Low-Rank Model for IVA Regression

Algorithm: Regression-assisted IVA-G, N (Number of Sources) = 10, K (Number of Datasets) = 100, T (Samples) = 10,000

Model: Low-Rank Model with parameters (μ, λ, η, Rank)

The rank deficient data has been generated by multivariate gaussian distribution using the covariance matrix (Σ_n) where

$$\Sigma_n = \mu 1 1^T + \lambda \frac{1}{R} Q Q^T + \eta I_k$$

Here $Q \in \mathcal{R}^{K \times R}$, $Q_{ij} \sim \mathcal{N}(0,1)$, QQ^T is the Wishart distribution. There are two versions of the middle term: the above (unnormalized) follows a Wishart distribution while the other one is the normalized version (rows of Q are normalized to unit norm - Trung's Slide)

For normalized version, Q is replaced with $\hat{Q}$ where $\hat{Q}_k = Q_k / \|Q_k\|$.

As $Q_{ij} \sim \mathcal{N}(0,1)$, each entry of $\hat{Q}$ will be Student's t distribution.

The benefit of using a normalized version is that each entry of the second term is bounded with a unit diagonal.

For all of my simulations, I have used the normalized version of the low-rank model.

μ is defined as mean and λ is defined as variance, both mean and variance are related to the correlation value of the covariance matrix as well the correlation value within each SCV for all the datasets.

R is defined as the effective rank which represents the complexity of the model.

η is a parameter to make sure $\mu + \lambda + \eta = 1$ and to make sure that the diagonal terms are 1 for the covariance matrix.

The first term of the equation of the covariance matrix is rank 1, the second term is rank R and the last term is rank K. So, the covariance matrix is $K \times K$ matrix.

The low mean means lower correlation values for entries in the covariance matrix.

The high mean means higher correlation values for entries in the covariance matrix.

The low variance means lower variability between entries in the covariance matrix.

The high variance means high variability between entries in the covariance matrix.

- For $\mu = 0.3 - 0.2$, $\lambda = 0.04$, low mean and low variability are achieved.
- For $\mu = 0.3 - 0.2$, $\lambda = 0.25$, low mean and high variability are achieved.
- For $\mu = 0.7 - 0.6$, $\lambda = 0.04$, high mean and low variability are achieved.
- For $\mu = 0.7 - 0.6$, $\lambda = 0.25$, high mean and high variability are achieved.

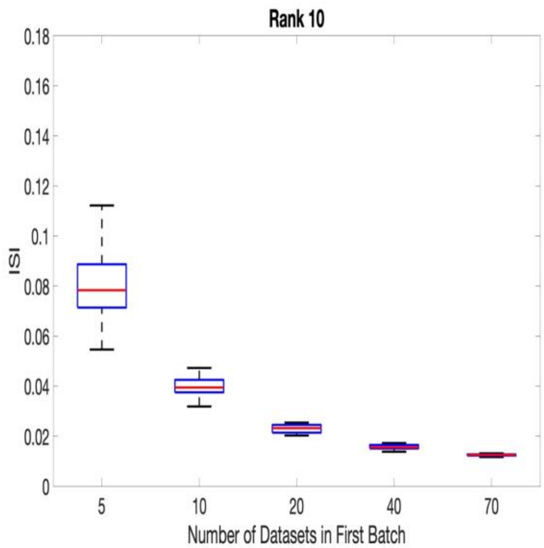
Low Rank (Rank 10):

I have done experiments for Regression-assisted IVA-G with varying mean(low and high) and variance (low and high) within each SCV for all datasets by tuning parameters (μ, λ, η) while keeping rank fixed (low rank). The observations I have found are given below

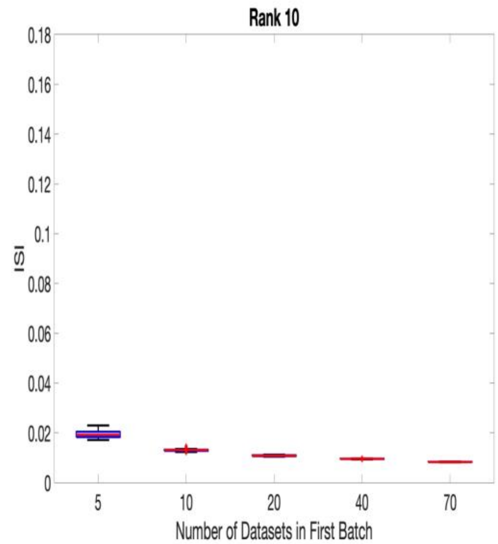
- For low mean and low variance within each SCV, Regression-assisted IVA-G performs worse (larger ISI) when the number of datasets in the first batch is smaller than the effective rank.
- For all the cases, Regression-assisted IVA-G performs better when we put more datasets in the first batch.
- For high mean and high variance within each SCV, Regression-assisted IVA-G performs best (smaller ISI) compared with other cases.
- The performance of Regression-assisted IVA-G depends on both mean and variance.
- Wall time and CPU time keep increasing with the number of datasets in the first batch.
- For low mean and low variance within each SCV, Wall time and CPU time are higher compared with other cases.

- For high mean and high variance within each SCV, Wall time and CPU time are lower compared with other cases.

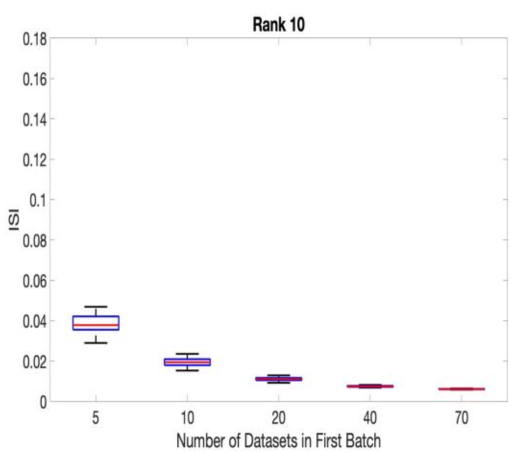
Low Mean and Low Variability



Low Mean and High Variability



High Mean and Low Variability



High Mean and High Variability

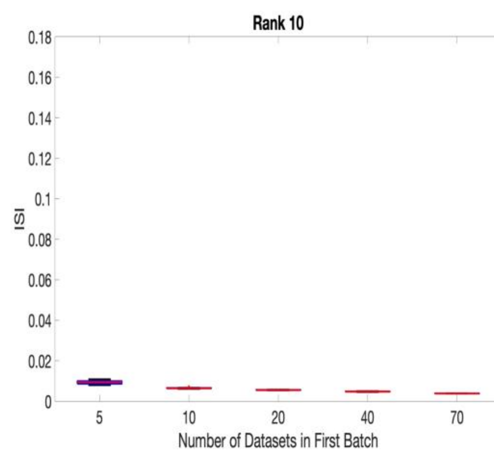
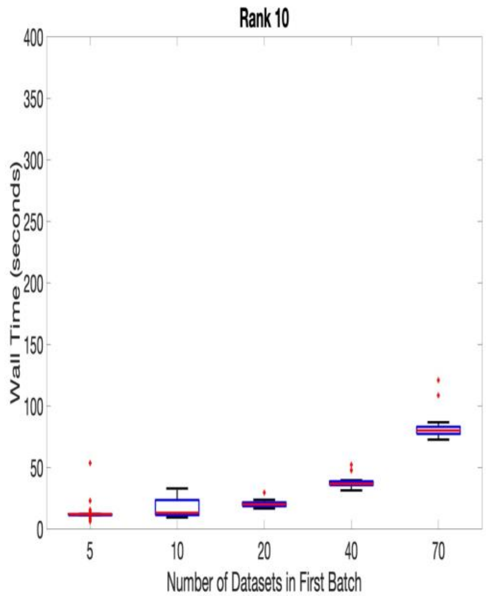
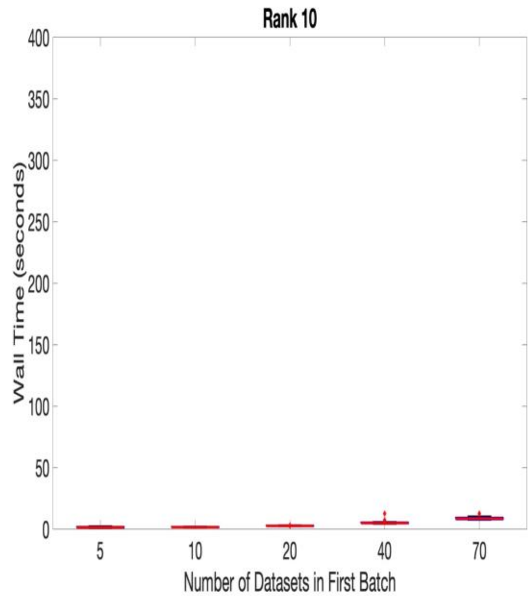


Figure: ISI plots for Regression-assisted IVA-G with rank 10

Low Mean and Low Variability

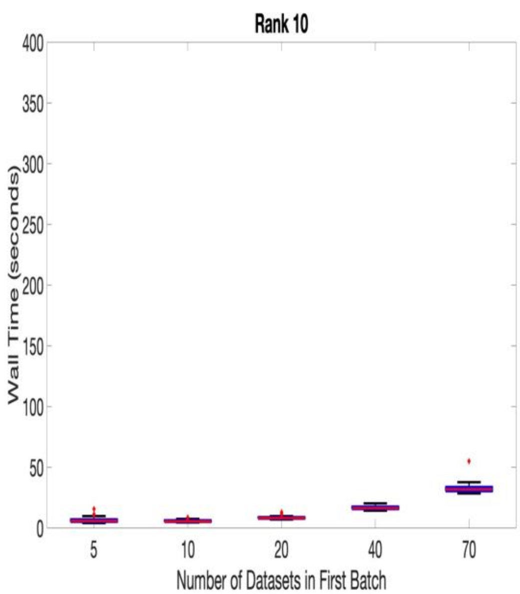
Low Mean and High Variability





High Mean and Low Variability

High Mean and High Variability



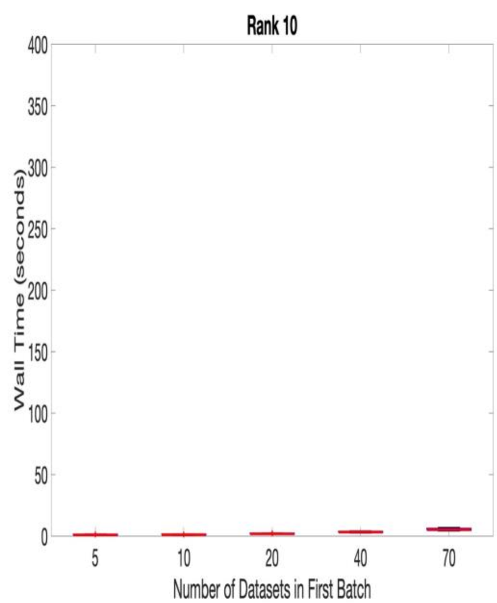


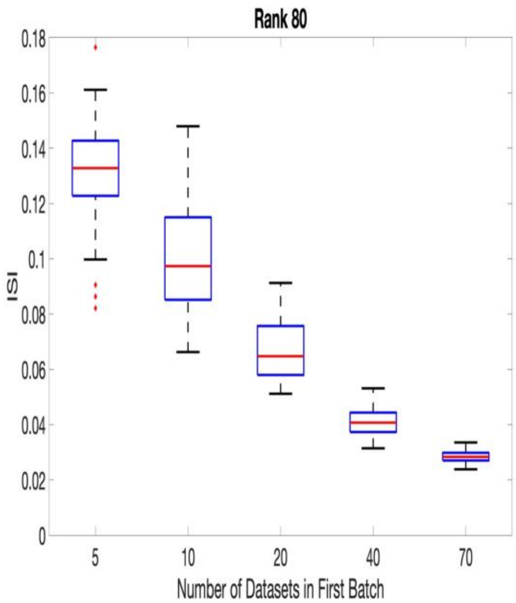
Figure: Wall Time plots for Regression-assisted IVA-G with rank 10

High Rank (Rank 80):

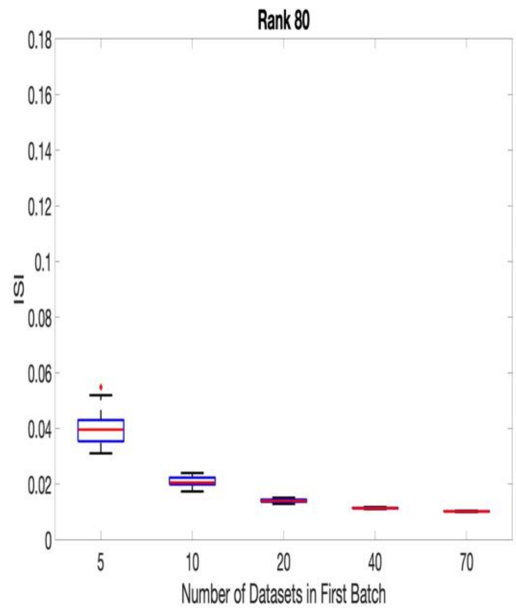
I have done experiments for Regression-assisted IVA-G with varying mean(low and high) and variance (low and high) within each SCV for all datasets by tuning parameters (μ, λ, η) while keeping rank fixed (high rank). The observations I have found are given below

- For low mean and low variance within each SCV, Regression-assisted IVA-G performs worse (larger ISI) when the number of datasets in the first batch is smaller than the effective rank.
- For all the cases, Regression-assisted IVA-G performs better when we put more datasets in the first batch.
- For high mean and high variance within each SCV, Regression-assisted IVA-G performs best (smaller ISI) compared with other cases.
- The performance of Regression-assisted IVA-G gets worse when the rank is increased.
- The performance of Regression-assisted IVA-G depends on both mean, variance, and rank.
- Wall time and CPU time keep increasing with the number of datasets in the first batch.
- For low mean and low variance within each SCV, Wall time and CPU time are higher compared with other cases.
- For high mean and high variance within each SCV, Wall time and CPU time are lower compared with other cases.
- Wall time and CPU time are higher when the rank is increased.

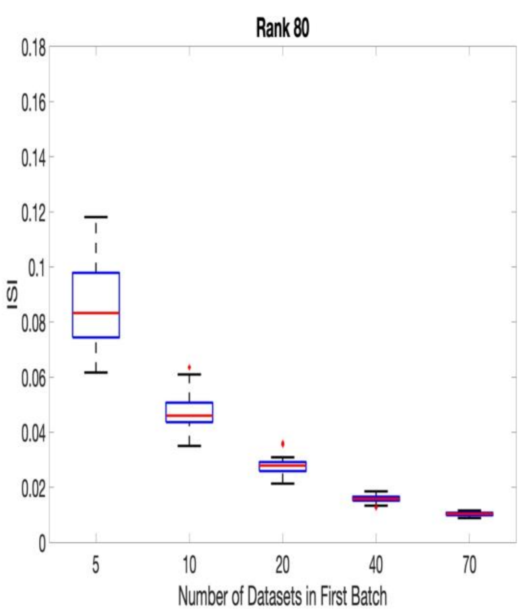
Low Mean and Low Variability



Low Mean and High Variability



High Mean and Low Variability



High Mean and High Variability

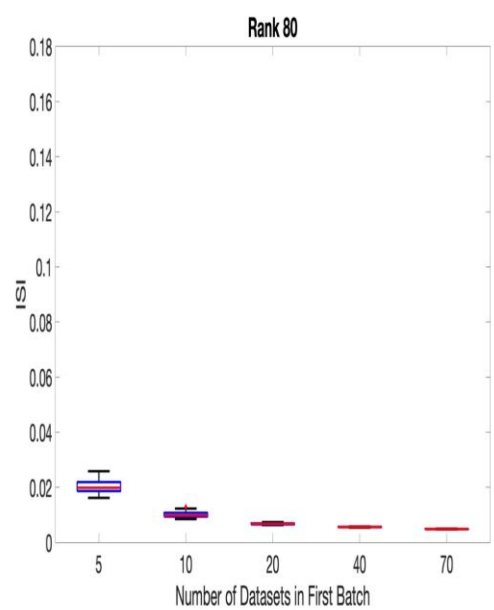
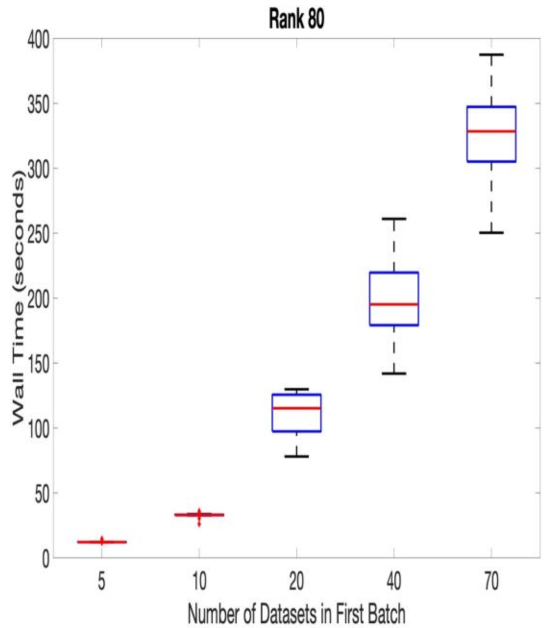
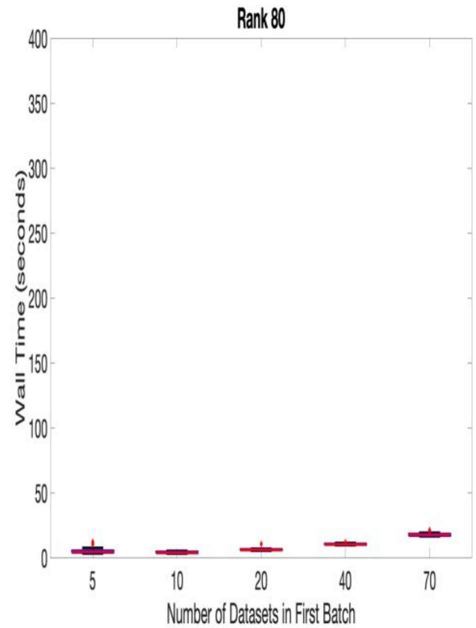


Figure: ISI plots for Regression-assisted IVA-G with rank 80

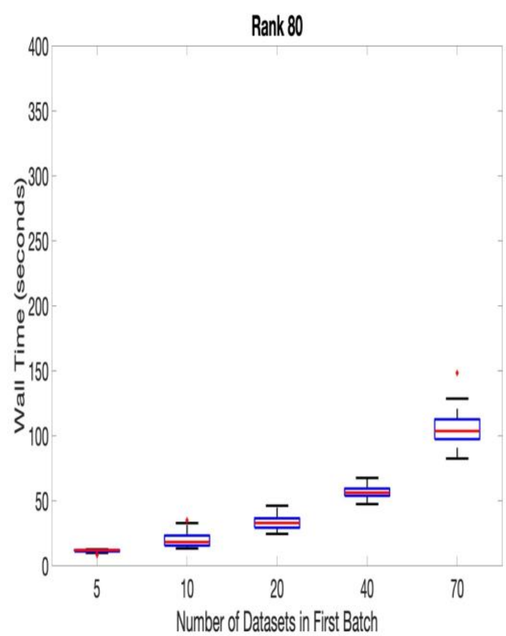
Low Mean and Low Variability



Low Mean and High Variability



High Mean and Low Variability



High Mean and High Variability

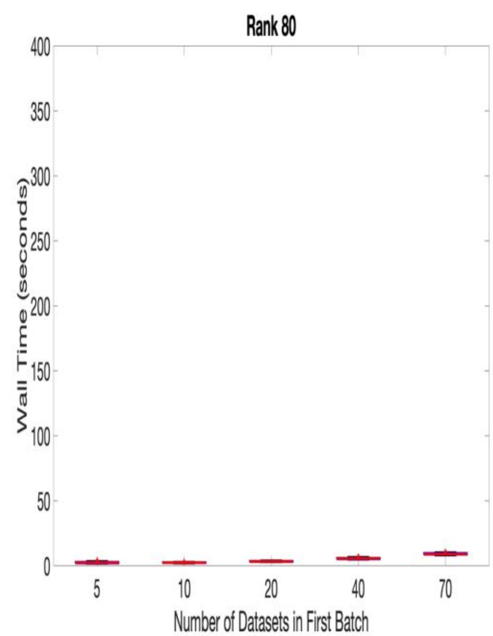


Figure: Wall Time plots for Regression-assisted IVA-G with rank 80

Simulation of IVA-G Only:

I have done experiments for IVA-G only (regression not included) with varying mean(low and high) and variance (low and high) within each SCV for all datasets by tuning parameters (μ, λ, η) while keeping rank 10. Then I did similar experiments keeping rank 80. The observations I have found are given below

- Regression-assisted IVA-G is always performing at very similar levels to IVA-G.
- Regression makes use of the sources estimated by the IVA-G.
- If IVA-G has problems estimating the sources, then regression IVA-G inherits those problems.

Next Step:

- I would like to investigate the importance of mean and variance. For example, I will choose a very high mean and a very low variance and vice versa to see the performance of Regression-assisted IVAG.
- I expect that Regression-assisted IVAG will perform poorly in both cases. These experiments will strengthen the importance of both mean and variance.
- I expect that the variance has a more important impact than the mean on the performance of IVA-G.
- I think the variance contributes more to the variability than the mean and variability is the key factor for the performance of IVA-G.
- It will be interesting to look at the performance when the mean and variance change from 0 to 1.