Given the equation $y'' = -(x+1)y' + 2y + (1-x^2)e^{-x}$, $0 \le x \le 1$, $y(0) = 1$,

$y(1) = 2$

use $h = 0.1$

Questions:

a. Use the shooting method to approximate the solution of the problem

b. Use the finite-difference method to approximate the solution

c. Use the variation approach to approximate the solution.

經過迭代:

a. Shooting method

$$y_1 = y, \quad y_2 = y'$$

$$\begin{cases} y_1' = y_2 \\ y_2' = -(x+1)y_2 + 2y_1 + (1-x^2)e^{-x} \end{cases}$$

$$y_1(0) = 1, \quad y_1(1) = 2$$

| $x$ | $y$ |
|-----|-----|
| 0.0 | 1 |
| 0.1 | 1.0166 |
| 0.2 | 1.0592 |
| 0.3 | 1.1243 |
| 0.4 | 1.2090 |
| 0.5 | 1.3105 |
| 0.6 | 1.4262 |
| 0.7 | 1.5545 |
| 0.8 | 1.6938 |
| 0.9 | 1.8427 |
| 1.0 | 2 |

b. Finite-difference method

| $x$ | $y$ |
|-----|-----|
| 0.0 | 1 |
| 0.1 | 1.1337 |
| 0.2 | 1.2685 |
| 0.3 | 1.4019 |
| 0.4 | 1.5310 |
| 0.5 | 1.6527 |
| 0.6 | 1.7634 |
| 0.7 | 1.8587 |
| 0.8 | 1.9339 |
| 0.9 | 1.9833 |
| 1.0 | 2 |

c. Variation approach

| $x$ | $y$ |
|-----|-----|
| 0.0 | 1 |
| 0.1 | -0.2740 |
| 0.2 | 0.4489 |
| 0.3 | 0.8298 |
| 0.4 | 0.7048 |
| 0.5 | 0.6131 |
| 0.6 | 0.8896 |
| 0.7 | 1.3248 |
| 0.8 | 1.3894 |
| 0.9 | 1.0248 |
| 1.0 | 2 |

程式執行結果:



Comparison of Numerical Methods for BVP