E94114057_古清賢_hw5

1. The initial-value problem

   $y' = 1 + (y/t) + (y/t)^2$, $1 \le t \le 2$, $y(1) = 0$ has the exact solution $y(t) = t\tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

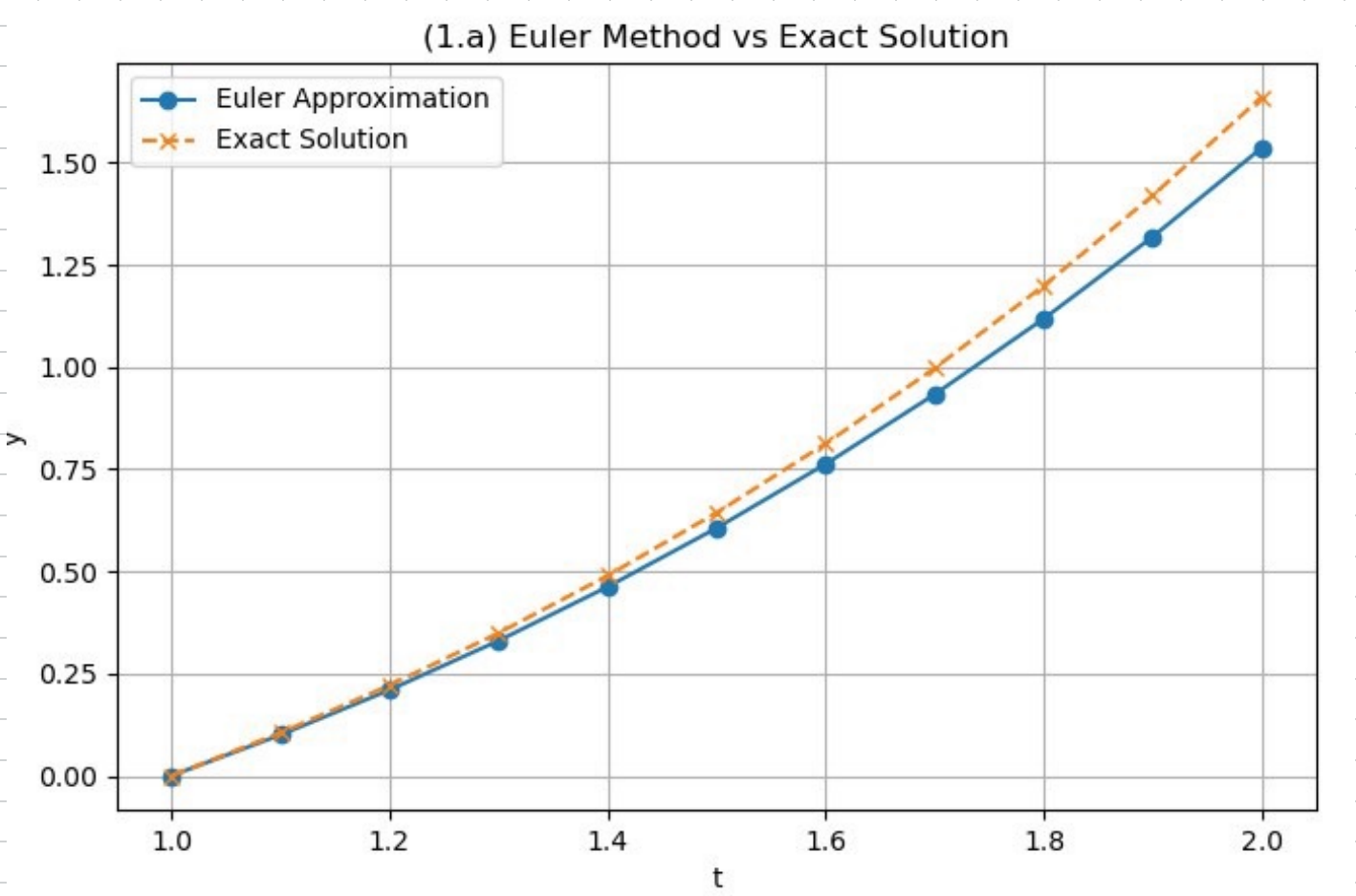
b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

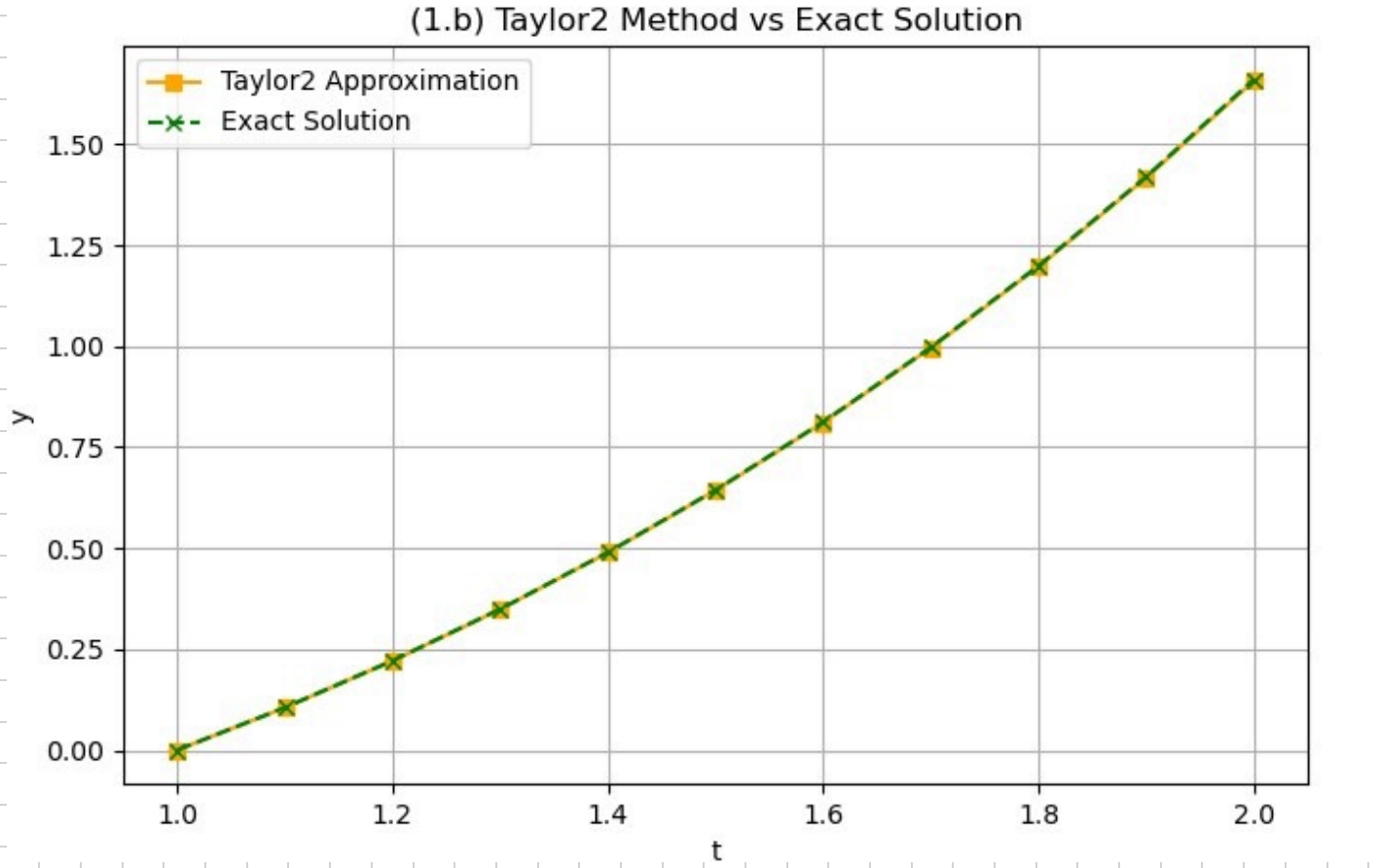
(1.a) Euler Method Results:

t	Euler y	Exact y	Error
1.000000	0.000000	0.000000	0.000000
1.100000	0.100000	0.105160	0.005160
1.200000	0.209917	0.221243	0.011325
1.300000	0.330471	0.349121	0.018651
1.400000	0.462354	0.489682	0.027328
1.500000	0.606285	0.643875	0.037590
1.600000	0.763041	0.812753	0.049711
1.700000	0.933475	0.997494	0.064019
1.800000	1.118537	1.199439	0.080902
1.900000	1.319293	1.420116	0.100823
2.000000	1.536943	1.661282	0.124338

(1.b) Taylor Method Results:

t	Taylor2 y	Exact y	Error
1.000000	0.000000	0.000000	0.000000
1.100000	0.105000	0.105160	0.000160
1.200000	0.220919	0.221243	0.000324
1.300000	0.348612	0.349121	0.000509
1.400000	0.488954	0.489682	0.000728
1.500000	0.642883	0.643875	0.000993
1.600000	0.811438	0.812753	0.001315
1.700000	0.995787	0.997494	0.001707
1.800000	1.197252	1.199439	0.002187
1.900000	1.417344	1.420116	0.002772
2.000000	1.657795	1.661282	0.003487


(1.a) Euler Method vs Exact Solution
Euler Approximation
Exact Solution
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
1.0
1.2
1.4
1.6
1.8
2.0
t


(1.b) Taylor2 Method vs Exact Solution
Taylor2 Approximation
Exact Solution
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
1.0
1.2
1.4
1.6
1.8
2.0
t

2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t, \quad u_2(0) = \frac{2}{3},$$

has the unique solution

$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t.$$

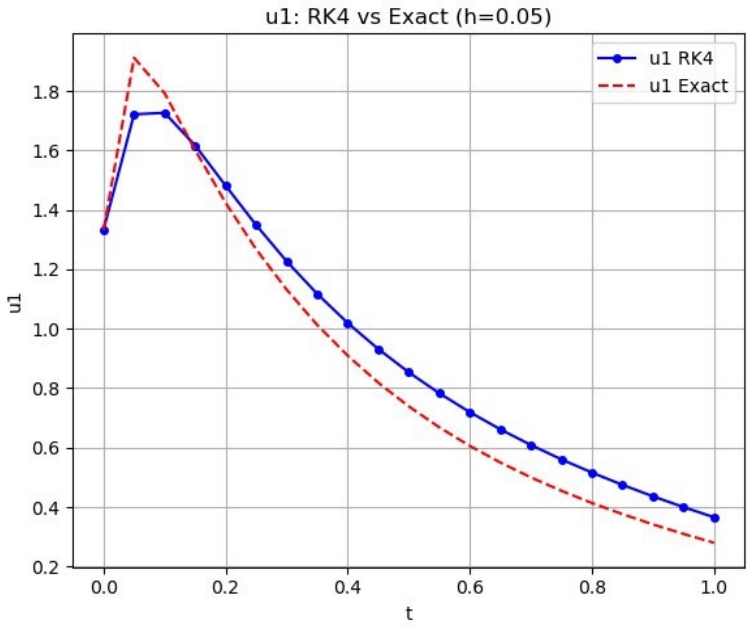
Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

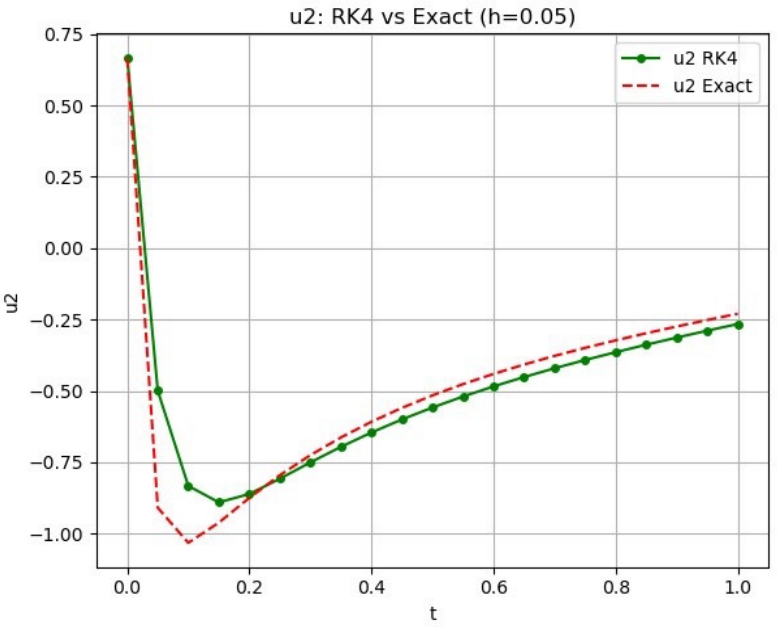
unge-Kutta with h = 0.05

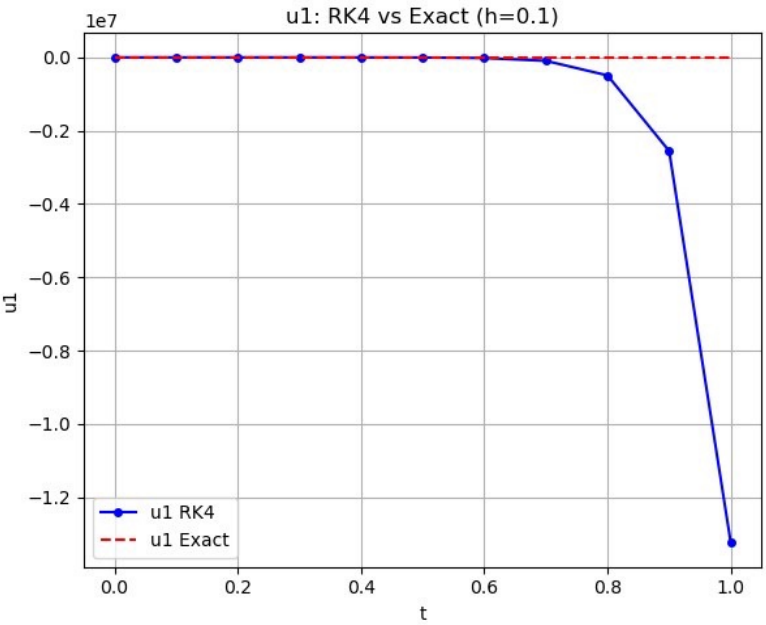
t	u1 (RK4)	u1 (Exact)	Error u1	u2 (RK4)	u2 (Exact)	Error u2
0.000000	1.333333	1.333333	0.000000	0.666667	0.666667	0.000000
0.050000	1.721880	1.912059	0.190178	-0.499599	-0.909077	0.409477
0.100000	1.726915	1.793063	0.066148	-0.832598	-1.032002	0.199405
0.150000	1.617161	1.601967	0.015194	-0.890373	-0.961459	0.071086
0.200000	1.481687	1.423902	0.057785	-0.861042	-0.874681	0.013639
0.250000	1.348945	1.267646	0.081299	-0.807505	-0.795221	0.012284
0.300000	1.227063	1.131577	0.095487	-0.750341	-0.724999	0.025342
0.350000	1.117478	1.012999	0.104480	-0.695886	-0.663060	0.032826
0.400000	1.019525	0.909409	0.110117	-0.645732	-0.608214	0.037518
0.450000	0.931977	0.818630	0.113347	-0.599934	-0.559389	0.040545
0.500000	0.853541	0.738788	0.114753	-0.558092	-0.515658	0.042435
0.550000	0.783017	0.668275	0.114743	-0.519706	-0.476225	0.043482
0.600000	0.719337	0.605710	0.113627	-0.484290	-0.440411	0.043880
0.650000	0.661560	0.549909	0.111651	-0.451407	-0.407635	0.043772
0.700000	0.608868	0.499860	0.109007	-0.420673	-0.377404	0.043269
0.750000	0.560547	0.454695	0.105852	-0.391754	-0.349296	0.042459
0.800000	0.515980	0.413671	0.102309	-0.364365	-0.322954	0.041411
0.850000	0.474633	0.376158	0.098475	-0.338259	-0.298076	0.040183
0.900000	0.436043	0.341614	0.094428	-0.313226	-0.274409	0.038817
0.950000	0.399812	0.309583	0.090229	-0.289089	-0.251739	0.037351
1.000000	0.365600	0.279675	0.085925	-0.265698	-0.229888	0.035810

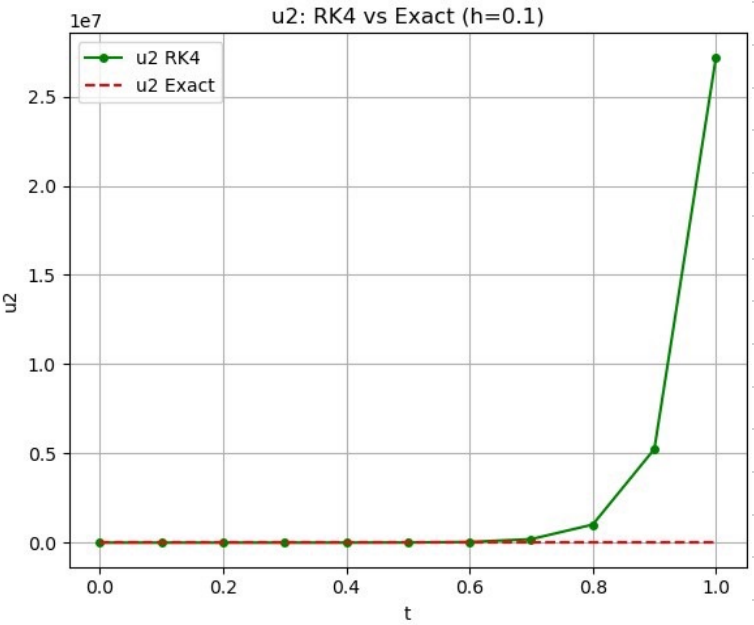
Runge-Kutta with h = 0.1

t	u1 (RK4)	u1 (Exact)	Error u1	u2 (RK4)	u2 (Exact)	Error u2
0.000000	1.333333	1.333333	0.000000	0.666667	0.666667	0.000000
0.100000	-3.052437	1.793063	4.845500	8.989305	-1.032002	10.021308
0.200000	-23.847795	1.423902	25.271697	51.192704	-0.874681	52.067385
0.300000	-130.165202	1.131577	131.296778	269.269193	-0.724999	269.994192
0.400000	-680.231485	0.909409	681.140894	1399.368584	-0.608214	1399.976798
0.500000	-3531.299585	0.738788	3532.038373	7258.241839	-0.515658	7258.757497
0.600000	-18312.795052	0.605710	18313.400762	37634.955483	-0.440411	37635.395894
0.700000	-94951.331907	0.499860	94951.831768	195131.871735	-0.377404	195132.249139
0.800000	-492306.465639	0.413671	492306.879311	1011721.872078	-0.322954	1011722.195031
0.900000	-2552513.623867	0.341614	2552513.965482	5245578.826590	-0.274409	5245579.100999
1.000000	-13234278.789168	0.279675	13234279.068843	27197287.206587	-0.229888	27197287.436475


u1: RK4 vs Exact (h=0.05)
u1 RK4
u1 Exact
u1
t


u2: RK4 vs Exact (h=0.05)
u2 RK4
u2 Exact
u2
t


u1: RK4 vs Exact (h=0.1)
1e7
u1 RK4
u1 Exact
u1
t


u2: RK4 vs Exact (h=0.1)
1e7
u2 RK4
u2 Exact
u2
t