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1.  $f(0.75) = 0.7317$   
 $f(0.698) = 0.7661$   
 $f(0.733) = 0.7432$   
 $f(0.768) = 0.7193$   
 $f(0.803) = 0.6946$

degree 1:

$$f(x) = 0.7661 \cdot \frac{(x-0.733)}{(0.698-0.733)} + 0.7432 \cdot \frac{(x-0.698)}{(0.733-0.698)}$$

$$f(0.75) \approx 0.7321 \#$$

$$\text{error} = \left| 0.7321 - 0.7317 \right| = 0.0004$$

$$|f(x) - P_1(x)| \leq \frac{M}{2!} \cdot |(x-x_0)(x-x_1)|$$

$$\text{error bound} = \frac{1}{2} \cdot |(0.75-0.698)(0.75-0.733)| = 4.42 \cdot 10^{-4} \#$$

degree 2:

$$f(x) = 0.7661 \cdot \frac{(x-0.733)(x-0.768)}{(0.698-0.733)(0.698-0.768)} + 0.7432 \cdot \frac{(x-0.698)(x-0.768)}{(0.733-0.698)(0.733-0.768)} + 0.7193 \cdot \frac{(x-0.698)(x-0.733)}{(0.768-0.698)(0.768-0.733)}$$

$$f(0.75) \approx 0.7317 \# \quad \text{error} = \left| 0.7317 - 0.7317 \right| = 0$$

$$|f(x) - P_2(x)| \leq \frac{M}{3!} \cdot |(x-x_0)(x-x_1)(x-x_2)|$$

$$\text{error bound} = \frac{1}{6} \left| (0.75-0.698)(0.75-0.733)(0.75-0.768) \right| = 2.657 \cdot 10^{-6} \#$$

degree 3:

$$f(x) = 0.7661 \cdot \frac{(x-0.733)(x-0.768)(x-0.803)}{(0.698-0.733)(0.698-0.768)(0.698-0.803)} + 0.7432 \cdot \frac{(x-0.698)(x-0.768)(x-0.733)}{(0.733-0.698)(0.733-0.768)(0.733-0.733)} + 0.7193 \cdot \frac{(x-0.698)(x-0.733)(x-0.803)}{(0.768-0.698)(0.768-0.733)(0.768-0.803)} \\ + 0.6946 \cdot \frac{(x-0.698)(x-0.733)(x-0.768)}{(0.803-0.698)(0.803-0.733)(0.803-0.768)}$$

$$f(0.75) \approx 0.7317 \#$$

$$|f(x) - P_3(x)| \leq \frac{M}{4!} \left| (x-x_0)(x-x_1)(x-x_2)(x-x_3) \right|$$

$$\text{error bound} = \frac{1}{24} \left| (0.75-0.698)(0.75-0.733)(0.75-0.768)(0.75-0.803) \right| = 3.51 \cdot 10^{-8} \#$$

degree 4:

只有三個點，算不出來

$$2. \quad y = x - e^{-x}$$

$x$	0.3	0.4	0.5	0.6	$x$
$y$	-0.440818	-0.27032	-0.16531	0.051188	0

$$X(y) = 0.3 \cdot \frac{(y+0.27032)(y+0.16531)(y-0.051188)}{(-0.440818+0.27032)(-0.440818+0.16531)(-0.440818-0.051188)} + 0.4 \cdot \frac{(y+0.440818)(y+0.16531)(y-0.051188)}{(-0.27032+0.440818)(-0.27032+0.16531)(-0.27032-0.051188)} \\ + 0.5 \cdot \frac{(y+0.440818)(y+0.27032)(y-0.051188)}{(-0.16531+0.440818)(-0.16531+0.27032)(-0.16531-0.051188)} + 0.6 \cdot \frac{(y+0.440818)(y+0.16531)(y-0.051188)}{(0.051188+0.440818)(0.051188+0.27032)(0.051188+0.16531)}$$

$$X(0) \approx 0.5671426235 \#$$

$$3. \quad \begin{array}{ccccc} z_0 & z_1 & z_2 & z_3 & z_4 z_5 \\ 0 & 3 & 5 & 8 & 13 \\ D & 0 & 200 & 375 & 620 & 990 \\ V & 75 & 17 & 80 & 74 & 72 \end{array}$$

$$\begin{array}{ccccccc} z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \\ 5 & 5 & 8 & 8 & 13 & 13 \\ f[z] & 375 & 375 & 620 & 620 & 990 & 990 \\ f' & 80 & 74 & 74 & 72 & 72 \end{array}$$

$$f[5,5] = 80, f[5,8] = \frac{620-375}{8-5} = 81.667, f[f,f] = 74, f[f,13] = \frac{990-620}{13-8} = 74, f[13,13] = 72$$

$$f[5,5,8] = \frac{81.667-80}{8-5} = 0.556, f[5,f,f] = \frac{74-81.667}{8-5} = -2.556, f[f,f,13] = 0, f[f,13,13] = \frac{72-74}{13-8} = -0.4$$

$$f[5,5,f,f] = \frac{-2.556-0.556}{8-5} = -1.037, f[5,f,f,13] = \frac{0-(-2.556)}{13-8} = 0.3195, f[f,f,13,13] = \frac{-0.4-0}{13-8} = -0.04$$

$$a. \quad H(x) = f[z_0] + (x-z_0)f[z_0, z_1] + (x-z_0)^2 f[z_0, z_1, z_2] + (x-z_0)^3 f[z_0, z_1, z_2, z_3] + \dots$$

$$z_0 = 5, x = 10$$

$$H(10) = 375 + (10-5) \cdot 80 + (10-5)^2 \cdot 0.556 + (10-5)^3 \cdot (-1.037) = \underline{737.05} \text{ ft}$$

$$H'(x) = f[z_0, z_1] + 2(x-z_0)f[z_0, z_1, z_2] + [2(x-z_0)(x-z_1) + (x-z_0)^2]f[z_0, z_1, z_2, z_3] = \underline{38.9 \text{ ft/s}}$$

$$\left\{ \begin{array}{l} D(10) = 737.05 \text{ ft} \\ V(10) = 38.9 \text{ ft/s} \end{array} \right. \#$$

b. 不會超過  $55 \text{ mi/h} (80.67 \text{ ft/s})$ , 最接近的時候  $t=5$ ,  $V=80 \text{ ft/s}$

c. 最快速度 =  $80 \text{ ft/s}$ ,  $t=5$