

1.

$$\begin{aligned}
 & \left[\begin{array}{cccc|c} 1.19 & 2.11 & -100 & 1 & 1.12 \\ 14.2 & -0.112 & 12.2 & -1 & 3.44 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 15.3 & 0.110 & -13.1 & -1 & 4.16 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 14.2 & -0.112 & 12.2 & -1 & 3.44 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 1.19 & 2.11 & -100 & 1 & 1.12 \end{array} \right] \\
 & \begin{aligned} R_2 - \frac{14.2}{15.3} R_1 \\ R_4 - \frac{1.19}{15.3} R_1 \end{aligned} \rightarrow \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & -0.2141 & 24.392 & -0.0719 & -0.4305 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 2.1014 & -99.0 & 1.0718 & 0.7928 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & -0.2141 & 24.392 & -0.0719 & -0.4305 \\ 0 & 2.1014 & -99.0 & 1.0718 & 0.7928 \end{array} \right] \\
 & \begin{aligned} R_3 + \frac{0.2141}{100} R_2 \\ R_4 - \frac{2.1014}{100} R_2 \end{aligned} \rightarrow \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & 24.1586 & -0.0934 & -0.4351 \\ 0 & 0 & -96.8994 & 1.0567 & 0.3395 \end{array} \right] \xrightarrow{R_3 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & -96.8994 & 1.0567 & 0.3395 \\ 0 & 0 & 24.1586 & -0.0934 & -0.4351 \end{array} \right] \\
 & R_4 + \frac{24.1586}{96.8994} R_3 \rightarrow \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & -96.8994 & 1.0567 & 0.3395 \\ 0 & 0 & 0 & 0.1701 & -0.3505 \end{array} \right] \Rightarrow \begin{cases} 15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16 \\ 100x_2 - 99.9x_3 + x_4 = 2.15 \\ -96.8994x_3 + 1.0567x_4 = 0.3395 \\ 0.1701x_4 = -0.3505 \end{cases} \Rightarrow \begin{cases} x_1 = 0.1148 \\ x_2 = 0.0161 \\ x_3 = -0.026 \\ x_4 = -2.061 \end{cases} \times
 \end{aligned}$$

2.

$$\begin{aligned}
 [A|I] &= \left[\begin{array}{cccc|cccc} 4 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 6 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{aligned} R_1 \div 4 \\ R_2 - \frac{R_1}{4} \\ R_3 + \frac{R_1}{4} \end{aligned}} \left[\begin{array}{cccc|cccc} 1 & \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{11}{4} & -\frac{3}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & -\frac{3}{4} & \frac{23}{4} & 2 & \frac{1}{4} & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array} \right] \\
 \begin{aligned} R_1 - R_2 \div \frac{11}{4} \times \frac{1}{4} \\ R_2 \div \frac{11}{4} \\ R_3 + R_2 \div \frac{11}{4} \times \frac{3}{4} \end{aligned} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{2}{11} & 0 & \frac{3}{11} & -\frac{1}{11} & 0 & 0 \\ 0 & 1 & -\frac{3}{11} & 0 & -\frac{1}{11} & \frac{4}{11} & 0 & 0 \\ 0 & 0 & \frac{6}{11} & 2 & \frac{2}{11} & \frac{3}{11} & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{aligned} R_1 + R_3 \div \frac{6}{11} \times \frac{2}{11} \\ R_2 + R_3 \div \frac{6}{11} \times \frac{3}{11} \\ R_3 \div \frac{6}{11} \\ R_4 - R_3 \div \frac{6}{11} \times 2 \end{aligned}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{4}{61} & \frac{73}{261} & -\frac{7}{81} & \frac{10}{261} & -\frac{4}{261} \\ 0 & 1 & 0 & \frac{6}{61} & -\frac{5}{61} & \frac{23}{61} & \frac{3}{61} & 0 \\ 0 & 0 & 1 & \frac{22}{61} & \frac{2}{61} & \frac{3}{61} & \frac{11}{61} & 0 \\ 0 & 0 & 0 & \frac{261}{61} & -\frac{4}{61} & -\frac{6}{61} & -\frac{22}{61} & 1 \end{array} \right] \\
 \begin{aligned} R_1 - R_4 \div \frac{261}{61} \times \frac{4}{61} \\ R_2 - R_4 \div \frac{261}{61} \times \frac{6}{61} \\ R_3 - R_4 \div \frac{261}{61} \times \frac{22}{61} \\ R_4 \div \frac{261}{61} \end{aligned} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{73}{261} & -\frac{7}{81} & \frac{10}{261} & -\frac{4}{261} \\ 0 & 1 & 0 & 0 & -\frac{7}{81} & \frac{11}{24} & \frac{5}{81} & -\frac{2}{81} \\ 0 & 0 & 1 & 0 & \frac{10}{261} & \frac{5}{81} & \frac{55}{261} & -\frac{22}{261} \\ 0 & 0 & 0 & 1 & -\frac{4}{261} & -\frac{2}{81} & -\frac{22}{261} & \frac{61}{261} \end{array} \right] \\
 \therefore A^{-1} &= \left[\begin{array}{cccc} \frac{73}{261} & -\frac{7}{81} & \frac{10}{261} & -\frac{4}{261} \\ -\frac{7}{81} & \frac{11}{24} & \frac{5}{81} & -\frac{2}{81} \\ \frac{10}{261} & \frac{5}{81} & \frac{55}{261} & -\frac{22}{261} \\ -\frac{4}{261} & -\frac{2}{81} & -\frac{22}{261} & \frac{61}{261} \end{array} \right] \times
 \end{aligned}$$

3.

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \quad \begin{aligned} a &= [3, 3, 3, 3] \\ b &= [-1, -1, -1] \\ c &= [-1, -1, -1] \end{aligned}$$

$$\lambda_1 = a_1 = 3, \quad u_1 = \frac{c_1}{\lambda_1} = -\frac{1}{3}$$

$$\lambda_2 = a_2 - b_1 \cdot u_1 = 3 - (-1) \left(-\frac{1}{3}\right) = \frac{8}{3}, \quad u_2 = \frac{c_2}{\lambda_2} = -\frac{3}{8}$$

$$\lambda_3 = a_3 - b_2 \cdot u_2 = 3 - (-1) \left(-\frac{3}{8}\right) = \frac{21}{8}, \quad u_3 = \frac{c_3}{\lambda_3} = -\frac{8}{21}$$

$$\lambda_4 = a_4 - b_3 \cdot u_3 = 3 - (-1) \left(-\frac{8}{21}\right) = \frac{55}{21}$$

$$Ly = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & 0 \\ 0 & 0 & -1 & \frac{55}{21} \end{bmatrix} y = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \Rightarrow y = \begin{bmatrix} \frac{2}{3} \\ \frac{11}{8} \\ \frac{43}{21} \\ \frac{64}{55} \end{bmatrix}$$

$$Ux = y \Rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{2}{3} \\ \frac{11}{8} \\ \frac{43}{21} \\ \frac{64}{55} \end{bmatrix} \Rightarrow x = \begin{bmatrix} 4.008 \\ 11.2914 \\ 6.4883 \\ 1.1636 \end{bmatrix} \Rightarrow \begin{aligned} x_1 &= 4.008 \\ x_2 &= 11.2914 \\ x_3 &= 6.4883 \\ x_4 &= 1.1636 \end{aligned} \quad \#$$