

HW12 E94116198 劉羿伶

1. Given the problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = xy, \quad 0 < x < \pi, \quad 0 < y < \pi/2$$

$$u(0, y) = \cos y, \quad u(\pi, y) = -\cos y, \quad 0 \leq y \leq \pi/2,$$

$$u(x, 0) = \cos x, \quad u(x, \pi/2) = 0, \quad 1 \leq y \leq 2$$

To calculate $u(x, y)$ by using $h = k = 0.1\pi$.

✅ Converged after 94 iterations. Error = 8.84e-07

數值表格:

	y=0.00	y=0.31	y=0.63	y=0.94	y=1.26	y=1.57
x=0.00	1.000000e+00	0.951057	0.809017	0.587785	0.309017	0.0
x=0.31	9.510565e-01	0.753225	0.564621	0.368084	0.172806	0.0
x=0.63	8.090170e-01	0.555908	0.347640	0.176348	0.053086	0.0
x=0.94	5.877853e-01	0.333231	0.132647	-0.004976	-0.058882	0.0
x=1.26	3.090170e-01	0.085806	-0.086863	-0.182347	-0.166748	0.0
x=1.57	6.123234e-17	-0.173197	-0.305633	-0.353912	-0.269910	0.0
x=1.88	-3.090170e-01	-0.424258	-0.511151	-0.511645	-0.364162	0.0
x=2.20	-5.877853e-01	-0.645220	-0.686178	-0.642021	-0.441312	0.0
x=2.51	-8.090170e-01	-0.814475	-0.809947	-0.724389	-0.486321	0.0
x=2.83	-9.510565e-01	-0.915787	-0.858894	-0.725487	-0.467874	0.0
x=3.14	-1.000000e+00	-0.951057	-0.809017	-0.587785	-0.309017	0.0

2. Given the problem

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{4K} \frac{\partial T}{\partial t}, \quad \frac{1}{2} \leq r \leq 1, \quad 0 \leq t,$$

$$T(1, t) = 100 + 40t, \quad 0 \leq t \leq 10; \quad \frac{\partial T}{\partial r} + 3T = 0 \quad \text{at} \quad r = \frac{1}{2}$$

$$T(r, 0) = 200(r - 0.5), \quad 0.5 \leq r \leq 1,$$

and use $\Delta t = 0.5$, $\Delta r = 0.1$, and $K = 0.1$ to calculate $T(r, t)$

By (a) the forward-difference method

(b) the backward-difference method

© the Crank-Nicolson algorithm.

(a) Forward-Difference Method:

	r=0.50	r=0.60	r=0.70	r=0.80	r=0.90	r=1.00	r=1.10
t=0.0	0.000000e+00	2.000000e+01	4.000000e+01	6.000000e+01	8.000000e+01	1.000000e+02	120.0
t=0.5	3.358974e+02	4.366667e+02	3.971429e+02	3.725000e+02	3.577778e+02	3.500000e+02	120.0
t=1.0	-1.266307e+04	-1.646199e+04	1.684345e+03	1.305040e+03	1.069583e+03	-2.891389e+04	140.0
t=1.5	2.212421e+06	2.876147e+06	-2.155387e+06	1.448336e+04	-3.927286e+06	7.344946e+06	160.0
t=2.0	-5.804072e+08	-7.545294e+08	8.724695e+08	-7.777834e+08	1.948729e+09	-2.295236e+09	180.0
t=2.5	1.842459e+11	2.395197e+11	-4.089916e+11	5.547262e+11	-8.798988e+11	8.029253e+11	200.0
t=3.0	-7.224091e+13	-9.391318e+13	2.039340e+14	-3.029171e+14	3.905248e+14	-3.044164e+14	220.0
t=3.5	3.286372e+16	4.272284e+16	-1.022495e+17	1.511914e+17	-1.731678e+17	1.221745e+17	240.0
t=4.0	-1.593741e+19	-2.071863e+19	5.066787e+19	-7.262787e+19	7.708801e+19	-5.098513e+19	260.0
t=4.5	7.841578e+21	1.019405e+22	-2.474809e+22	3.426023e+22	-3.449624e+22	2.184950e+22	280.0
t=5.0	-3.839314e+24	-4.991108e+24	1.193394e+25	-1.601250e+25	1.551710e+25	-9.536954e+24	300.0
t=5.5	1.860706e+27	2.418918e+27	-5.695406e+27	7.446484e+27	-7.012469e+27	4.217357e+27	320.0
t=6.0	-8.925828e+29	-1.160358e+30	2.696220e+30	-3.452948e+30	3.181660e+30	-1.882852e+30	340.0
t=6.5	4.244362e+32	5.517671e+32	-1.268492e+33	1.598312e+33	-1.448305e+33	8.466524e+32	360.0
t=7.0	-2.004089e+35	-2.605316e+35	5.939584e+35	-7.389840e+35	6.610286e+35	-3.828027e+35	380.0
t=7.5	9.410825e+37	1.223407e+38	-2.771070e+38	3.414044e+38	-3.023460e+38	1.738150e+38	400.0
t=8.0	-4.400347e+40	-5.720451e+40	1.289237e+41	-1.576385e+41	1.385228e+41	-7.918353e+40	420.0
t=8.5	2.050791e+43	2.666028e+43	-5.985411e+43	7.275779e+43	-6.355009e+43	3.616628e+43	440.0
t=9.0	-9.533684e+45	-1.239379e+46	2.774252e+46	-3.357109e+46	2.918537e+46	-1.655198e+46	460.0
t=9.5	4.423424e+48	5.750451e+48	-1.284258e+49	1.548646e+49	-1.341435e+49	7.587205e+48	480.0
t=10.0	-2.049316e+51	-2.664111e+51	5.939345e+51	-7.142712e+51	6.169526e+51	-3.482169e+51	500.0

(b) Backward-Difference Method:

	r=0.50	r=0.60	r=0.70	r=0.80	r=0.90	r=1.00	r=1.10
t=0.0	0.000000e+00	2.000000e+01	4.000000e+01	6.000000e+01	8.000000e+01	1.000000e+02	120.000000
t=0.5	1.696232e+02	1.924452e+02	2.527877e+02	2.849230e+02	2.681804e+02	1.808164e+02	357.321528
t=1.0	8.935280e+00	1.013748e-01	1.256584e-01	1.304280e-01	1.106171e-01	6.535247e-02	148.577512
t=1.5	4.286491e-01	4.863219e-03	5.950165e-03	6.052454e-03	5.013241e-03	2.895704e-03	160.380061
t=2.0	2.015294e-02	2.286443e-04	2.789106e-04	2.824034e-04	2.327438e-04	1.338843e-04	180.017572
t=2.5	9.432702e-04	1.070183e-05	1.304580e-05	1.319577e-05	1.086378e-05	6.244221e-06	200.000820
t=3.0	4.410702e-05	5.004142e-07	6.099271e-07	6.168020e-07	5.076836e-07	2.917540e-07	220.000038
t=3.5	2.061989e-06	2.339420e-08	2.851298e-08	2.883298e-08	2.373098e-08	1.363715e-08	240.000002
t=4.0	9.639285e-08	1.093621e-09	1.332901e-09	1.347846e-09	1.109333e-09	6.374800e-10	260.000000
t=4.5	4.506080e-09	5.112353e-11	6.230909e-11	6.300759e-11	5.185772e-11	2.980007e-11	280.000000
t=5.0	2.106455e-10	2.389868e-12	2.912758e-12	2.945409e-12	2.424186e-12	1.393060e-12	300.000000
t=5.5	9.847025e-12	1.117190e-13	1.361624e-13	1.376888e-13	1.133232e-13	6.512122e-14	320.000000
t=6.0	4.603180e-13	5.222517e-15	6.365173e-15	6.436525e-15	5.297509e-15	3.044215e-15	340.000000
t=6.5	2.151844e-14	2.441365e-16	2.975522e-16	3.008876e-16	2.476421e-16	1.423077e-16	360.000000
t=7.0	1.005921e-15	1.141263e-17	1.390964e-17	1.406557e-17	1.157650e-17	6.652443e-18	380.000000
t=7.5	4.702368e-17	5.335050e-19	6.502328e-19	6.575217e-19	5.411658e-19	3.109811e-19	400.000000
t=8.0	2.198212e-18	2.493971e-20	3.039637e-20	3.073711e-20	2.529783e-20	1.453741e-20	420.000000
t=8.5	1.027596e-19	1.165854e-21	1.420937e-21	1.436865e-21	1.182595e-21	6.795788e-22	440.000000
t=9.0	4.803693e-21	5.450009e-23	6.642439e-23	6.716898e-23	5.528267e-23	3.176821e-23	460.000000
t=9.5	2.245578e-22	2.547710e-24	3.105135e-24	3.139942e-24	2.584294e-24	1.485065e-24	480.000000
t=10.0	1.049738e-23	1.190976e-25	1.451555e-25	1.467826e-25	1.208078e-25	6.942222e-26	500.000000

(c) Crank-Nicolson Method:

	r=0.50	r=0.60	r=0.70	r=0.80	r=0.90	r=1.00	r=1.10
t=0.0	0.000000e+00	2.000000e+01	4.000000e+01	6.000000e+01	8.000000e+01	1.000000e+02	1.200000e+02
t=0.5	8.085855e+02	1.834754e+01	2.196701e+01	2.230598e+01	1.902998e+01	1.174961e+01	8.910679e+02
t=1.0	7.272166e+04	1.650121e+03	1.544843e+03	1.418047e+03	1.278038e+03	1.130669e+03	7.434014e+04
t=1.5	6.454832e+06	1.464660e+05	1.367013e+05	1.247271e+05	1.111673e+05	9.641432e+04	6.327350e+06
t=2.0	5.693507e+08	1.291908e+07	1.201936e+07	1.091938e+07	9.674236e+06	8.318321e+06	5.458900e+08
t=2.5	5.005770e+10	1.135855e+09	1.055025e+09	9.563423e+08	8.446444e+08	7.229368e+08	4.744273e+10
t=3.0	4.393780e+12	9.969887e+10	9.252578e+10	8.377458e+10	7.386951e+10	6.307358e+10	4.139204e+12
t=3.5	3.853287e+14	8.743459e+12	8.110833e+12	7.339301e+12	6.466057e+12	5.514129e+12	3.618647e+14
t=4.0	3.377772e+16	7.664472e+14	7.108299e+14	6.430130e+14	5.662565e+14	4.825773e+14	3.166914e+16
t=4.5	2.960252e+18	6.717080e+16	6.228919e+16	5.633737e+16	4.960102e+16	4.225681e+16	2.773103e+18
t=5.0	2.594028e+20	5.886085e+18	5.457982e+18	4.936049e+18	4.345321e+18	3.701275e+18	2.428961e+20
t=5.5	2.272969e+22	5.157573e+20	4.782303e+20	4.324796e+20	3.806984e+20	3.242431e+20	2.127845e+22
t=6.0	1.991582e+24	4.519081e+22	4.190199e+22	3.789251e+22	3.335454e+22	2.840690e+22	1.864203e+24
t=6.5	1.745001e+26	3.959566e+24	3.671372e+24	3.320029e+24	2.922377e+24	2.488825e+24	1.633292e+26
t=7.0	1.528936e+28	3.469295e+26	3.216771e+26	2.908915e+26	2.560481e+26	2.180590e+26	1.431012e+28
t=7.5	1.339618e+30	3.039715e+28	2.818453e+28	2.548709e+28	2.243411e+28	1.910551e+28	1.253799e+30
t=8.0	1.173739e+32	2.663321e+30	2.469454e+30	2.233108e+30	1.965610e+30	1.673961e+30	1.098537e+32
t=8.5	1.028399e+34	2.333531e+32	2.163668e+32	1.956587e+32	1.722210e+32	1.466674e+32	9.625047e+33
t=9.0	9.010555e+35	2.044577e+34	1.895747e+34	1.714307e+34	1.508952e+34	1.285057e+34	8.43186e+35
t=9.5	7.894802e+37	1.791402e+36	1.661001e+36	1.502028e+36	1.322101e+36	1.125930e+36	7.388918e+37
t=10.0	6.917208e+39	1.569577e+38	1.455324e+38	1.316036e+38	1.158388e+38	9.865085e+37	6.473962e+39

3. Given the problem

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0, \quad \frac{1}{2} \leq r \leq 1, \quad 0 \leq t \leq \pi/3,$$

$$T(r, 0) = 0, \quad T(r, \pi/3) = 0, \quad T(1/2, \theta) = 50, \quad T(1, \theta) = 100.$$

Converged in 2894 iterations, final max error = 9.99e-06

	0. 0°	1. 02°	2. 03°	3. 05°	4. 07°	5. 08°	6. 1°	7. 12°	8. 14°	9. 15°	...	50. 85°
r=0. 5	0. 0	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	...	50. 000000
r=0. 51	0. 0	23. 349925	33. 444155	38. 495471	41. 379635	43. 213516	44. 480078	45. 410804	46. 127326	46. 698744	...	46. 698751
r=0. 52	0. 0	13. 891907	23. 555822	29. 943178	34. 250330	37. 274622	39. 491655	41. 181769	42. 513759	43. 592646	...	43. 592662
r=0. 531	0. 0	9. 727337	17. 819427	24. 071007	28. 809875	32. 426672	35. 236912	37. 467667	39. 276251	40. 770951	...	40. 770973
r=0. 541	0. 0	7. 543464	14. 332341	20. 069584	24. 777731	28. 608885	31. 737005	34. 315192	36. 465379	38. 280588	...	38. 280617
r=0. 551	0. 0	6. 245157	12. 087722	17. 295009	21. 802736	25. 649492	28. 918161	31. 700781	34. 082051	36. 133657	...	36. 133693
r=0. 561	0. 0	5. 404906	10. 569493	15. 325885	19. 595248	23. 367923	26. 675290	29. 567237	32. 098030	34. 318918	...	34. 318961
r=0. 571	0. 0	4. 829702	9. 503103	13. 898702	17. 944042	21. 611339	24. 904830	27. 847703	30. 472030	32. 812228	...	32. 812276
r=0. 582	0. 0	4. 421490	8. 734294	12. 848179	16. 701335	20. 260869	23. 517468	26. 478190	29. 159869	31. 584095	...	31. 584149
r=0. 592	0. 0	4. 125640	8. 171512	12. 068449	15. 764620	19. 227286	22. 440797	25. 402757	28. 119878	30. 604400	...	30. 604460
	51. 86°	52. 88°	53. 9°	54. 92°	55. 93°	56. 95°	57. 97°	58. 98°	60. 0°			
r=0. 5	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	50. 000000	0. 0			
r=0. 51	46. 127334	45. 410810	44. 480084	43. 213521	41. 379639	38. 495474	33. 444157	23. 349926	0. 0			
r=0. 52	42. 513773	41. 181782	39. 491667	37. 274632	34. 250338	29. 943184	23. 555827	13. 891910	0. 0			
r=0. 531	39. 276272	37. 467687	35. 236929	32. 426687	28. 809888	24. 071017	17. 819434	9. 727340	0. 0			
r=0. 541	36. 465406	34. 315217	31. 737028	28. 608905	24. 777748	20. 069597	14. 332350	7. 543468	0. 0			
r=0. 551	34. 082085	31. 700812	28. 918190	25. 649517	21. 802757	17. 295025	12. 087733	6. 245163	0. 0			
r=0. 561	32. 098070	29. 567274	26. 675324	23. 367952	19. 595272	15. 325904	10. 569506	5. 404913	0. 0			
r=0. 571	30. 472076	27. 847746	24. 904868	21. 611372	17. 944069	13. 898724	9. 503118	4. 829710	0. 0			
r=0. 582	29. 159920	26. 478238	23. 517511	20. 260906	16. 701366	12. 848204	8. 734311	4. 421498	0. 0			
r=0. 592	28. 119934	25. 402809	22. 440844	19. 227327	15. 764654	12. 068476	8. 171530	4. 125650	0. 0			

4. Given the problem

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}, \quad 0 \leq x \leq 1, \quad 0 \leq t$$

$$p(0, t) = 1, \quad p(1, t) = 2, \quad p(x, 0) = \cos(2\pi x), \quad \frac{\partial p}{\partial t}(x, 0) = 2\pi \sin(2\pi x), \quad 0 \leq x \leq 1$$

To calculate p by using $\Delta x = \Delta t = 0.1$.

p(x, t):

	x=0. 0	x=0. 1	x=0. 2	x=0. 3	x=0. 4	x=0. 5	x=0. 6	x=0. 7	x=0. 8	x=0. 9	x=1. 0
t=0. 0	1. 0	0. 8090	0. 3090	-0. 3090	-0. 8090	-1. 000	-0. 8090	-0. 3090	0. 3090	0. 8090	2. 0
t=0. 1	1. 0	1. 0238	0. 8476	0. 3476	-0. 2852	-0. 809	-1. 0238	-0. 8476	-0. 3476	0. 2852	2. 0
t=0. 2	1. 0	1. 0385	1. 0624	0. 8714	0. 3476	-0. 309	-0. 8476	-1. 0624	-0. 8714	0. 8434	2. 0
t=0. 3	1. 0	1. 0385	1. 0624	1. 0624	0. 8476	0. 309	-0. 3476	-0. 8714	0. 1286	0. 8434	2. 0
t=0. 4	1. 0	1. 0238	1. 0385	1. 0385	1. 0238	0. 809	0. 2852	0. 8434	0. 8434	1. 2852	2. 0
t=0. 5	1. 0	1. 0000	1. 0000	1. 0000	1. 0000	1. 000	2. 0000	2. 0000	2. 0000	2. 0000	2. 0
t=0. 6	1. 0	0. 9762	0. 9615	0. 9615	0. 9762	2. 191	2. 7148	3. 1566	3. 1566	2. 7148	2. 0
t=0. 7	1. 0	0. 9615	0. 9376	0. 9376	2. 1524	2. 691	3. 3476	3. 8714	3. 8714	3. 1566	2. 0
t=0. 8	1. 0	0. 9615	0. 9376	2. 1286	2. 6524	3. 309	3. 8476	4. 0624	3. 8714	3. 1566	2. 0
t=0. 9	1. 0	0. 9762	2. 1524	2. 6524	3. 2852	3. 809	4. 0238	3. 8476	3. 3476	2. 7148	2. 0
t=1. 0	1. 0	2. 1910	2. 6910	3. 3090	3. 8090	4. 000	3. 8090	3. 3090	2. 6910	2. 1910	2. 0