1. Use Gaussian elimination and pivoting technique to solve

E94111061 趙芯苡

$$1.19x_1 + 2.11x_2 - 100x_3 + x_4 = 1.12$$

$$14.2x_1 - 0.112x_2 + 12.2x_3 - x_4 = 3.44$$

$$100x_2 - 99.9x_3 + x_4 = 2.15$$

$$15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16$$

$$\left[\begin{array}{cccc|c} 1.19 & 2.11 & -100 & 1 & 1.12 \\ 14.2 & -0.112 & 12.2 & -1 & 3.44 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 15.3 & 0.11 & -13.1 & -1 & 4.16 \end{array}\right] = \left[\begin{array}{cccc|c} 15.3 & 0.11 & -13.1 & -1 & 4.16 \\ 0 & -0.214 & 24.37 & -0.071 & -0.426 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 2.101 & -99 & 1.077 & 0.797 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 15.3 & 0.11 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & 23.144 & -0.049 & -0.369 \\ 0 & 0 & -77.001 & 1.055 & 0.352 \end{array}\right] = \left[\begin{array}{cccc|c} 15.3 & 0.11 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & -77.001 & 1.055 & 0.352 \\ 0 & 0 & 0 & 0.266 & -0.264 \end{array}\right]$$

$$x_4 = \frac{-0.264}{0.266} = -0.994$$

$$x_3 = \frac{0.352 - (-0.994) \times 1.055}{-77.001} = -0.01371$$

$$x_2 = \frac{2.15 - (-0.994) - (-99.9)(-0.01371)}{100} = 0.0442$$

$$x_1 = \frac{4.16 + (-0.994) + (13.1)(-0.1371) - (0.11)(0.0442)}{15.3} = 0.366 \quad \#$$

2. Find the inverse of the matrix A where

$$A=\begin{bmatrix}4&1&-1&0\\1&3&-1&0\\-1&-1&6&2\\0&0&2&5\end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc}4&1&-1&0&1&0&0&0\\1&3&-1&0&0&1&0&0\\-1&-1&6&2&0&0&1&0\\0&0&2&5&0&0&0&1\end{array}\right]$$

$$=\left[\begin{array}{cccc|cccc}1&\frac{1}{4}&-\frac{1}{4}&0&\frac{1}{4}&0&0&0\\1&3&-1&0&0&1&0&0\\-1&-1&6&2&0&0&1&0\\0&0&2&5&0&0&0&1\end{array}\right]$$

$$\xrightarrow{-\frac{1}{11}\,\text{row}_2+\text{row}_1,\ \frac{3}{11}\,\text{row}_2+\text{row}_3}\left[\begin{array}{cccc|cccc}1&\frac{1}{4}&-\frac{1}{4}&0&\frac{1}{4}&0&0&0\\0&\frac{11}{4}&-\frac{3}{4}&0&-\frac{1}{4}&1&0&0\\0&-\frac{3}{4}&\frac{23}{4}&2&\frac{1}{4}&0&1&0\\0&0&2&5&0&0&0&1\end{array}\right]$$

$$\xrightarrow{\frac{2}{61}\,\text{row}_4+\text{row}_1,\ \frac{3}{61}\,\text{row}_4+\text{row}_2,\ -\frac{22}{61}\,\text{row}_4+\text{row}_3}\left[\begin{array}{cccc|cccc}1&0&-\frac{27}{11}&0&\frac{3}{11}&-\frac{1}{11}&0&0\\0&1&-\frac{7}{11}&0&-\frac{1}{11}&\frac{4}{11}&0&0\\0&0&\frac{61}{11}&2&\frac{2}{11}&\frac{3}{11}&1&0\\0&0&2&5&0&0&0&1\end{array}\right]$$

$$=\left[\begin{array}{cccc|cccc}1&0&0&\frac{45}{61}&\frac{11}{61}&\frac{5}{61}&\frac{2}{61}&0\\0&1&0&\frac{6}{61}&\frac{1}{61}&\frac{23}{61}&\frac{3}{61}&0\\0&0&\frac{61}{11}&2&\frac{2}{11}&\frac{3}{11}&1&0\\0&0&0&\frac{261}{61}&-\frac{4}{61}&-\frac{6}{61}&-\frac{22}{61}&1\end{array}\right]$$

$$\xrightarrow{-\frac{4}{61}\,\text{row}_4+\text{row}_1,\ -\frac{6}{61}\,\text{row}_4+\text{row}_2,\ \frac{22}{61}\,\text{row}_4+\text{row}_3}\left[\begin{array}{cccc|cccc}1&0&0&\frac{45}{61}&\frac{11}{61}&\frac{5}{61}&\frac{2}{61}&0\\0&1&0&\frac{6}{61}&\frac{1}{61}&\frac{23}{61}&\frac{3}{61}&0\\0&0&1&\frac{22}{61}&\frac{2}{61}&\frac{3}{61}&\frac{11}{61}&0\\0&0&0&1&-\frac{4}{261}&-\frac{6}{261}&-\frac{22}{261}&\frac{61}{261}\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc}1&0&0&0&\frac{22}{261}&-\frac{11}{87}&\frac{10}{261}&\frac{4}{261}\\0&1&0&0&-\frac{7}{87}&\frac{11}{29}&\frac{5}{87}&\frac{2}{87}\\0&0&1&0&\frac{10}{261}&\frac{5}{87}&\frac{55}{261}&\frac{22}{261}\\0&0&0&1&-\frac{4}{261}&-\frac{2}{87}&-\frac{22}{261}&\frac{61}{261}\end{array}\right]$$

逆矩阵：$\begin{bmatrix}\frac{22}{261}&-\frac{11}{87}&\frac{10}{261}&\frac{4}{261}\\-\frac{7}{87}&\frac{11}{29}&\frac{5}{87}&\frac{2}{87}\\\frac{10}{261}&\frac{5}{87}&\frac{55}{261}&\frac{22}{261}\\-\frac{4}{261}&-\frac{2}{87}&-\frac{22}{261}&\frac{61}{261}\end{bmatrix}$

3. Use Crout factorization for a tri-diagonal system to solve the problem

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{Bmatrix}.$$

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} = LU \Rightarrow L = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & 0 \\ 0 & 0 & -1 & \frac{55}{21} \end{bmatrix} \quad U = \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ly = b \quad \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & 0 \\ 0 & 0 & -1 & \frac{55}{21} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 1.375 \\ 2.0476 \\ 1.1636 \end{bmatrix}$$

$$Ux = y \quad \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 1.375 \\ 2.0476 \\ 1.1636 \end{bmatrix} \Rightarrow \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 1.14364 \\ 2.3091 \\ 2.4909 \\ 1.1636 \end{bmatrix} \#$$