HW4 $\int_1^2 e^x \sin(4x)\,dx$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
-2.05720491	-2.85877	-3.307383	-3.241657	-2.559912449	-1.2522534	0.577272	2.7047505	4.80141028	6.471408256	7.310423586

(a) Trapezoidal

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right] = \frac{0.1}{2}\cdot(2\times 1.334866137 + 7.310423586 - 2.05720491)$$

$$= 0.3961568528$$

(b) simpson's rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{even} f(x_i) + 4\sum_{odd} f(x_i) + f(b)\right]$$

$$= 0.3856639$$

(c) midpoint

$$\int_a^b f(x)\,dx = 2h\left[\sum_{odd} f(x_i)\right]$$

$$= 0.3646957712$$

2. $\int_1^{1.5} x^2 \ln x \, dx$

$a=1 \quad b=1.5$

$x = \frac{b-a}{2}\eta + \frac{a+b}{2}$

$= \frac{1}{4}\eta + \frac{5}{4}$

$\Rightarrow dx = \frac{1}{4} d\eta$

$n=3$

$\Rightarrow \begin{cases} \eta_1 = -\sqrt{\frac{3}{5}} \\ \eta_2 = 0 \\ \eta_3 = \sqrt{\frac{3}{5}} \end{cases}$ $\quad C_1 = \frac{5}{9} \quad C_2 = \frac{8}{9} \quad C_3 = \frac{5}{9}$

$\Rightarrow \begin{cases} x_1 = -\frac{1}{4}\sqrt{\frac{3}{5}} + \frac{5}{4} = 1.056350833 \\ x_2 = \frac{5}{4} \\ x_3 = \frac{1}{4}\sqrt{\frac{3}{5}} + \frac{5}{4} = 1.443649167 \end{cases}$

$\int_1^{1.5} x^2 \ln x \, dx$

$\frac{1.5-1}{2} \sum_{i=1}^{3} C_i f(x_i) = 0.192259399$

$n=4$

$\begin{cases} \eta_1 = -\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}} \Rightarrow x_1 = 1.034715922 \\ \eta_2 = -\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}} \Rightarrow x_2 = 1.165004739 \\ \eta_3 = \sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}} \Rightarrow x_3 = 1.334995261 \\ \eta_4 = \sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}} \Rightarrow x_4 = 1.465284078 \end{cases}$ $\quad C_1 = \frac{18-\sqrt{30}}{36} \quad C_2 = \frac{18+\sqrt{30}}{36} \quad C_3 = \frac{18+\sqrt{30}}{36} \quad C_4 = \frac{18-\sqrt{30}}{36}$

$\frac{b-a}{2} \sum_{i=1}^{4} C_i f(x_i) = \frac{1}{4} \cdot (0.01270996 + 0.1351793 + 0.33580949$

$+ 0.28533887) = 0.19225935$

3. $\int_0^{\frac{\pi}{4}} \int_{\sin x}^{\cos x} (2y\sin x + \cos^2 x)\, dy\, dx$

(a)

for $\int_{\sin x}^{\cos x} (2y\sin x + \cos^2 x)\, dy$

$m=4 \Rightarrow k(x) = \frac{\cos x - \sin x}{4} \qquad y_j = \sin x + \left(\frac{\cos x - \sin x}{4}\right) j$

$f(x, y_0) = 2\sin^2 x + \cos^2 x = 1 + \frac{1-\cos 2x}{2} = \frac{3}{2} - \frac{1}{2}\cos 2x$ (✓)

$f(x, y_1) = 1 + \frac{1}{4}\sin 2x + \frac{1}{4}(1-\cos 2x) = \frac{5}{4} + \frac{1}{4}(\sin 2x - \cos 2x)$ (✓)

$f(x, y_2) = 1 + \frac{1}{2}\sin 2x$ (✓)

$f(x, y_3) = \frac{1}{2} + \frac{3}{4}\sin 2x + \frac{1}{4}(1+\cos 2x)$

$= \frac{3}{4} + \frac{3}{4}\sin 2x + \frac{1}{4}\cos 2x$

$f(x, y_4) = \sin 2x + \frac{1}{2}(1+\cos 2x) = \frac{1}{2} + \sin 2x + \frac{1}{2}\cos 2x$

$\frac{(\cos x - \sin x)}{12}\Big[\frac{3}{2} - \cancel{\frac{1}{2}\cos 2x} + 4\left(\frac{5}{4} + \frac{1}{4}(\sin 2x - \cancel{\cos 2x}) + \frac{3}{4} + \frac{3}{4}\sin 2x + \cancel{\frac{1}{4}\cos 2x}\right)$

$+ 2\left(1 + \frac{1}{2}\sin 2x\right) + \frac{1}{2} + \sin 2x + \cancel{\frac{1}{2}\cos 2x}\Big]$

$= \frac{(\cos x - \sin x)}{\cancel{12}\ 2}\left(\overset{2}{\cancel{12}} + \cancel{6}\sin 2x\right) = \frac{1}{2}(\cos x - \sin x)(2 + \sin 2x)$

$n=4 \Rightarrow h = \frac{\frac{\pi}{4} - 0}{4} = \frac{\pi}{16} \Rightarrow x_i = \frac{\pi}{16} i$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2}(\cos x-\sin x)(2+\sin 2x)\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{\pi}{16}\left\{2+2\left(\cos\frac{\pi}{8}-\sin\frac{\pi}{8}\right)\left(2+\sin\frac{\pi}{4}\right)+4\left(\cos\frac{\pi}{16}\frac{1}{3}-\sin\frac{\pi}{16}\frac{1}{3}\right)\left(2+\sin\frac{\pi}{8}\frac{1}{3}\right)\right.$$

$$\left. + \; 0\right]$$

$$= 0.511987544$$

4. (a) $x = t^{-1}$

$\Rightarrow dx = -t^{-2}dt$

$\Rightarrow \underline{\int_0^1 x^{-\frac{1}{4}}} \sin x dx$

$\frac{4}{7}x^{\frac{7}{4}} - \frac{4}{15}x^{\frac{15}{4}} \times \frac{1}{6}$

$P_4 = x - \frac{1}{3!}x^3$

$x^{\frac{3}{4}} \quad x^{\frac{11}{4}}$

$$\int_0^1 \frac{\sin x - x + \frac{1}{3!}x^3}{x^{\frac{1}{4}}} dx + \int_0^1 \frac{x - \frac{1}{3!}x^3}{x^{\frac{1}{4}}} dx$$

$= 0.526984129$

$x_i = 0.25 i$

$$\Rightarrow \int_0^1 \frac{\sin x}{x^{\frac{1}{4}}} dx \approx 0.526984129$$

$(\sin x - p_4(x) \approx 0)$

b) $\int_1^0 t^4 \sin(t^{-1}) \cdot \frac{-1}{t^2} dt$

$= \int_0^1 t^2 \sin(t^{-1}) dt$

$n=4 \Rightarrow t_i = \frac{i}{8}$

$\frac{1}{3} \cdot \frac{1-0}{8} \Big[0 + 2\left(\frac{1}{16}\sin 4 + \frac{1}{4}\sin 2 + \frac{9}{16}\sin\frac{4}{3} \right)$

$+ 4\left(\frac{1}{64}\sin 8 + \frac{9}{64}\sin\frac{8}{3} + \frac{25}{64}\sin\frac{8}{5} + \frac{49}{64}\sin\frac{8}{7} \right) + \sin 1 \Big]$

$= 0.2909$