E94114057 古清賢

1. Determine the values $\int_1^2 e^x \sin(4x)dx$ with $h = 0.1$ by

a. Use the composite trapezoidal rule

b. Use the composite Simpsons' method

c. Use the composite midpoint rule

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PS C:\Users\古淯賢> & D:/anaconda/python.exe c:/Users/古淯賢/E94114057_numerical_hw4.py
1.a) Trapezoidal Rule result: 0.39614759
1.b) Simpson's Method result: 0.38566360
1.c) Midpoint Rule result: 0.38080480
```

2. Approximate $\int_1^{1.5} x^2 \ln x dx$ using Gaussian Quadrature with $n = 3$ and $n = 4$. Then compare the result to the exact value of the integral.

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PS C:\Users\古淯賢> & D:/anaconda/python.exe c:/Users/古淯賢/Desktop/E94114057_numerical_hw4-2.py
2) Gaussian Quadrature results:
   n=3 approximation: 0.19225938
   n=4 approximation: 0.19225936
   Exact value:       0.19225936
   Error (n=3):       1.95e-08
   Error (n=4):       7.21e-11
```

3. Approximate $\int_0^{\pi/4} \int_{\sin x}^{\cos x} (2y \sin x + \cos^2 x) dy dx$ using

a. Simpson's rule for $n = 4$ and $m = 4$

b. Gaussian Quadrature, $n = 3$ and $m = 3$

c. Compare these results with the exact value.

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PS C:\Users\古淯賢> & D:/anaconda/python.exe c:/Users/古淯賢/Desktop/E94114057_numerical_hw4-3.py
3) Double Integral results:
   a) Simpson 2D (4x4):     0.51198754
   b) Gaussian 2D (3x3):    0.51186554
   c) Exact value:          0.51184464
      Simpson error:        1.43e-04
      Gauss error:          2.09e-05
```

4. Use the composite Simpson's rule and $n=4$ to approximate the improper integral a) $\int_0^1 x^{-1/4}\sin x dx$, b) $\int_1^{\infty} x^{-4}\sin x dx$ by use the transform $t=x^{-1}$

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PS C:\Users\古清賢> & D:/anaconda/python.exe c:/Users/古清賢/Desktop/E94114057_numerical_hw4-4.py
4) Improper Integral results:
  a) ∫₀¹ x^(-1/4)sin(x) dx ≈ 0.52593128 (using Simpson's rule)
  b) ∫₁^∞ x^(-4)sin(x) dx ≈ 0.27465825 (after substitution t=1/x)
```