac{116}{25}$$

a)  $H_3(10) = 768.96 \text{ (ft)}$   $H'(10) = 74.64 \text{ (ft/s)}$

b)  $55 \text{ mi/hr} = 55 \cdot \frac{5280}{3600} = 80.6 \text{ ft}$

$$H_3(x) (x \in (0,3)) = 0 + 75(x-0) + -\frac{25}{9}(x-0)^2 + \frac{56}{27}(x-0)^2(x-3)$$

$$H_3'(x) (x \in (0,3)) = 75 + -\frac{50}{9}x + \frac{56}{27}(2x(x-3) + x^2) \quad \frac{56}{9}(x^2-2x)$$

$$= 75 - \frac{162}{9}x + \frac{56}{9}x^2$$

$$H_3''(x) (x \in (0,3)) = -\frac{162}{9} + \frac{112}{9}x$$

$$H_3'''(x) (x \in (0,3)) = \frac{112}{9} > 0 \Rightarrow \text{极小值为 min} \Rightarrow [H_3(x)]_{\max} = 77 (x \in (0,3)) < 80.6$$

$t$	$D(t)$			
3	200			
3	200	77		
5	375	87.5	5.25	
5	375	80	-3.75	-4.5

$t$				
5	375			
5	375	80		
8	620	$\frac{245}{3}$	$\frac{5}{9}$	
8	620	74	$-\frac{23}{9}$	$-\frac{28}{27}$

$x \in (3, 5)$

$$H_3(x) = 200 + 77(x-3) + 5.25(x-3)^2 + -4.5(x-3)^2(x-5)$$

$$H_3'(x) = 77 + 10.5(x-3) - 4.5[2(x-3)(x-5) + (x-3)^2]$$

$$H_3''(x) = 10.5 - 4.5[2(x-5) + 2(x-3) + 2(x-3)]$$

$$H_3'''(x) = -13.5 \rightarrow \text{極值為 max}$$

$$H_3''(x) = 0 \rightarrow x = \frac{73}{18}$$

$$\Rightarrow H_3'(x) \Big|_{x=\frac{73}{18}} = \frac{209}{24} = 92.041\bar{6} \approx 80.6$$

$\Rightarrow$  在 3-5 秒時第一次超速

$$77 + 10.5(x-3) - 4.5[3x^2 - 22x + 39] = 80.6$$

$$x = 3.137627 \vee 4.97348$$

$t = 3.137627$  第一次超速

$$c) x \in (5, 8)$$

$$H_3(x) = 375 + 80(x-5) + \frac{5}{9}(x-5)^2 - \frac{28}{27}(x-5)^2(x-8)$$

$$\begin{aligned} H'_3(x) &= 80 + \frac{10}{9}(x-5) - \frac{28}{27} [2(x-5)(x-8) + (x-5)^2] \\ &= -\frac{28}{9}x^2 + \frac{346}{9}x - \frac{260}{9} \end{aligned}$$

$2x^2 - 26x + 80$        $x^2 = 10x + 25$   
 $3x^2 - 36x + 105$

$$H''_3(x) = -\frac{56}{9}x + \frac{346}{9}$$

$$H'''_3(x) = -\frac{56}{9} < 0 \Rightarrow \text{Extremumwert max} \Rightarrow H''_3(x) = 0 \Rightarrow x = \frac{193}{28}$$

$$H'_3(x) \Big|_{x=\frac{193}{28}} = \frac{22649}{252} = 89.87698$$

$$x \in (8, 13)$$

$$H'''_3(x) < 0 \quad H''_3(x) = 0 \Rightarrow x = \frac{29}{3} \quad H'_3(x) \Big|_{x=\frac{29}{3}} = \frac{224}{3} = 74.\bar{6}$$

$$\max(H_3(x)) \quad (x \in (0, 13)) = 92.041\bar{6} \text{ (f/s)}$$

$$t = \frac{73}{18} \text{ s}$$