1.
$$\begin{bmatrix} 1.19 & 2.11 & -100 & 1 \\ 14.2 & -0.112 & 12.2 & -1 \\ 0 & 100 & -99.9 & 1 \\ 15.3 & 0.11 & -13.1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 3.44 \\ 2.15 \\ 4.66 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1.19 & 2.11 & -100 \\ -1 & 14.2 & -0.112 & 12.2 \\ 1 & 0 & 100 & -99.9 \\ -1 & 15.3 & 0.11 & -13.1 \end{bmatrix} \begin{bmatrix} x_4 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 3.44 \\ 2.15 \\ 4.66 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1.19 & 2.11 & -100 \\ 0 & 15.39 & 1.998 & -87.8 \\ 0 & -1.19 & 99.89 & 0.1 \\ 0 & 16.49 & 2.22 & -113.1 \end{bmatrix} \begin{bmatrix} x_4 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 4.56 \\ 1.03 \\ 5.28 \end{bmatrix}$$

$x_1 = 0.176776\,3296$

$x_2 = 0.012692\,10289$

$x_3 = -0.020661\,20121$

$x_4 = -1.183264\,29$

(用計算機可算三元一次eq)

$$\Rightarrow \begin{bmatrix} 1 & 1.19 & 2.11 & -100 \\ 0 & 15.39 & 1.998 & -89.8 \\ 0 & 0 & 98.0449123 & -6.688953866 \\ 0 & 0 & 0.09719298246 & -19.02449643 \end{bmatrix} \begin{bmatrix} x_4 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 4.56 \\ 1.38259289 \\ \frac{266}{625} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1.19 & 2.11 & -100 \\ 0 & 15.39 & 1.998 & -89.8 \\ 0 & 0 & 98.045 & -6.689 \\ 0 & 0 & 0 & -19.01904683 \end{bmatrix} \begin{bmatrix} x_4 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 4.56 \\ 1.386 \\ 0.392959243 \end{bmatrix}$$

$\Rightarrow x_3 = -0.0206612526$

$x_2 = 0.01240717046$

$x_1 = 0.1767740822$

$x_4 = -1.169193554$

2 $A = \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$

$\because AA^{-1} = I^{4\times 4}$

$A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$

$$\begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 5.2 & 0 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$a_{11} = \frac{43}{261}$

$a_{21} = -\frac{7}{87}$

$a_{31} = \frac{10}{261}$

$a_{41} = \frac{-4}{261}$

同理 $a_{12}=-\frac{7}{87}$ $a_{13}=\frac{10}{261}$

$a_{22}=\frac{11}{29}$ $a_{23}=\frac{5}{87}$

$a_{32}=\frac{5}{87}$ $a_{33}=\frac{55}{261}$

$a_{42}=-\frac{2}{87}$ $a_{43}=\frac{-22}{261}$

$$\begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 5/2 & 0 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \\ a_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{2}{5} \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} a_{14} &= -\frac{4}{261} \\ a_{24} &= -\frac{2}{87} \\ a_{34} &= -\frac{22}{261} \\ a_{44} &= \frac{44}{1305} \end{aligned}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{73}{261} & -\frac{7}{87} & \frac{10}{261} & -\frac{4}{261} \\ -\frac{7}{87} & \frac{11}{29} & \frac{5}{87} & -\frac{2}{87} \\ \frac{10}{261} & \frac{5}{87} & \frac{55}{261} & -\frac{22}{261} \\ -\frac{4}{261} & -\frac{2}{87} & \frac{-22}{261} & \frac{44}{1305} \end{bmatrix}$$

3.

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ 0 & l_{32} & l_{33} & 0 \\ 0 & 0 & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & 0 & 0 \\ 0 & 1 & u_{23} & 0 \\ 0 & 0 & 1 & u_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$l_{11} = 3 \quad 3u_{12} = -1 \Rightarrow u_{12} = -\frac{1}{3}$

$l_{21} = -1 \quad u_{12}l_{21} + l_{22} = 3 \Rightarrow l_{22} = \frac{8}{3}$

$u_{23}l_{22} = -1 \quad u_{23} = -\frac{3}{8}$

$l_{32} = -1 \quad l_{32}u_{23} + l_{33} = 3 \Rightarrow l_{33} = \frac{21}{8}$

$l_{33}u_{34} = -1 \Rightarrow u_{34} = -\frac{8}{21}$

$l_{43} = -1 \quad l_{43}u_{34} + l_{44} = 3 \Rightarrow l_{44} = \frac{55}{21}$

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & 0 \\ 0 & 0 & -1 & \frac{55}{21} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & 0 \\ 0 & 0 & -1 & \frac{55}{21} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & 0 \\ 0 & 0 & -1 & \frac{55}{21} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} y_1 &= \frac{2}{3} \\ y_2 &= \frac{11}{8} \\ y_3 &= \frac{43}{21} \\ y_4 &= \frac{64}{55} \end{aligned}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{11}{8} \\ \frac{43}{21} \\ \frac{64}{55} \end{bmatrix}$$

$$\Rightarrow \begin{aligned} x_4 &= \frac{64}{55} \\ x_3 &= \frac{107}{55} \\ x_2 &= \frac{129}{55} \\ x_1 &= \frac{79}{55} \end{aligned}$$