1. $y' = 1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2$ $\quad y = t\tan(\ln t) \quad y(1) = 0$

(a) $h = 0.1 \quad t_0 = 1 \quad y(t_{i+1}) = y(t_i) + h\,y'|_{t=t_i}$

	Euler's method	Exact value	absolute error	relative error
1.1	0.1	0.105159815 8	0.005159815 8	4.9066%
1.2	0.209917355 4	0.221242997 8	0.011325 4146	5.119%
1.3	0.330470558 3	0.349121132 3	0.018650574	5.3421%
1.4	0.462335354 4	0.489681663 8	0.027328 1164	5.5808%
1.5	0.606285 4744	0.643875 3319	0.037589 8575	5.8381%
1.6	0.763044897 2	0.812752 7406	0.049711 2534	6.002%
1.7	0.933475 0298	0.999494 129	0.066019 0992	6.418%
1.8	1.118536 432	1.199438 64	0.080901 908	6.745%
1.9	1.319292 612	1.420115 837	0.100823 225	7.0996%
2	1.536943 276	1.661281 756	0.124338 48	7.4845%
2.1	1.772845 306	1.924961 651	0.152116 345	7.902%
2.2	2.028535 909	2.213501 813	0.184965 904	8.3563%
2.3	2.305761 879	2.527633 322	0.221871 443	8.8499%
2.4	2.606514 056	2.876551 42	0.270037 364	9.3875%
2.5	2.933068 931	3.258015 365	0.324946 674	9.9738%
2.6	3.288039 755	3.678475 37	0.390435 776	10.614%

(b) $y' = 1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2 \quad \frac{\partial f}{\partial t} = \frac{-y(t+2y)}{t^3}$

$$\Rightarrow y(t_{i+1}) = y(t_i) + h\left(1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2\right) + \frac{h^2}{2}\left[-\frac{y(t+2y)}{t^3} + \left(1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2\right)\left(\frac{1}{t} + \frac{2y}{t^2}\right)\right]$$

$$= y(t_i) + \left[h + \frac{h^2}{2}\left(\frac{1}{t} + \frac{2y}{t^2}\right)\right]\left(1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2\right) - \frac{h^2}{2}\frac{y(t+2y)}{t^3}$$

這項開始計算搜尋法

	Taylor's method of order 2	abs error	relative error
1.1	0.105	0.000159.8168	0.15197%
1.2	0.2209586017	0.000284171	0.12844%
1.3	0.3487845489	0.000336834	0.09641%
1.4	0.4893859088	0.000029575	0.0604%
1.5	0.6437298549	0.000014794792	0.0229%
1.6	0.8128921832	0.000011944326	0.0147%
1.7	0.9980094906	0.000051536	0.05169%
1.8	1.200489695 9	0.000108510559	0.08963%
1.9	1.42185434 98	0.000173851 28	0.12242%
2	1.6638732834	0.000255078	0.15354%
2.1	1.9285876653	0.000362601 43	0.18887%
2.2	2.2183628353	0.000486079 23	0.21959%
2.3	2.5359506706	0.000631734 86	0.24973%
2.4	2.8845725552	0.000802110 52	0.24884%
2.5	3.2680164651	0.001000110 01	0.30697%
2.6	3.6907660432	0.012290712	0.33413%

2. $u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t \quad u_1(0) = \frac{4}{3}$

$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t \quad u_2(0) = \frac{2}{3}$

$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t$

```
--- Runge-Kutta 4階, h = 0.1 ---
t        u1 (RK4)        u1 (Exact)      Error(u1)       u2 (RK4)        u2 (Exact)      Error(u2)
0.00     1.333333        1.333333        0.00e+00        0.666667        0.666667        1.11e-16
0.10     -3.052437       1.793063        4.85e+00        8.989305        -1.032002       1.00e+01
0.20     -23.847795      1.423902        2.53e+01        51.192704       -0.874681       5.21e+01
0.30     -130.165202     1.131577        1.31e+02        269.269193      -0.724999       2.70e+02
0.40     -680.231485     0.909409        6.81e+02        1399.368584     -0.608214       1.40e+03
0.50     -3531.299585    0.738788        3.53e+03        7258.241839     -0.515658       7.26e+03
0.60     -18312.795052   0.605710        1.83e+04        37634.955483    -0.440411       3.76e+04
0.70     -94951.331907   0.499860        9.50e+04        195131.871735   -0.377404       1.95e+05
0.80     -492306.465639  0.413671        4.92e+05        1011721.872078  -0.322954       1.01e+06
0.90     -2552513.623867         0.341614        2.55e+06        5245578.826590  -0.274409       5.25e+06
1.00     -13234278.789168        0.279675        1.32e+07        27197287.206587         -0.229888       2.72e+07

--- Runge-Kutta 4階, h = 0.05 ---
t        u1 (RK4)        u1 (Exact)      Error(u1)       u2 (RK4)        u2 (Exact)      Error(u2)
0.00     1.333333        1.333333        0.00e+00        0.666667        0.666667        1.11e-16
0.05     1.721880        1.912059        1.90e-01        -0.499599       -0.909077       4.09e-01
0.10     1.726915        1.793063        6.61e-02        -0.832598       -1.032002       1.99e-01
0.15     1.617161        1.601967        1.52e-02        -0.890373       -0.961459       7.11e-02
0.20     1.481687        1.423902        5.78e-02        -0.861042       -0.874681       1.36e-02
0.25     1.348945        1.267646        8.13e-02        -0.807505       -0.795221       1.23e-02
0.30     1.227063        1.131577        9.55e-02        -0.750341       -0.724999       2.53e-02
0.35     1.117478        1.012999        1.04e-01        -0.695886       -0.663060       3.28e-02
0.40     1.019525        0.909409        1.10e-01        -0.645732       -0.608214       3.75e-02
0.45     0.931977        0.818630        1.13e-01        -0.599934       -0.559389       4.05e-02
0.50     0.853541        0.738788        1.15e-01        -0.558092       -0.515658       4.24e-02
0.55     0.783017        0.668275        1.15e-01        -0.519706       -0.476225       4.35e-02
0.60     0.719337        0.605710        1.14e-01        -0.484290       -0.440411       4.39e-02
0.65     0.661560        0.549909        1.12e-01        -0.451407       -0.407635       4.38e-02
0.70     0.608868        0.499860        1.09e-01        -0.420673       -0.377404       4.33e-02
0.75     0.560547        0.454695        1.06e-01        -0.391754       -0.349296       4.25e-02
0.80     0.515980        0.413671        1.02e-01        -0.364365       -0.322954       4.14e-02
0.85     0.474633        0.376158        9.85e-02        -0.338259       -0.298076       4.02e-02
0.90     0.436043        0.341614        9.44e-02        -0.313226       -0.274409       3.88e-02
0.95     0.399812        0.309583        9.02e-02        -0.289089       -0.251739       3.74e-02
1.00     0.365600        0.279675        8.59e-02        -0.265698       -0.229888       3.58e-02
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