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1. $\int_1^2 e^x \sin(4x)dx \quad h=0.1 \quad x_0=1,\ x_i=1+i\cdot 0.1$

$f(x)=e^x\sin(4x)$

a. $\int_1^2 e^x\sin(4x)dx = \frac{0.1}{2}\left[f(1)+2\left(f(1.1)+f(1.2)+f(1.3)+f(1.4)+\cdots+f(1.8)+f(1.9)\right)+f(2)\right]$

≈ 0.3961 #

b. $\int_1^2 e^x\sin(4x)dx = \frac{0.1}{3}\left[f(1)+2\left(f(1.2)+f(1.4)+f(1.6)+f(1.8)\right)+4\left(f(1.1)+f(1.3)+f(1.5)+f(1.7)+f(1.9)\right)\right.$
$\left.+f(2)\right]$

≈ 0.3857 #

c. $\int_1^2 e^x\sin(4x)dx = 2\cdot 0.1\left[f(1.1)+f(1.3)+f(1.5)+f(1.7)+f(1.9)\right] \approx 0.3808$ #

2. $\int_1^{1.5} x^2\ln x\,dx,\quad f(x)=x^2\ln x$

$n=3$:

$\int_1^{1.5} x^2\ln x\,dx = \frac{1.5-1}{2}\cdot\left(0.556 f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}\cdot(-0.775)\right)+0.889 f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}\cdot 0\right)+0.556\cdot f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}\cdot(0.775)\right)\right)$

$=0.25\cdot\left(0.556\cdot f(1.05625)+0.889 f(1.25)+0.556\cdot f(1.44375)\right)$

≈ 0.1923 #

$n=4$:

$\int_1^{1.5} x^2\ln x\,dx = \frac{1.5-1}{2}\cdot\left(0.348\cdot f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}\cdot(-0.861)\right)+0.652 f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}\cdot(0.34)\right)+0.652 f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}\cdot(-0.34)\right)\right.$
$\left.+0.348\cdot f\left(\frac{1.5+1}{2}+\frac{1.5-1}{2}(0.861)\right)\right)$

$=0.25\cdot\left[0.348\cdot f(1.03475)+0.652 f(1.335)+0.652 f(1.165)+0.348 f(1.46525)\right]$

≈ 0.1923 #

exact: $\int_1^{1.5} x^2\ln x\,dx = \frac{1}{3}x^3\ln x-\frac{1}{9}x^3\Big]_1^{1.5} \approx 0.1923$ #

3. $f(x,y) = 2y\sin x + \cos^2 x \quad \int_0^{\frac{\pi}{4}} \int_{\sin x}^{\cos x} f(x,y)\,dy\,dx$

Simpson's Rule : ans = 0.5118 #

Gauss Quadrature : ans = 0.5119 #

Exact : ans = 0.5118 #

4.

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 2\sum_{i=2}^{n} f(x_{2i-2}) + 4\sum_{i=1}^{n} (x_{2i-1}) + f(x_n) \right]$$

$\int_0^1 x^{-\frac{1}{4}} \sin x\,dx \quad , t = x^{-1}$

a. Composite Simpson's Rule = 0.5259 #

b. $\int_1^{\infty} x^{-4} \sin x\,dx \quad , \quad t = x^{-1} \, , \, dx = -\frac{1}{t^2}\,dt$

$= \int_1^0 t^4 \sin\frac{1}{t} \cdot \left(-\frac{1}{t^2}\right) dt$

$= \int_0^1 t^2 \sin\left(\frac{1}{t}\right) dt$

$= 0.2745$ #

1.

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1
a. Trapezoidal Rule: 0.396147592214901
b. Simpson's Rule: 0.38566359602374467
c. Midpoint Rule: 0.3808047983772932
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2.

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2
n = 3: 0.19225937725687903
n = 4: 0.1922593578048632
Exact: 0.19225935773279604
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3.

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3
a. Bode's Rule: 0.5118230071056833
b. Gauss-Legendre 2D: 0.5118655399452959
c. Exact: 0.5118446353109126
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4.

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4
a. Simpson's ∫ x^(-1/4) sin(x) dx: 0.5259312819330653
b. Transformed ∫ x^(-4) sin(x) dx: 0.27448161270510074
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