Counterfactually Fair Dynamic Assignment: A Case Study on Policing

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Resource assignment algorithms for decision-making in dynamic environments have been shown to sometimes lead to negative impacts on individuals from minority populations. We propose a framework for algorithmic assignment of scarce resources in a dynamic setting that seeks to minimize concerns around unfairness and the potential for runaway feedback loops that create injustices. Our model estimates an underlying true latent confounder in a biased dataset, and makes allocation decisions based on a notion of fair intervention. We present evidence for the plausibility of our model by analyzing a novel dataset obtained from the City of Chicago through FOIA requests, and plan to release this dataset along with a visualization tool for use by various stakeholders. We also show that, in a simulated environment, our counterfactually fair policy can allocate limited resources near optimally, and better than baseline alternatives.

CCS Concepts: • **Computing methodologies** → **Model development and analysis**.

Additional Key Words and Phrases: Resource Allocation, Fairness, Causal Models.

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1 INTRODUCTION

Algorithmic methods are increasingly used for decision-making in several societal resource allocation domains such as child welfare, homelessness, and policing services [15, 20, 26]. However, there is now widespread recognition that such methods impact differently demographic groups defined by ages, genders, races, etc [4, 25]. Additionally, the data collection process is dynamic and future data collected depends on past decisions of the algorithms, resulting in a feedback loop. For example, in the case of predictive policing, the algorithm may send more officers to neighborhoods with more reports of crime if it is continuously trained on previous data [10, 18]. This in turn can lead to more stops and arrests even without a true increase in crime in those neighborhoods. Such feedback loops may end up with the policing rate (and hence arrest rates) in neighborhoods becoming divorced from the "true" crime rate in those neighborhoods [10].

A theoretically well-grounded approach to fairness that has received considerable attention lately is to tie the notion of fairness to an explicit causal model [13, 16, 19, 21, 27]. This is particularly appealing in dynamic settings, because we can explicitly reason about the effects of interventions within a specific causal model. We argue in this paper that the correct notion of fairness in such settings is to require counterfactual fairness [16] in terms of both sensitive variables (as traditionally defined) and the variables corresponding to prior interventions in the system (for example, the level of past policing in a neighborhood). The latter requirement can help ensure that bias does not perpetuate dynamically through the system, resulting in runaway feedback loops of the kind hypothesized by Ensign et al. [10].

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We demonstrate the need for, and viability of, this approach through a combination of data and modeling. We first analyze a novel dataset for the City of Chicago collected by merging three different sources: (1) population demographics from the American Community Survey[1]; (2) publicly available data on crimes, arrests, and stops from the website of the Chicago Police Department[2]; (3) police deployment levels obtained using a Freedom of Information Act (FOIA) request from the police department (the deployment levels will be made publicly available). The dataset demonstrates significant variability in terms of observed measures related to crime and policing (rates of crimes, arrests, and stops), and also in terms of allocation of policing resources within and across districts. Of particular interest is the fact that there is a significant positive relationship between the discretionary stops of civilians and the number of police officers deployed in a beat within a district. While unsurprising, this provides empirical evidence that policing is likely to drive up stops (and hence arrests), while the effect on truly concerning crime is not necessarily evident. Though Horel et al. [12] present a novel dataset on complaints, use of force histories in the Chicago Police Department (CPD), our dataset has features on crime, arrests, stops and assignments on a beat level that allows for a study of resource allocation across police beats by district.

Based on this, we turn to constructing a model that aims to capture both the complexity of dynamically allocating limited police resources across beats and the constraints on doing so in a fair manner. The model is a stylized example with two beats and a constant total measure of police that can be allocated across the two beats. Our main idea is to introduce a causal model with a latent variable (which we label "Criminality") that captures all aspects of what would drive the truly concerning crime rate in a neighborhood (or beat), and to "tie" the causal models of the two beats together through a resource allocation constraint. Across a variety of representative scenarios, we show that an approach that estimates the latent Criminality variable in a counterfactually fair manner (with respect to both sensitive attributes and prior policing allocations) performs close to the optimal allocation decision if the true values of Criminality were known, and significantly better than a number of alternative baselines that are motivated by standard predictive policing. Finally, we discuss possible implications if a model like ours were used in a setting like Chicago by considering the decisions it would make on some beats and how they differ from what happened in reality.

Contributions. Our main contributions are as follows:

- We propose a causal model where the outcome of interest is affected neither by the dynamics of the data collection process nor by sensitive attributes.
- We show that estimating this model from observations leads to a fair allocation for which intervention on the sensitive attributes would not affect the outcome.
- In a simulated environment, we also show that the resulting allocation is near optimal under resource constraints.
- We construct a novel real-world dataset that allows us to examine police force allocation at a granular level in the City of Chicago. We identify areas where it would or would not have made a difference if our causal model had been used in prior allocation decisions. We will make this dataset public to provide a new benchmark in fair dynamic resource allocation.
- We have also developed a visualization tool to explore key characteristics for each police beat and compare the allocation of police officers in both the real world and in our model. We envision use by multiple stakeholders – researchers, citizens, and police departments. The screenshots of this tool are in the appendix and will be released after publication.

[1] https://www.census.gov/programs-surveys/acs
[2] https://home.chicagopolice.org/statistics-data/

2 RELATED WORK

2.1 Causal Models and Fairness

Recent contributions have proposed different definitions of fairness in the context of decision making algorithms [5, 8, 11, 28]. Definitions include fairness through unawareness that ensures that protected attributes are not used in decision making [5, 9]; demographic parity [28] that equalizes the algorithm's positive rate across demographic groups; or, individual fairness [8] that enforces that similar individuals would be treated similarly.

Of particular interest in this paper is the notion of counterfactual fairness [16] that has been applied to hiring decisions or college admission decisions [19, 21, 24]. The notion of counterfactual fairness relies on explicitly modeling the causal relationships between parent and children variables [22] and by identifying and controlling the dependencies between sensitive/protected variables and the outcome variables [6, 14, 16]. This allows decision-making algorithms to conveniently detect and mitigate any unfair/biased effect incurred due to the protected variables [16].

We extend counterfactual fairness to dynamic settings by imposing that current allocations are not affected by both interventions on protected attributes and prior allocation decisions. These constraints relate to recent contributions in dynamical fairness (see [7] for a unifying framework). Our novelty is to evaluate allocation policy while mitigating the effect of both past decisions and protected attributes on predicted outcomes. Our work also relates to learning causal latent variable models (see e.g. [17, 19]). Likewise, we infer latent confounders from observed proxies. However, while Madras et al. [19] assume that the treatment decision is independent across agents, we argue that in scenarios where there is a limited resource to divide among multiple agents, the treatment variable for one agent depends on the treatment status of other agents. Our contribution is to show that in a stylized framework, the joint posterior distribution of the latent variables captures these dependencies across agents.

2.2 Predictive Policing

Predictive policing refers to decision algorithms that use historical data to allocate police officers across a city and minimize crime [3, 10]. Tools like PredPol [20] rely on attributes such as location, type, offenders, and characteristics of past crimes to identify crime hot spots [23]. However, these tools potentially exacerbate social biases because of feedback loops: the data is collected according to the algorithm's past decisions [10, 18, 20]. Using data on drug-related arrests, Lum and Isaac [18] showed empirically how feedback loops result in over-policing neighborhoods with a larger fraction of minority populations.

This paper relates to modeling efforts that attempt to mitigate the detrimental effect of feedback loops on minority populations. Ensign et al. [10] propose an arrest-based randomization procedure to decouple current allocation from past decisions. However, their model assumes the crime rates across neighborhoods without accounting for the geographic heterogeneity of criminal activities within a city. Our model captures into a latent variable the factors that would drive local criminality. Therefore, we can mitigate the feedback loop while proposing an allocation of police patrols that reflect the socio-economic diversity of a city. We believe that our approach is a step toward practical implementation of a fair allocation of police officers.

3 CAUSAL INFERENCE FOR PREDICTIVE POLICING

We follow Pearl [22] and define a structural causal model as a triple (Z, X, F) where Z is a set of latent background variables, X a set of observed variables, and F a set of functions or structural equations that represents causal relations between Z and X. Let $S \in X$ denote the set of protected attributes, variables that must not be discriminated against

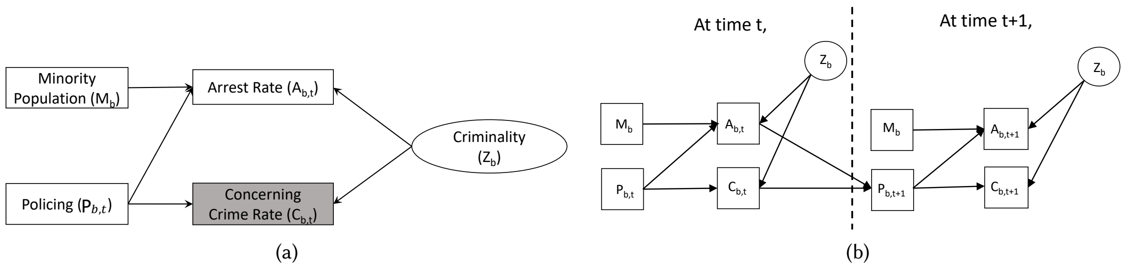


Fig. 1. (a) A causal model for policing and crime. Sensitive attributes: Policing, Minority Population; Observed features: Arrest Rate; Outcome Variable: Concerning Crime Rate; Latent Variable: Criminality. (b) In our causal model, there is no direct arrow between $(A_{b,t}, C_{b,t})$ and $(A_{b,t+1}, C_{b,t+1})$. We assume that time t variables influence time $t+1$ variables only through the observable $P_{b,t+1}$.

based on a fairness definition. We follow Counterfactual Fairness [16] to define the fairness, which states that changing S while holding everything else constant should not change the outcome of the model. The objective is to estimate the function F from data formed by observed variables X.

Our approach to predictive policing is to construct a causal model that could reasonably describe the underlying data generating process and to use this causal model to estimate an optimal policy allocation that is not affected by the biases introduced by the data collection.

3.1 Causal Model for Predictive Policing

The objective of predictive policing is to choose a police allocation $\{P_{b,t}\}$ for each beat b and period of time t. The decision maker can access historical arrest rates $A_{b,t'}$, stop rates $S_{b,t'}$, concerning crime rates $C_{b,t'}$ and protected demographic characteristics M_b collected at time $t^{'} < t$. Protected demographic characteristics include the fraction of minority populations that live in the beat. The adjective protected, as is conventional in the fairness literature, captures the fact that allocation decisions should not discriminate among populations based on these demographic characteristics.

In our causal model, we assume that there exists a latent variable Z_b that represents the true level of criminal activity in a beat b and that affects arrest rates $A_{b,t}$, and concerning crime rates $C_{b,t}$ (Figure 1a). Policing $P_{b,t}$ is what the causal literature considers as a treatment and affects $A_{b,t}$, and $C_{b,t}$. Our causal model first assumes that protected demographic characteristics M_b affects the outcomes $A_{b,t}$. Secondly, we assume that criminal activity Z_b does not fluctuate with the short-term allocation of police officers and thus does not depend on t. Thirdly, we assume that dynamic effects between time t and time $t+1$ are propagated only through the policing variable P_{t+1} (see Figure 1b). For example, arrest rates at time t affect outcomes at time $t+1$ only through their effect on policing at time $t+1$. Note that since M_b and Z_b are not time-dependent, we rule out the possibility that past criminal activities would affect the demographic composition of a neighborhood. We also rule out true feedback loops (e.g. crime itself begets more crime) that are not intermediated by policing.

3.2 Allocation

In this paper, we argue the need to carefully choose the objective that the decision maker attempts to optimize. First, in our setting, we assume that there exists a resource constraint on the total amount of police force $\overline{P}_t$. that can be allocated at time t:

$$\sum_b P_{b,t} \leq \overline{P}_t. \tag{1}$$

This expands previous work in fair causal modeling [16, 19] that learn optimal policies without such resource constraints on the treatment variable.

Second, we propose a dynamic resource allocation that optimizes an outcome that is not affected by the dynamics of the data collection process nor by the effect of the protected attribute M. This differs from previous approaches (e.g. [19]) that optimize an outcome possibly affected by both protected attributes and dynamic data collection and mitigate potential unfair outcomes via interventions that remove these unwanted effects. For example, choosing to minimize arrest rates would lead to sub-optimal policies since, in that data, arrest rates are mechanically larger in places with more police to make the arrests; and empirical evidence shows that arrest rates are larger in places with a larger fraction of minority populations (Figure 4a). Previous approaches [19] minimize arrest rates after an intervention $do(M \to 0)$ and/or $do(P_{t-1} \to 0)$. In this paper, we argue instead in favor of minimizing

$$\min_{P_{b,t}} \sum_b C_{b,t} \text{ s.t } \sum_b P_{b,t} \leq \overline{P}_t. \tag{2}$$

3.3 Fairness

The solution of the optimization problem $\boldsymbol{P_t^*} = \{P_{b,t}^*\}$ depends only on total resources $\overline{P}$ and criminal activity $\boldsymbol{Z} = \{Z_b\}$. Since in our causal model (Figure 1a), criminal activity and percentage of minority populations are independent, the allocation policy $\boldsymbol{P_t^*}$ is counterfactually fair with respect to minority populations in the sense of Kusner et al. [16]. An intervention or do-operation [22] on the percentage of minority populations would not affect the allocation of police force in a given beat: for any value of m and any beat b,

$$E[P_{b,t}^* | \text{do}(M_b) = m] = E[P_b^*]. \tag{3}$$

Moreover, since in our causal model (see Figure 1b), conditional on past data, M and P are independent, the counterfactually fair property of the optimal allocation $\boldsymbol{P_t^*}$ is akin to a statistical parity [8] condition with $E[P_{b,t}^* | M_b = m] = E[P_b^*]$. In this paper, we aim at generating fair allocation policies by learning from the data a policy that asymptotically converges to $\boldsymbol{P_t^*}$ and thus, retains the counterfactually fair property of $\boldsymbol{P_t^*}$.

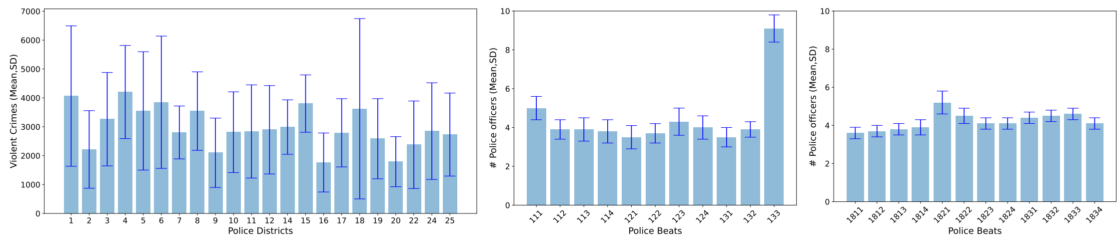
4 DATASET AND VALIDATION

As part of this work, we construct a novel dataset to analyze the relationships between police allocation and observed crimes in the city of Chicago. This is an important new benchmark to analyze the relationship between the allocation of scarce resources and outcomes at a granular level. We merge three sources of data: (i) population-level demographics (US Census Bureau's 2015-2019 American Community Survey [3]; (ii) crime-related historical data (crimes, arrests and police stops between 2017 and 2021); and, (iii) allocation of police officers at the district and beat levels in 2019 – 2021 [4].

Information on the allocation of police officers is critical to understand dynamic resource constraints on the decision variable within a given geographic area. The Chicago Police Department is organized into 22 police districts. A district is overseen by a police commander and is divided into 3 to 5 police sectors, which are further divided into 3 to 5 police beats. In total there are 273 police beats in the City of Chicago. The average standard deviation of district police counts

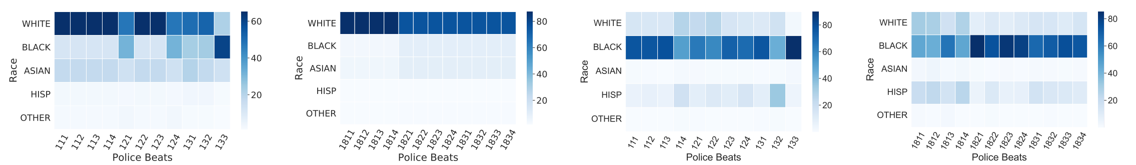
[3]https://www.census.gov/programs-surveys/acs

[4]We obtained data on allocation of police through a Freedom of Information Act (FOIA) request to the Chicago Police Department.



(a) The mean and std of Violent crimes (b) District 1 (c) District 18

Fig. 2. (a) The mean and standard deviation of observed Violent crimes in each district. Weekly assignment of police officers in Districts 1 (b) and 18 (c). X-axis represents the police beats and Y-axis represents the number of police officers assigned each week.



(a) District 1 (Population %) (b) District 18 (Population %) (c) District 1 (Stops %) (d) District 18 (Stops %)

Fig. 3. Demographic analysis of District 1 and 18. Y-axis represents the different racial groups and X-axis represents the police beats under District 1 and 18. The color scale represents the percentage of population in (a), (b), while it represents the percentage of stops in (c), (d).

within a year is about 24.5, while the average standard deviation of police counts within a beat is 2.4. Both numbers suggest significant dynamics in police counts at a granular level.

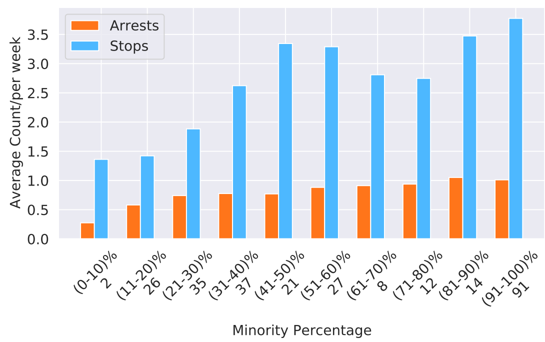
Data on allocation of police officers includes daily counts of police officer allocations at the district level between 2017 and 2019; and daily counts between 2019 and 2021 at the beat level. In addition to the allocation of police, our combined dataset includes every record of reported and discovered crimes, arrests, and stops between 2017 and 2021. Crimes can be categorized as violent or property-related crimes. The Chicago Police Department defines as violent the following crimes [5]: aggravated assault, robbery, homicide, human trafficking, rape, and aggravated battery. We use violent crime rate as a proxy for concerning crime rate.

We join this data with 2019 population-level demographics for each police beat. We define as protected attributes the percentage of non-white population in a beat.

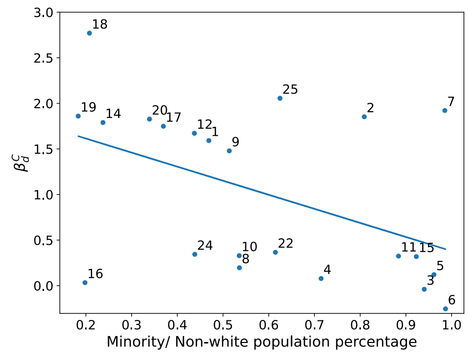
4.1 Dataset Characteristics

Figure 2a shows the average and variance of violent crime rates across districts. District 1 and 18 have the highest variance in violent crime rates and thus, potentially the most interesting allocation dynamics. Heterogeneous crime rates across beats within a district open the possibility of a more efficient allocation of police officers than an allocation based on a constant number of police officers per capita.

[5] https://home.chicagopolice.org/statistics-data/statistical-reports



(a) Weekly arrests and stops for each concerning crime.



(b) β_d^c vs Minority population percentage

Fig. 4. (a) Average arrests (orange) and stops (blue) per week for each concerning crime within police beats by minority population percentage. The X-axis shows different ranges of minority population percentage (and the number of beats in that range) and the Y-axis shows the weekly counts. (b) We estimate the effectiveness (β_d^c) of police officers in different police districts. There is a clear negative relationship between our measure of effectiveness and the nonwhite population percentage in a district, consistent with several reports on policing in Chicago.

Interestingly, Figure 2b, which shows the means and standard deviations of weekly assignments of officers per beat in the two districts, reveals clear differences in district-level decision-making. While District 18 seems to allocate a relatively constant number of police officers per thousand residents, District 1 follows a more heterogeneous policy. Although there might be some contextual explanations (e.g different district commanders) for these different approaches, this variation could be interesting to exploit to understand the effects of different types of allocation strategies. For example, we explore in the next section how these allocations of police forces compare to the ones that minimize concerning crime rates. We can also do some analysis of how different policing statistics vary with racial composition.

Figures 3a and 3b show the population-level demographics for Districts 1 and 18, respectively. Figures 3c and 3d show the distribution of race of individuals stopped in different beats within these two districts. We find that while the majority of the population in all the beats within these two districts is white, most of the individuals being stopped are black.

Figure 4a shows the number of arrests and stops normalized by the number of concerning crimes as a function of the percentage of the minority population in a district. Both the average number of arrests and stops per concerning crime trends upward with the increase of minority population percentage.

Our data allows a more systematic analysis of data of so-called investigatory stops. We classify as investigatory the stops that are not directly related to a specific crime and do not follow the description made by a flash message, a victim, or a witness. We posit that these investigatory stops are discretionary and that higher deployment of police in a beat would lead to more discretionary stops. We find a positive and significant correlation between the number of police officers assigned to a beat and investigatory stops per week across beats. For statistical testing, we consider the correlation of weekly investigatory stops in a beat with minority population percentage, and, the number of police assigned in a beat each week. We do the statistical tests for each year from 2019 to 2021, the coefficients and p-values are presented in Table 1.

Table 1. Statistical Testing

Year	2019		2020		2021	
y = investigatory stops	**Coefficient**	**p value**	**Coefficient**	**p value**	**Coefficient**	**p value**
x = minority population percentage	0.24	7.82e-222	0.14	2.94e-77	0.13	1.25e-54
x = the number of police assigned	0.29	0.0	0.27	7.09e-293	0.21	1.37e-152

From all of these results, we can see that, over the years, investigatory stops had a positive and significant correlation with minority population percentage and the amount of policing. This is an indication of the minority population being disproportionately affected by discretionary stops.

We have also developed a visualization tool to explore the demographic information for each police beat and compare the allocation of police officers both in the real world and in our model. Multiple allocation test cases for different districts and beats are presented in the appendix, and we will release the tool when this paper becomes public.

5 ALLOCATION

5.1 Optimal Allocation of Police Force

To make progress toward solving (2), we parameterize the structural causal model in Figure 1a as follows:

$$\begin{aligned} Z_b &\sim \mathcal{N}(0,1) \\ C_{b,t} &= \alpha_d^C + \gamma_t^C + (1 - \beta_d^C \theta(P_{b,t}))Z_b + \epsilon_{b,t}^C \\ A_{b,t} &= \alpha_d^A + \gamma_t^A + \beta_d^A \theta(P_{b,t})Z_b + \delta_d M_b + \epsilon_{b,t}^A \end{aligned} \tag{4}$$

where $\epsilon_{b,t}^C, \epsilon_{b,t}^A$ are unobserved independent zero-mean Gaussian noise.

5.1.1 Validation of Causal Model. Validating the specific causal model we have developed is challenging, because the "true" rate of concerning crime is both unobserved (although we may be able to proxy for it using the violent crime rate, for example), and a function of endogenous past policing decisions. However, one approach to validation is to use our estimated β parameter at a district-wise level as a measure of policing effectiveness. A district would demonstrate more effective policing if allocating more police successfully reduces concerning crime; thus higher values of the β parameter in the model would indicate more effective policing; at the extreme, negative values would indicate a reversal (commonly speculated on in journalistic reports) where more policing increases (relatively) the rate of crime.[6]

Figure 4b demonstrates a clear negative relationship between our measure of effectiveness and the nonwhite population percentage in a district, consistent with several reports on policing in Chicago [1]. The cluster of relatively high-crime, almost exclusively minority population districts at the bottom right (3, 5, 6, 11, 15) also features 5 of the 7 leaders in the per-capita number of complaints against the police department, which we collected separately. There are a couple of outliers, District 7, also high minority, high crime, and high complaint, but revealing a high estimate of effectiveness, and District 16, a high majority, low-complaint district that has a low estimated policing effectiveness. Further investigation into whether these are statistical anomalies or reveal something deeper about policing practices in those districts would be interesting, but the overall results are reassuring that our causal model is capturing important

[6]Note that for the purposes of the theoretical modeling of optimal allocation in this paper, we ignore the case of negative β as we take this to be indicative of ineffective or problematic policing and not a truly negative relationship in the presence of an effective police force. We also note that negative β values are few in the data.

relationships between policing and crime. One possible concern is that District 18 might influence the regression and create the negative slope. We removed District 18 from the data and regressed for the other districts. The slope did not change significantly and remained negative.

5.1.2 Solving the Allocation Problem. For a given district d, we want to allocate in B beats at time t the total police force $\overline{P_d}$ to minimize the total criminal activities in the district:

$$\min_{P_1,\ldots,P_B} \sum_b^B C_{b,t} \text{ s. t } \sum_b^B P_{b,t} \leq \overline{P_d} \text{ and } \forall b, t, P_{b,t} > 0.$$

We assume that the criminal activity in beat b at time, $t = 1$ given by,

$$C_b = \kappa + (1 - \beta \frac{1}{1 + e^{-P_b}}) Z_b \tag{5}$$

where κ regroups the district and time fixed effects. For nonnegative assignment, the objective function is convex. The Lagrangian of the optimization problem can be written

$$\mathcal{L} = \sum_b^B C_b + \lambda \sum_b^B (\lambda - \mu_b) P_b \tag{6}$$

where λ and μ_b are the Lagrangian multipliers associated with the resource constraint and the nonnegativity constraint, respectively. Since the constraints are linear, the linear constraint qualification applies and a solution $P^* = P_1, \ldots P_B$ of the allocation problem satisfies the following Kuhn Tucker first order conditions:

$$\forall b, \beta Z_b \theta(P_b^*)(1 - \theta(P_b^*)) = \lambda - \mu_b \tag{7}$$

and

$$\lambda \geq 0 \text{ and } \lambda(\sum_b^B P_b^* - \overline{P_d}) = 0 \tag{8}$$

and

$$\forall b, \mu_b \geq 0 \text{ and } \mu_b P_b^* = 0 \tag{9}$$

where $\theta(.)$ is the sigmoid function.

Solution. Suppose that we order the beats by their criminality Z_b such that $Z_1, Z_2, \ldots, Z_B$.

Lemma 5.1. *If for beat b, the optimal solution implies $P_b^* = 0$ then for any $b' < b$, $P_{b'}^* = 0$.*

Proof. Suppose that for some $b' < b$, $P_{b'}^* = 0$. An allocation that will allocate $P_{b'}^*$ to beat b' and no policing to beat b will satisfy the resource constraint, but since the crime rate

$$\begin{aligned} &\beta Z_b (\frac{1}{2} - \theta(P_{b'}^*)) - \beta Z_{b'} (\theta(P_{b'}^*) - \frac{1}{2}) \\ &= \beta(\frac{1}{2} - \theta(P_{b'}^*))(Z_b - Z_{b'}) < 0 \end{aligned}$$

It is a contradiction, since the allocation P^* is supposed to be optimal. □

Theorem 5.2. *Let ξ_b denote $\frac{4}{\beta Z_b}$ for $b = 1, \ldots, B$. Let $\overline{b}$ be the first beat such that $\sum_{b \geq \overline{b}}^B -ln\left(\frac{1 - \sqrt{1 - Z_{\overline{b}}/Z_b}}{1 + \sqrt{1 - Z_{\overline{b}}/Z_b}}\right) \leq \overline{P}$. The allocation problem has a unique solution P^* such that:*

(1) *For* $b < \overline{b}, P_b = 0$;

(2) *For* $b \geq \overline{b}, \mu_b = 0$ *and* $P_b = \sum_{b \geq \overline{b}}^{B} -ln\left(\frac{1-\sqrt{1-\lambda^*\xi_b}}{1+\sqrt{1-\lambda^*\xi_b}}\right)$ *where* λ^* *is the unique solution of*

$$\overline{P} = \sum_{b \geq \overline{b}}^{B} -ln\left(\frac{1-\sqrt{1-\lambda^*\xi_b}}{1+\sqrt{1-\lambda^*\xi_b}}\right) \tag{10}$$

Proof. First, because of the linear constraint qualification, we know that a solution P^* to the allocation problem is solution of the Kuhn-Tucker first conditions (7), (8) and (9). We show then that P^* is uniquely defined by 5. and 6. in Theorem 5.2. By Lemma 5.1, we can define $\underline{b}$ such that for $b < \underline{b}, P^*_b = 0$. For $b < \underline{b}$, we have $\mu_b = 0$ because of the third set of Kuhn-Tucker conditions (9) and we can invert the first Kuhn-Tucker condition to obtain $P_b = -ln\left(\frac{1-\sqrt{\lambda\xi_b}}{1+\sqrt{\lambda\xi_b}}\right)$ Moreover, as long as policing decreases crime rate ($\beta > 0$, see below for further discussion of this issue), a solution of the allocation problem will exhaust all policing resources and the allocation constraint is binding, that is λ satisfies

$$\overline{P} = \sum_{b \geq \underline{b}}^{B} -ln\left(\frac{1-\sqrt{1-\lambda\xi_b}}{1+\sqrt{1-\lambda\xi_b}}\right) \tag{11}$$

and $\lambda \in [0, \frac{1}{\xi_{\underline{b}}})$. It remains to show that $\underline{b} = \overline{b}$. Suppose $\underline{b} < \overline{b}$, then

$$\overline{P} = \sum_{b \geq \overline{b}}^{B} -ln\left(\frac{1-\sqrt{1-\lambda^*\xi_b}}{1+\sqrt{1-\lambda^*\xi_b}}\right) \geq \sum_{b \geq \underline{b}}^{B} -ln\left(\frac{1-\sqrt{1-Z_{\underline{b}}/Z_b}}{1+\sqrt{1-Z_{\underline{b}}/Z_b}}\right) > \overline{P} \tag{12}$$

where the last inequality comes from $\underline{b} < \overline{b}$, which leads to a contradiction. Therefore, $\underline{b} \geq \overline{b}$. Suppose that $\underline{b} > \overline{b}$. Then, $P^*_{\overline{b}} = 0$. Moreover, $\lambda^* < \beta Z_{\overline{b}}/4$. Hence, by the first Kuhn-Tucker condition (7),

$$\mu_{\overline{b}} = \lambda^* - \beta Z_b \theta(P^*_b)(1-\theta(P^*_b)) = \lambda - \frac{\beta Z_{\overline{b}}}{4} < 0 \tag{13}$$

which is a contradiction since $\mu_b \geq 0$ Hence $\underline{b} = \overline{b}$.

Moreover, at P^*, the Hessian matrix of the Lagrangian $\mathcal{L}$ is a diagonal matrix with non-negative entries and thus is definite semi-positive. Therefore, the solution P^* of the Kuhn-Tucker first order conditions is a local minimum of the allocation problem. Since it is the unique local minimum. The objective function is continuous and the optimization problem is over a compact set $[0, \overline{P}]$, hence by the extreme value theorem, the allocation problem has a global minimum. Therefore, P^* is the unique solution of the allocation problem.

□

5.1.3 *Allocation Policies.*

Benevolent Omniscient Planner Policy. A benevolent planner that knows the structure of the causal model in Figure 1a and the values of its parameters would allocate police at time t according to equation (2). However, in practice, decision makers do not have direct knowledge of the parameter values in the model. We propose five allocation approaches that use past data to allocate future policing: *PropFair*, *PropArrest*, *PropCrime*, *PropPolyaUrnAR*, and *PropPolyaUrnCR*.

PropFair. This method estimates the parameters $\boldsymbol{\alpha}$ and the criminal activity $\boldsymbol{Z}$ from the data and then, derives an allocation of the police force using the policy (2). We estimate the posterior distribution of $\boldsymbol{Z}$ and other coefficients from past observed data $\boldsymbol{P}_{t'}, \boldsymbol{A}_{t'}, \boldsymbol{S}_{t'}, \boldsymbol{M}, \boldsymbol{C}_{t'}$ where $t^{'} < t$. Then, we plug in (2) the maximum a posteriori values for $\boldsymbol{Z}$ and $\boldsymbol{\alpha}$ given past data and compute an allocation of police force at time t.

Baselines: PropArrest and PropCrime. Our baseline is an allocation policy, *PropArrest*, that allocates police resources to a beat proportionally to past arrest rates $A_{b,t-1}$, that is $P_{b,t}^{PropArrest} = A_{b,t-1}/\sum_{b'} A_{b',t-1}$. This policy allocation is likely to generate a feedback loop [10, 18] since a larger fraction of the police force will be allocated to beats with proportionally larger shares of arrest rates at time $t-1$; and thus, if more policing leads to more arrests, future allocations would exacerbate initial differences in arrest rates A_{t-1}. An alternative policy, *PropCrime*, allocates police force proportionally to past concerning crime rates.

PropPolyaUrnAR and PropPolyaUrnCR. We compare our method with two baselines based on the Polya urn model proposed by Ensign et al. [10]: *PropPolyaUrnAR* and *PropPolyaUrnCR*. We model the two beats as two colored balls X and Y in an urn. Initially the urn contains equal numbers of balls for the two beats ($n_x = n_y = n$), meaning they have equal numbers of officers allocated. The urn is updated based on the past arrest (*PropPolyaUrnAR*) and crime (*PropPolyaUrnCR*) rates. Specifically the probability of adding a police officer in beat X is $\frac{n_x^t}{n_x^t+n_y^t}\lambda_X \frac{n_y^t}{n_x^t+n_y^t}$, where λ_X is the past crime or arrest rate and n_x^t, is number of police officers in beat X at time t.

5.2 Simulation Environment

For simplicity, we instantiate the causal model in Figure 1a using the parameter values set to 1. $\alpha_d^C = \gamma_t^C = \beta_d^C = \alpha_d^A = \gamma_t^A = \beta_d^A = \delta_d = 1$. We denote all parameters by $\boldsymbol{\alpha}$. In this setting, the benevolent planner's policy is given by (2). However, in practice, we do not observe $\boldsymbol{Z}$ and the parameters and need to estimate these quantities from observed data.

At each period t, our approach, *PropFair*, uses the observed data $\mathcal{D}_{t-1} = \{\boldsymbol{A}_\tau, \boldsymbol{S}_\tau, \boldsymbol{C}_\tau, \boldsymbol{M}, \boldsymbol{P}_\tau\}_{\tau<t}$ and computes the posterior distribution $P(\boldsymbol{Z}|\mathcal{D}_{t-1})$ and $P(\boldsymbol{\alpha}|\mathcal{D}_{t-1})$. We fix $\boldsymbol{\alpha}_t = \operatorname{argmax} P(\boldsymbol{\alpha}|\mathcal{D}_{t-1})$ and $\boldsymbol{Z}_t = \operatorname{argmax} P(\boldsymbol{Z}|\mathcal{D}_{t-1})$ and compute the allocation policy as in (2).

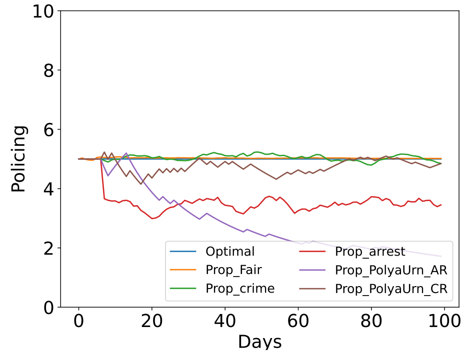
We compare the allocation policy from *PropFair* to the policy followed by a benevolent omniscient planner (who knows the parameters of the model) and to the policies obtained by *PropArrest* and *PropCrime* that allocate policing proportionally to past arrest and crime rates. Additionally, we compare the allocation to the randomized policies *PropPolyaUrnAR* and *PropPolyaUrnCR* based on the past arrest and crime rates.

5.3 Results

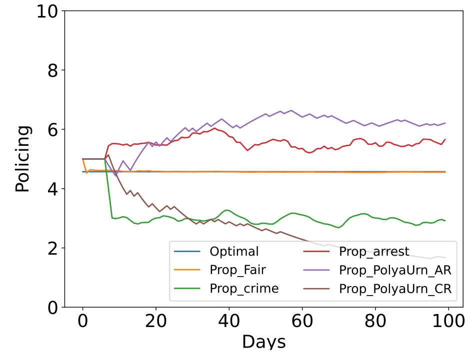
Figure 5 presents the results of an experiment where we allocate police force across two beats B_1 and B_2, which, in the data generating process, vary by their level of criminality Z and percentage of minority population M. The allocation is done under a resource constraint $P_{b_1,t} + P_{b_2,t} = 10$ over 100 time periods.

Case 1: Beats differ by percentage of minority populations. Figure 5a examines the case where the two beats differ only in the percentage of the minority population in the beat. We observe that the policies obtained from *PropFair*, *PropCrime* and *PropPolyaUrnCR* converge to the optimal policy. However, *PropArrest* is unable to allocate policing optimally: the allocation policy converges to a policy that allocates majority of police officers to B_2, instantiating the predicted runaway feedback loop from the initial allocation of more police to an area with a higher minority population. Similar to *PropArrest*, *PropPolyaUrnAR* allocates more police to Beat B_2.

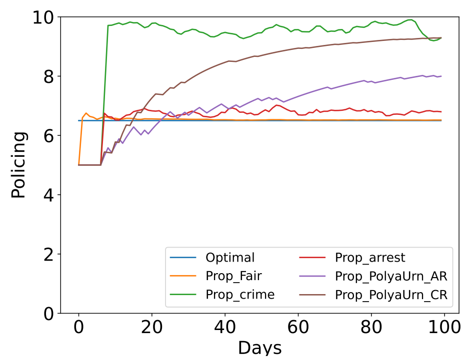
Case 2: Beats differ by level of criminal activity. In the case where level of criminal activity varies across beats, with lower criminal activity in the beat with a higher minority population, the allocation policy from *PropFair* is closer



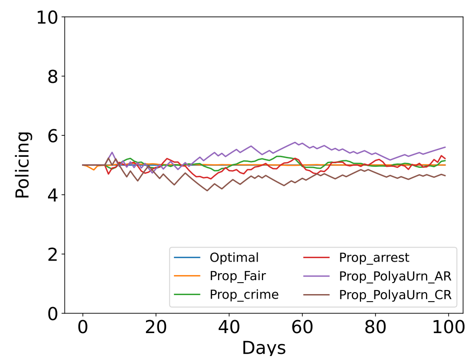
(a) Case 1: Beats differ by percentages of minority population ($Z_{b_1} = 0.5$, $Z_{b_2} = 0.5$, $M_{b_1} = 0.2$, $M_{b_2} = 0.8$).



(b) Case 2: Beats differ by level of criminal activity ($Z_{b_1} = 0.3$, $Z_{b_2} = 0.7$, $M_{b_1} = 0.8$, $M_{b_2} = 0.2$).



(c) Case 3: Beats differ by level of criminal activity ($Z_{b_1} = 0.95$, $Z_{b_2} = 0.05$, $M_{b_1} = 0.8$, $M_{b_2} = 0.8$).



(d) Case 4: Beats are the same ($Z_{b_1} = 0.5$, $Z_{b_2} = 0.5$, $M_{b_1} = 0.25$, $M_{b_2} = 0.25$).

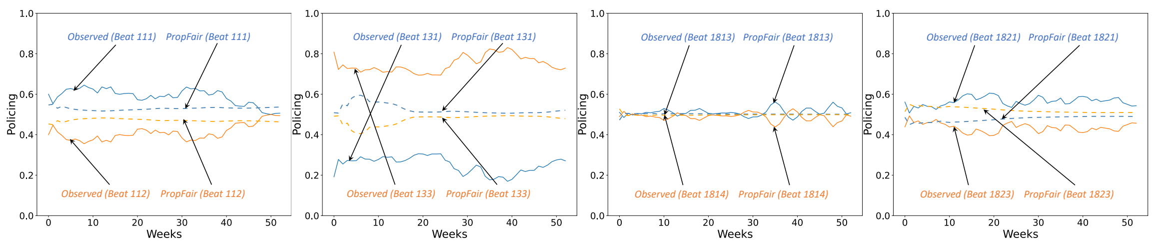
Fig. 5. Comparative analysis of baseline policies (*PropArrest*, *PropCrime*, *PropPolyaUrnAR*, *PropPolyaUrnCR*), optimal policy (Benevolent planner's policy) and proposed policy (*PropFair*). The graphs show the fraction of police allocated to Beat B_1 (the remaining fraction is allocated to B_2).

to the optimal policy generated by an omniscient and benevolent planner than the allocations offered by the baseline models *PropArrest* and *PropCrime*. We observe that both *PropArrest* and *PropPolyaUrnAR* exacerbate initial differences in the level of arrests and allocates too much policing to the beat (B_1) with the lower initial criminal activity ($Z_{B_1} < Z_{B_2}$). On the other hand, both *PropCrime* and *PropPolyaUrnCR* underestimate differences in initial criminal activity and under-allocates policing to beat B_1. Only *PropFair* is able to reach the correct balance.

Case 3: Beats differ by level of criminal activity. In Case 3, criminality is now higher in beat B_1 than beat B_2, but the minority populations are the same. *PropCrime* and *PropPolyaUrnCR* converge to allocating the majority of policing resources to B_1. Though *PropArrest* and *PropPolyaUrnAR* slightly overestimates the criminal activity in beat B_1, for *PropArrest*, it is not too far from the optimal allocation. However, *PropFair* provides the closest allocation policy that converges to the optimal policy.

Case 4: Beats are similar. Finally, Figure 5d shows that when criminal activity Z and percentages of the minority population are the same across beats, all methods, including *PropArrest*, generate an allocation policy that is close to optimal. Case 4 serves as a robustness check and illustrates that the baseline models fail to allocate policing appropriately only if there are differences in percentages of minority populations and criminality across beats.

5.4 Application to Police Allocation in Chicago.



(a) (111,112) (b) (131,133) (c) (1813,1814) (d) (1821,1823)

Fig. 6. Observed and predicted assignment of police officers as estimated by our method. Police Beats (111,112) and (131,133) are neighbouring beats of District 1 Police Beats (1813,1814) and (1821,1823) neighbouring beats of District 18 of Chicago police department. Y axis shows the proportion of policing in the pair of beats.

We now counterfactually consider whether applying *PropFair* in Chicago would have led to different allocations than were observed in reality, and use this as an example of how it can help provide insight into the data. We assume that the latent variable Z_b remains constant over a week at the beat level and apply *PropFair* to allocate police across all beats in a given district. We apply this procedure to Districts 1, 4, 7, 11, 18, and 22 as illustrative examples. In the main text we report results from Districts 1 and 18 and defer the remaining to the Appendix. We estimate the model in Equation (4) with the same procedure as in our previous simulations: (i) for each week t, we estimate $\boldsymbol{\alpha}$ and $\boldsymbol{Z}$ using the daily data observed before week t; and, (ii) we assign police P^* at week t according to (13).

Figure 6 shows the police assignment for four specific beats (for visual parsimony) in Districts 1 and 18. First, let us look at beats in District 1, which seems to be more dynamic in real-world allocation (6a, 6b). *PropFair* assigns similar fractions of police to beats 111 and 112 as the real-world allocation. These beats have roughly equal minority populations, and therefore the similarity of the optimal allocation to the real one implies that the differences in policing levels are probably driven by "true criminality," which one would consider fair. On the other hand, Figure 6b shows that the differences in observed police allocation between beats 131 and 133 cannot be explained only by differences between the criminality in both beats as measured by the latent variable Z. *PropFair*'s allocation indicates a more even split of policing between beats 131 and 133 than the one observed in the data. Indeed, 133 seems over-policed and does indeed have a higher fraction of the minority population.

For most police beats, District 18 shows less variation in real-world allocation across beats, with a less dynamic response. As we would expect, observed police allocation is very similar across beats (1813, 1814). Beats 1813 and 1814 have roughly equal minority populations. *PropFair* does not differentiate the allocation of police across Beats 1813 and 1814 and keeps the fraction of allocated police officers similar to observed policing. Though the real-world allocation of

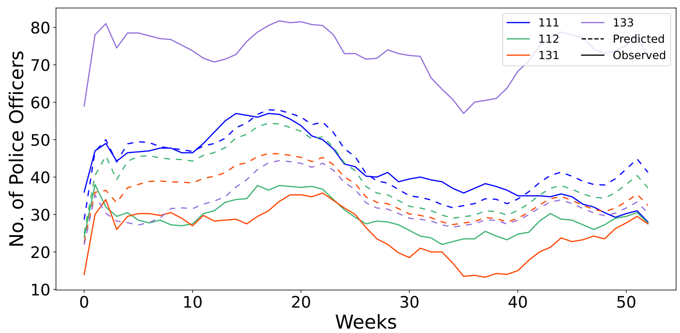


Fig. 7. *PropFair* (dashed) and Observed (solid) allocation of police officers in four beats (111,112,131,133) of district 1. Similar to our results in Figure 6, we can see Beat 133 has more police allocated than the other beats, and *PropFair* suggests a more non-discriminatory allocation that is based on the true-criminality of the beats.

police in Beat 1821 and 1823 differs, *PropFair*'s allocation suggests that similarity in the latent "true criminality" in Beats 1821 and 1823 should lead to similar allocations. We note that these two beats have similar minority population.

Figure 7 shows the true and predicted (*PropFair)* allocations of police officers across four beats of District 1 (the Appendix shows results across all beats in the district), namely 111,112,131 and 133. The neighboring beats are pair wise compared in Figure 6.

6 CONCLUSION

In this paper, we propose a method to dynamically allocate scarce resources. Our method is counterfactually fair with respect to sensitive attributes and avoids pitfalls related to data collection and runaway feedback loops. We apply our model to the real-world problem of optimally allocating policing in Chicago.

One limitation of our work lies in the stylized nature of our model, which cannot capture all the complexities observed in the data, including potential feedback loops of past crimes and/or policing on a neighborhood's latent criminality and demographic composition. Future work could explore whether our framework can be extended to other dynamic resource allocation problems, including the allocation of community-based services to vulnerable populations and public health interventions.

6.1 Research Ethics and Social Impact

Clearly, technologies for predictive policing can have massive social impact, and there are concerns across society about the role of policing itself in propagating inequities and injustice, appropriate domains of jurisdiction for policing, and funding and oversight of police. We do not view our paper as a proposal for a system to automate or replace existing automation of "predictive policing" systems. Instead we view it as both a diagnostic and a formalizer, in the language of Abebe and colleagues [2]; by posing a notion of what could be considered fair, we can analyze existing methods and real-world data, like we do here, and point out potential problems like the overpolicing we observe in some beats. Finally, we also note that there were no human subjects involved in this research and all data used were obtained through open government initiatives and FOIA requests.

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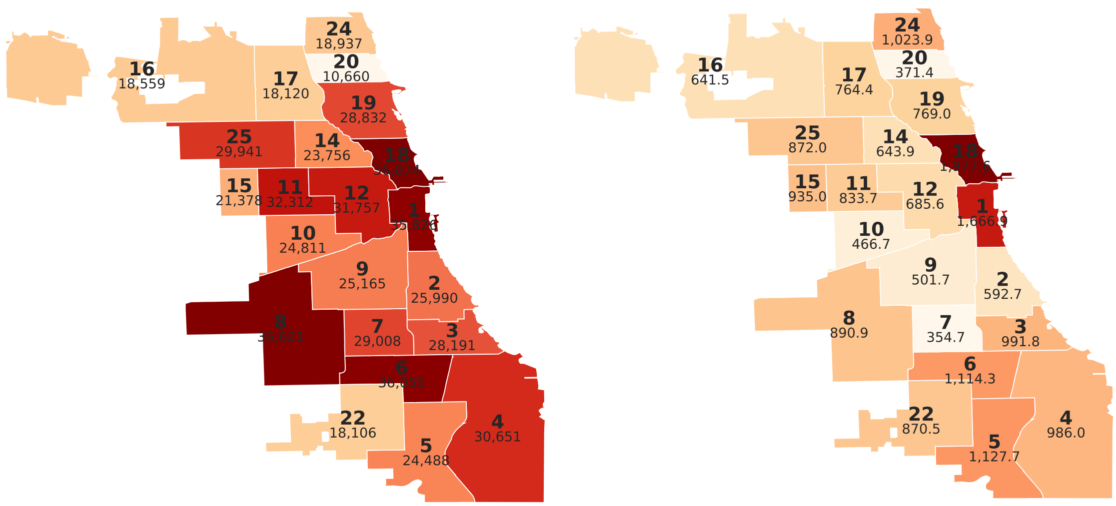
A ADDITIONAL ANALYSIS AND RESULTS

A.1 District Level Analysis—Violent Crimes and Demographic Information

Figure 8 shows the distribution of violent crimes in the City of Chicago. The color scale indicates the magnitude of the values; darker shades denote higher values. Districts 1 and 18 have the maximum values for the total number and variance of violent crimes in the City of Chicago. We presented the policing allocations for pairs of beats in districts 1 and 18. Here, we show the allocations of police officers for police beats in other districts. We select four police Districts (4, 6, 11, 22) based on their differences in population or violent crime distribution, or both, and compare the proposed and true policing allocations in police beats of these districts.

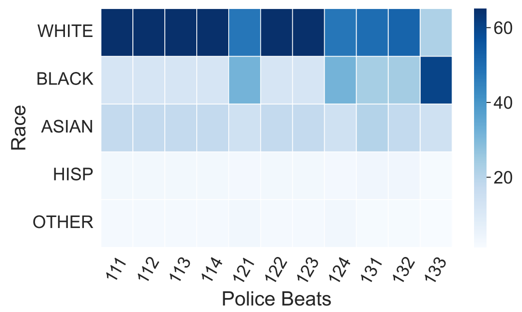


(a) Total number of violent crimes

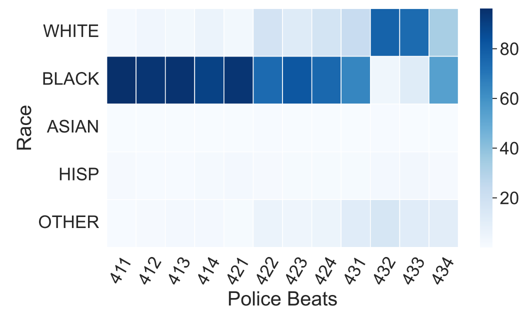
(b) Variance of violent crimes

Fig. 8. Violent crime analysis of the police districts in the City of Chicago. The color scale represents the total number of violent crimes in (a) and the variance of violent crimes in (b). Darker shades denotes higher values.

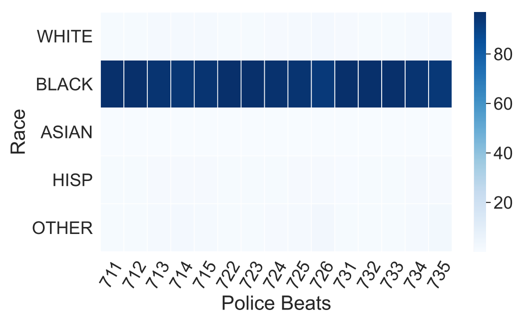
In the next sections, we present the population distributions, policing allocations, and beat-pair comparisons for districts with varying violent crime rates. As in our paper, we treat the concerning crime rate as a proxy of the violent crime rate and define our objective to reduce the concerning crime rate in a police beat, it is interesting to observe how our predicted allocation changes for districts with varying crime rates.



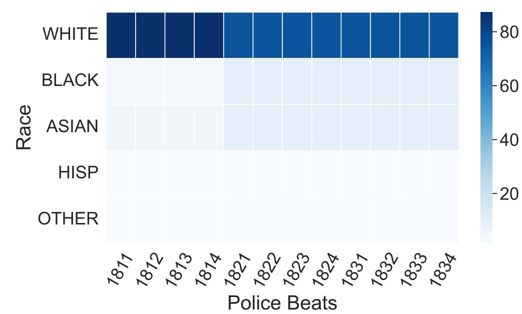
(a) District 1



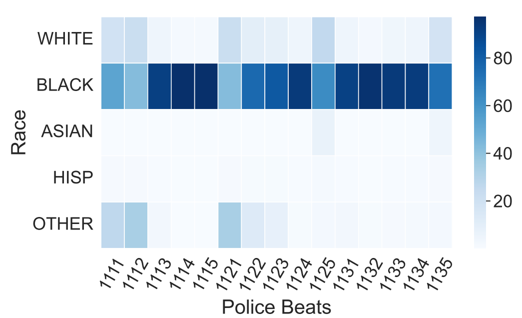
(b) District 4



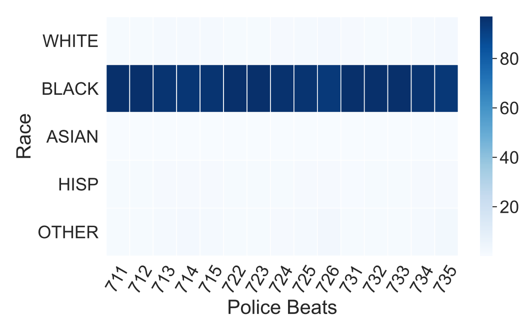
(c) District 7



(d) District 18



(e) District 11



(f) District 22

Fig. 9. Population demographics for police districts. X-axis represents the police beats of the a district while Y-axis represents the races. Darker shades represent higher value.

B DISTRICT LEVEL ANALYSIS—RESULTS

In this section, we demonstrate the differences between the observed and the policing predicted by our model for all police beats in the police districts. We note that the proposed model can be applied to allocate policing for all of the districts and is not limited to the districts presented.

B.1 District 1

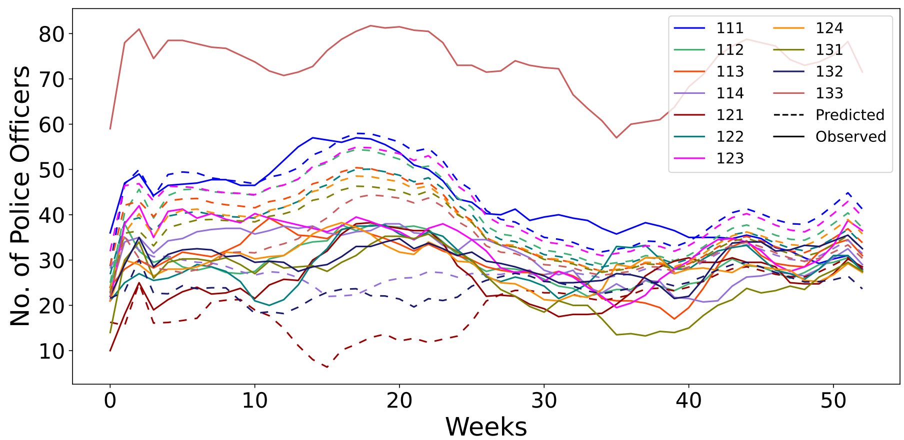


Fig. 10. Observed and predicted (*PropFair*) allocation of police officers for all police beats in District 1. The predicted allocation is more consistent across beats than the true allocation.Over time the change in allocation follows the "true criminality" of that beats.

B.2 District 4

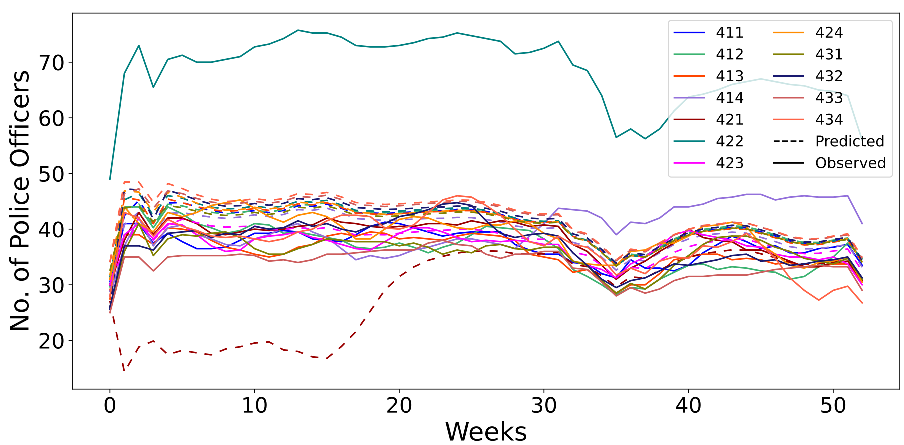


Fig. 11. Predicted (*PropFair*) and observed allocation of police officers in District 4. *PropFair* suggests similar allocation of police officer for all beats except Beat 421, indicating difference in "true criminality" from other beats at least for earlier weeks.

B.3 District 7

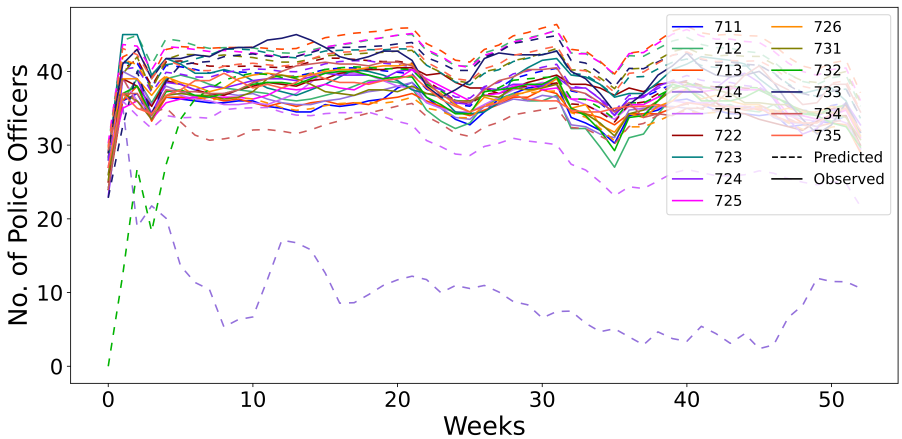


Fig. 12. Observed and predicted allocation of police officers in District 7. *PropFair* suggests that several of the beats (724,725,713) should be allocated less police officers than their observed allocation. All of the beats in District 7 is non-white population majority beats. The observed policing is also similar across the beats, however *PropFair* allocates less officers in beats with lower "true criminality".

B.4 District 11

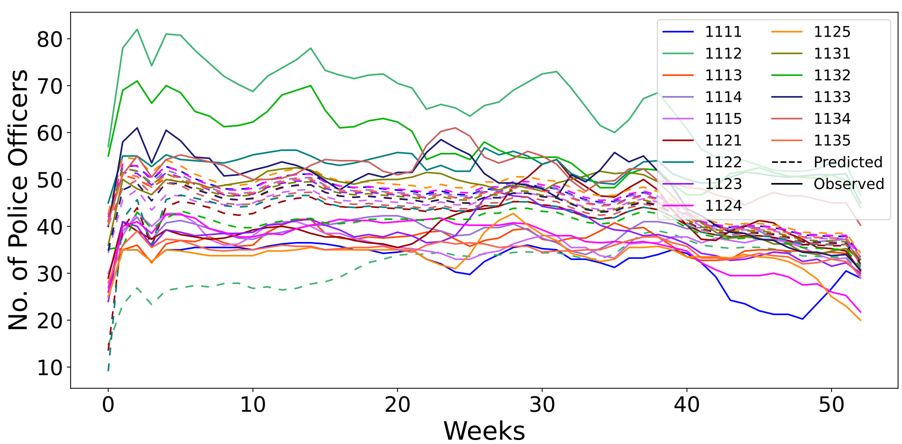


Fig. 13. Observed and predicted allocation of police officers in District 11. District 11 has diverse demographic population in its beats. Some of the non-white majority beats (1132,1112) show higher values of observed policing, *PropFair* suggests that these beats should be allocated less police officers than the observed.

B.5 District 18

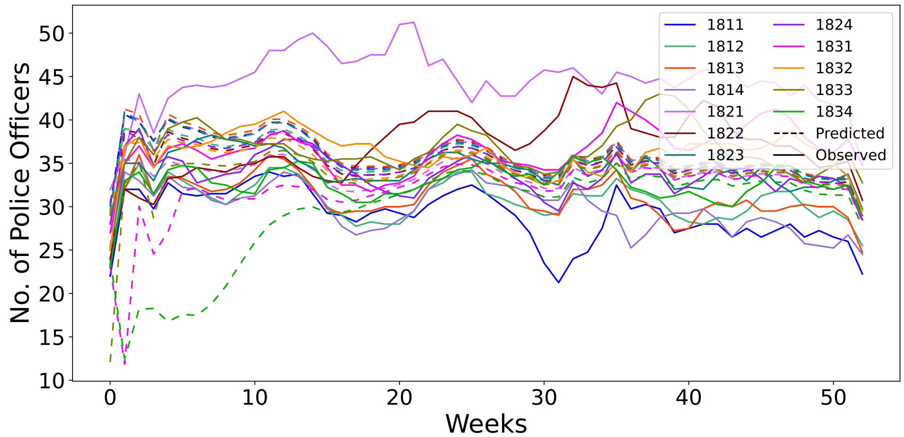


Fig. 14. Observed and predicted allocation of police officers in District 18.

B.6 District 22

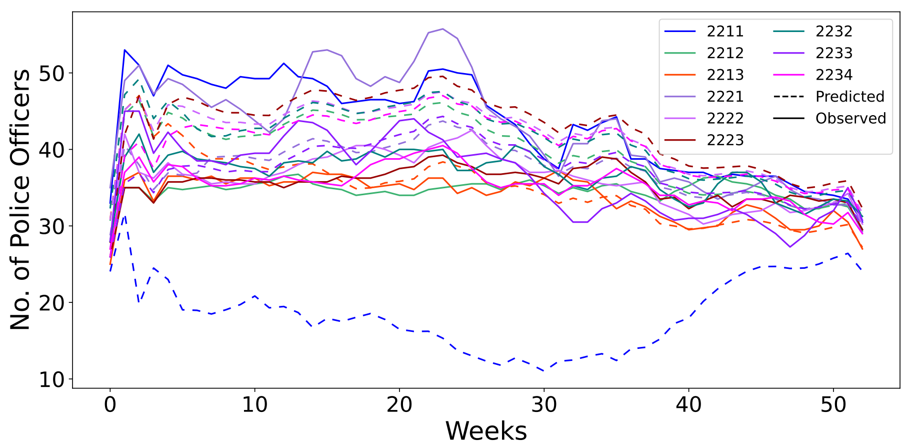
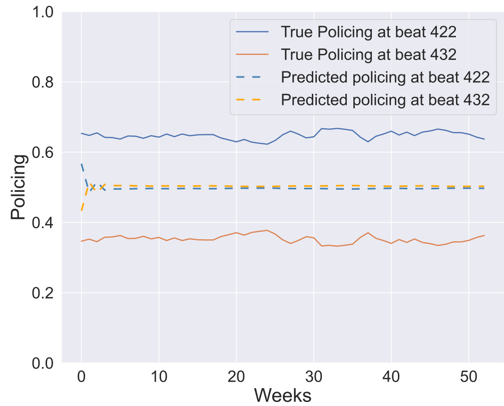


Fig. 15. Observed and predicted allocation of police officers in District 22.

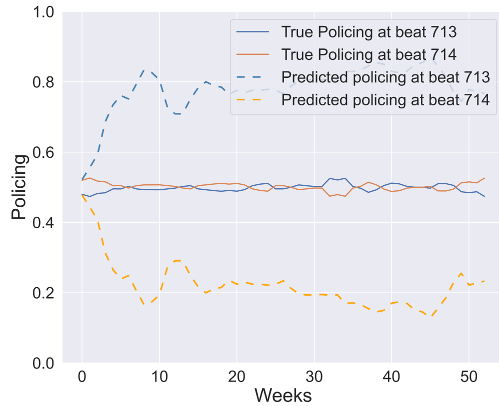
C PAIR-WISE COMPARISON

In this section, we show the pairwise comparison of different police beats in the police districts. From District 4, we select Beat 422, and Beat 432. Beat 422 is a non-white majority beat and Beat 432 is a white majority beat. The true policing in beat 422, is higher than beat 432. Our model suggests an allocation that allocates similar policing to both beats (Figure 16a). From District 7, we selected 713 and 714 Percentage of the minority population in Beat 713 and 714 is 96%. The difference in predicted allocation can imply the difference in "true criminality” Figure 16b.

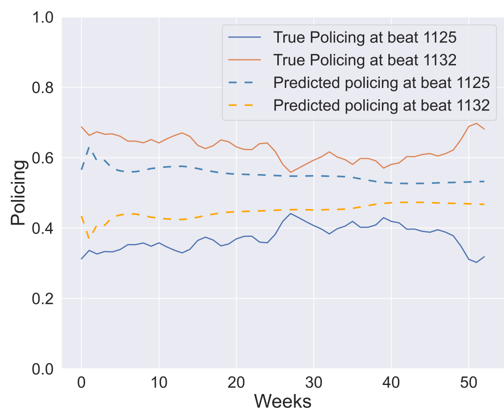
From District 22, we selected beats 2211 and 2222. The percentage of the minority population in Beat 2111 is 10% and Beat 2222 is 90%. There is a significant difference in the fraction of the minority population in beats 2211 and 2222. However, the observed policing is higher for Beat 2222 than for Beat 2211. The predicted policing suggests a more diverse allocation in Beat 2222, and 2211 (Figure 16d).



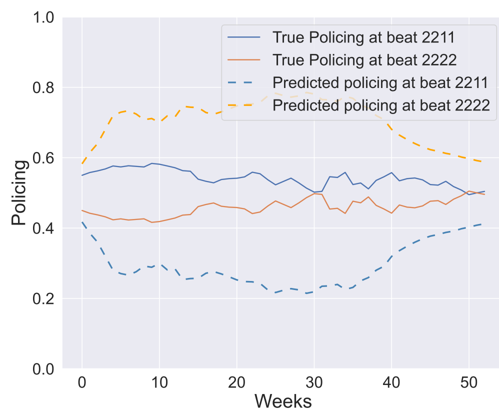
(a) Policing in beat (422,432)



(b) Policing in beat (713,714)



(c) Policing in beat (1125,1132)



(d) Policing in beat (2211,2222)

Fig. 16. Pairwise comparison of predicted and observed policing allocation in neighboring beats of police districts 4,7,11,22.

C.1 Visualization Tool

We developed a visualization tool to explore both the observed and predicted policing assignments in different districts. Figure 17 shows the interface of the tool.

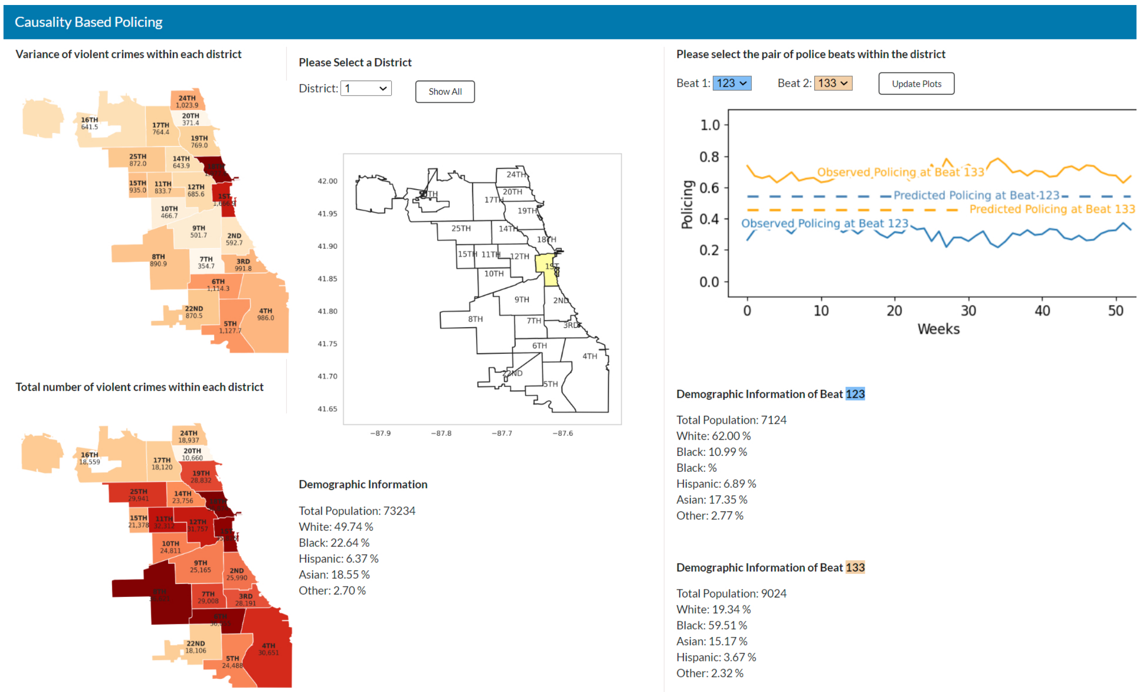


Fig. 17. Interface for exploring police assignment using our method. The left panel presents two heat maps showing population and variance in violent crimes for Chicago. User can choose a district of interest from the dropdown in the central panel and see the selected district in the map. In the right most panel, a user can select a pair of beats from the selected district. Upon selection, the panel shows the observed policing and predicted policing for both beats.