數值方法 HW8 吳博軒 E94111130

1. Given the data as listed below

x	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3
y	102.6	113.2	130.1	142.1	167.5	195.1	224.9	256.8

a. Construct the least squares approximation of degree two and compute the error.
b. Construct the least squares approximation of the form be^{ax} and compute the error.
c. Construct the least squares approximation of the form bx^n and compute the error.

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(a) Quadratic Polynomial Least Squares Approximation:
  y ≈ 6.6912x^2 + -1.8837x + 3.0864
  Sum of squared errors: 0.00525

(b) Exponential Fit (y = b * e^(a * x)):
  a ≈ 0.3985
  b ≈ 21.4445
  Sum of squared errors: 94.98302

(c) Power Fit (y = b * x^n):
  n ≈ 2.0196
  b ≈ 6.2390
  Sum of squared errors: 0.01172
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2. Find the least squares polynomial approximation of degree two on the interval $[-1,1]$ for the function $f(x)=\frac{1}{2}\cos x+\frac{1}{4}\sin 2x$

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Least Squares Polynomial Approximation (degree 2) for f(x) on [-1, 1]:
  p(x) ≈ -0.232631x^2 + 0.326548x + 0.498279
  Mean squared error (integral of squared error): 0.003240
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3. Determine the discrete least squares trigonometric polynomial S_4

using $m=16$ for $f(x)=x^2\sin x$ on the interval $[0,1]$.

b. Compute $\int_0^1 S_4(x)dx$

c. Compare the integral in part (b) to $\int_0^1 x^2\sin x dx$

d. Compute the error $E(S_4)$

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(a)
a0 = 0.45921
a1 = -0.14676 , b1 = 0.23229
a2 = 0.05461 , b2 = -0.12494
a3 = -0.03893 , b3 = 0.08293
a4 = 0.03354 , b4 = -0.06091

(b)
∫₀¹ S4(x)dx = 0.22960
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(c)
     x   f(x)   S4(x)  Error
0.0000 0.0000  0.5034 0.5034
0.0323 0.0000  0.2910 0.2910
0.0645 0.0003  0.0931 0.0929
0.0968 0.0009 -0.0360 0.0369
0.1290 0.0021 -0.0760 0.0781
0.1613 0.0042 -0.0456 0.0498
0.1935 0.0072  0.0114 0.0042
0.2258 0.0114  0.0523 0.0409
0.2581 0.0170  0.0570 0.0400
0.2903 0.0241  0.0338 0.0097
0.3226 0.0330  0.0093 0.0237
0.3548 0.0437  0.0075 0.0362
0.3871 0.0566  0.0352 0.0214
0.4194 0.0716  0.0786 0.0070
0.4516 0.0890  0.1156 0.0266
0.4839 0.1089  0.1316 0.0227
0.5161 0.1315  0.1299 0.0015
0.5484 0.1568  0.1288 0.0279
0.5806 0.1849  0.1483 0.0367
0.6129 0.2161  0.1951 0.0210
0.6452 0.2503  0.2575 0.0072
0.6774 0.2876  0.3133 0.0257
0.7097 0.3282  0.3455 0.0173
0.7419 0.3720  0.3562 0.0157
0.7742 0.4190  0.3679 0.0512
0.8065 0.4695  0.4089 0.0605
0.8387 0.5232  0.4934 0.0298
0.8710 0.5803  0.6060 0.0258
0.9032 0.6407  0.7036 0.0629
0.9355 0.7044  0.7347 0.0303
0.9677 0.7713  0.6662 0.1052
1.0000 0.8415  0.5034 0.3380

True Integral: 0.22324 , S4 Integral: 0.22960
Absolute Error: 0.00636 , Relative Error: 2.84817%

(d)
E(S4): 0.50556
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