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(a)

$$A=\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 1 & -1 \\ -1 & 0 & 0 & 4 & -1 & -1 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \quad b=\begin{bmatrix} 0 \\ -1 \\ 9 \\ 4 \\ 8 \\ 6 \end{bmatrix}$$

$$x^{(k)} = -b^{-1}(L+U)x^{(k-1)} + b^{-1}b$$

$$L=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \quad U=\begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad D=\begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$L+U=\begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \quad D^{-1}=\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$x^{(k)} = -\begin{bmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{bmatrix} x^{(k-1)} + \begin{bmatrix} 0 \\ -0.25 \\ 2.25 \\ 1 \\ 2 \\ 1.5 \end{bmatrix}$$

$$x^{1}=\begin{bmatrix} 0 \\ -0.25 \\ 2.25 \\ 1 \\ 2 \\ 1.5 \end{bmatrix} \quad x^{2}=\begin{bmatrix} 0.19 \\ 0.81 \\ 2.06 \\ 1.88 \\ 2.56 \\ 2.56 \end{bmatrix} \quad \cdots \quad x^{n}=\begin{bmatrix} 1.175 \\ 1.643 \\ 2.448 \\ 3.056 \\ 3.949 \\ 3.099 \end{bmatrix}$$

4

(b)

$$X_1^k = \frac{X_2^{(k-1)} + X_4^{(k-1)}}{4}$$

$$X_2^k = \frac{-1 + X_1^k + X_3^{(k-1)} + X_5^{(k-1)}}{4}$$

$$X_3^k = \frac{9 + X_2^k - X_5^{(k-1)} + X_6^{(k-1)}}{4}$$

$$X_4^k = \frac{4 + X_1^k + X_5^{(k-1)} + X_6^{(k-1)}}{4}$$

$$X_5^k = \frac{8 + X_2^k + X_4^k + X_6^{(k-1)}}{4}$$

$$X_6^k = \frac{6 + X_3^k + X_5^k}{4}$$

X_n \ k	0	1	2	...	n
X_1	0	0	0,1875		1.175
X_2	0	-0,25	0,890625		1.643
X_3	0	2,1875	2.5 7421875		2,448
X_4	0	1	2,2421875		3,056
X_5	0	2,1875	3,431640625		3.949
X_6	0	2.59375	3,001464844		3,099

#

(C)

$$W = 1.25 \quad 1 < W < 2$$

$$X_1^k = W\left(\frac{X_2^{(k-1)} + X_4^{(k-1)}}{4}\right) + (1-W)X_1^{(k-1)}$$

$$X_2^k = W\left(\frac{-1 + X_1^k + X_3^{(k-1)} + X_5^{(k-1)}}{4}\right) + ((-W)X_2^{(k-1)}$$

$$X_3^k = W\left(\frac{9 + X_2^k - X_5^{(k-1)} + X_6^{(k-1)}}{4}\right) + (1-W)X_3^{(k-1)}$$

$$X_4^k = W\left(\frac{4 + X_1^k + X_5^{(k-1)} + X_6^{(k-1)}}{4}\right) + ((1-W)X_4^{(k-1)}$$

$$X_5^k = W\left(\frac{8 + X_2^k + X_4^k + X_6^{(k-1)}}{4}\right) + (1-W)X_5^{(k-1)}$$

$$X_6^k = W\left(\frac{6 + X_3^k + X_5^k}{4}\right) + (1-W)X_6^{(k-1)}$$

X_n \ k	1	2	...	n
X_1	0	0,421875		1.175
X_2	-0,375	2,466796875		1.643
X_3	3,234375	3.179443359		2,448
X_4	1,5	3.971191406		3,056
X_5	3,421875	5.483093262		3.949
X_6	4.946093750	3.125439453		3.099

(d)

x_n \ k	1	2	…	n
x_1	0	0		1.17806665
x_2	0	-0.32673269		1.64269366
x_3	0	2.94059406		2.44433267
x_4	0	1.36693069		3.06002082
x_5	0	2.61386139		3.95260785
x_6	0	1.96039604		3.09922059

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=== Jacobi Method ===
Iteration 1: [ 0.    -0.25  2.25  1.     2.     1.5 ]
Iteration 2: [0.1875 0.8125 2.0625 1.875  2.5625 2.5625]
Final result : [1.1747883  1.64317298 2.44824825 3.05598016 3.94965738 3.09947604]
Exact result : [1.17478856 1.64317358 2.44824809 3.05598067 3.94965767 3.09947644]
Max error    : 6.051242815718894e-07

=== Gauss-Seidel Method ===
Iteration 1: [ 0.       -0.25     2.1875   1.       2.1875   2.59375]
Iteration 2: [0.1875     0.890625   2.57421875 2.2421875  3.43164062 3.00146484]
Final result : [1.17478836 1.64317351 2.44824812 3.05598056 3.94965762 3.09947644]
Exact result : [1.17478856 1.64317358 2.44824809 3.05598067 3.94965767 3.09947644]
Max error    : 2.055332473105409e-07

=== SOR Method ===
Iteration 1: [ 0.          -0.375       3.234375    1.5         3.421875    4.74609375]
Iteration 2: [0.421875   2.46679688 3.17944336 3.97119141 5.48309326 3.12540436]
Final result : [1.17478833 1.64317366 2.44824823 3.05598074 3.94965791 3.09947654]
Exact result : [1.17478856 1.64317358 2.44824809 3.05598067 3.94965767 3.09947644]
Max error    : 2.348160426102197e-07

=== Conjugate Gradient Method ===
Iteration 1: [0. 0. 0. 0. 0. 0.]
Iteration 2: [ 0.         -0.32673267  2.94059406  1.30693069  2.61386139  1.96039604]
Final result : [1.17656665 1.64269366 2.44433267 3.06002082 3.95260785 3.09922059]
Exact result : [1.17478856 1.64317358 2.44824809 3.05598067 3.94965767 3.09947644]
Max error    : 0.0040401475461226966
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