HW4

1. Determine the values $\int_1^2 e^x \sin(4x)dx$ with $h = 0.1$ by

a. Use the composite trapezoidal rule

b. Use the composite Simpsons' method

c. Use the composite midpoint rule

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========== 題目 1 ==========
1.a Trapezoidal Rule: 0.3961475922149067
1.b Simpson's Method: 0.3856635960237502
1.c Midpoint Rule: 0.3808047983772992
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2. Approximate $\int_1^{1.5} x^2 \ln x dx$ using Gaussian Quadrature with $n = 3$ and $n = 4$. Then compare the result to the exact value of the integral.

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========== 題目 2 ==========
2. Gaussian n=3 : 0.1922593772568790
2. Gaussian n=4 : 0.1922593578048631
2. Exact Value  : 0.1922593577327960
2. Error n=3    : 1.95e-08
2. Error n=4    : 7.21e-11
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3. Approximate $\int_0^{\pi/4} \int_{\sin x}^{\cos x} (2y \sin x + \cos^2 x)dydx$ using

a. Simpson's rule for $n = 4$ and $m = 4$

b. Gaussian Quadrature, $n = 3$ and $m = 3$

c. Compare these results with the exact value.

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========== 題目 3 ==========
3.a Simpson 2D (n=4, m=4): 0.5119875440121252
3.b Gaussian 2D (n=3, m=3): 0.5118655399452960
3.c Exact Value           : 0.5118446353109126
3.c Simpson Error         : 1.43e-04
3.c Gauss Error           : 2.09e-05
```

4. Use the composite Simpson's rule and $n = 4$ to approximate the improper integral a) $\int_0^1 x^{-1/4} \sin x dx$, b) $\int_1^\infty x^{-4} \sin x dx$ by use the transform $t = x^{-1}$

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========== 題目 4 ==========
4.a Improper Simpson x^(-1/4): 0.5259312819330653
4.b Improper Simpson (∞→1 by t=1/x): 0.2744816127051007
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