

HW 11

Given the equation $y'' = -(x+1)y' + 2y + (1-x^2)e^{-x}$, $0 \leq x \leq 1$, $y(0) = 1$,

$$y(1) = 2$$

use $h = 0.1$

Questions:

- Use the shooting method to approximate the solution of the problem
- Use the finite-difference method to approximate the solution
- Use the variation approach to approximate the solution.

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--- HW11: Boundary-value problem for O.D.E. ---
Given equation: y'' = -(x+1)y' + 2y + (1-x^2)e^(-x), 0 <= x <= 1, y(0) = 1, y(1) = 2, h = 0.1

--- a. Shooting Method ---
C = 0.02415761802882529
y values: [1.          1.01664898  1.05929008  1.12447309  1.20911769  1.31052469
1.42637382  1.55470926  1.69391549  1.8426876  2.          ]

--- b. Finite-Difference Method ---
y values: [1.          1.01653219  1.05910229  1.12425112  1.2088902  1.31031341
1.42619408  1.55457046  1.69382243  1.84264199  2.          ]

--- c. Variation Approach (Galerkin Method) ---
Coefficients c: [-0.19690203 -0.01747145 -0.00884708 -0.00223296 -0.00196785]
y values: [1.          1.0176355  1.05792097  1.12463303  1.20978944  1.30997721
1.42608101  1.5552407  1.69377865  1.84242177  2.          ]

--- Summary Table ---
  x    Shooting      FD      Variation
0.0    1.000000    1.000000    1.000000
0.1    1.016649    1.016532    1.017635
0.2    1.059290    1.059102    1.057921
0.3    1.124473    1.124251    1.124633
0.4    1.209118    1.208890    1.209789
0.5    1.310525    1.310313    1.309977
0.6    1.426374    1.426194    1.426081
0.7    1.554709    1.554570    1.555241
0.8    1.693915    1.693822    1.693779
0.9    1.842688    1.842642    1.842422
1.0    2.000000    2.000000    2.000000
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