# Joint Migration Inversion including Q effects: towards Q estimation

## Introduction

Seismic waves traveling through the subsurface undergo various forms of attenuation effects. Geometric spreading is one such effect, which is typically compensated for in some of the migration algorithms. Reflection and transmission effects also cause changes in wavefield amplitudes due to contrasts in the medium. Furthermore, the earth attenuation effect (quality factor) causes wave energy to diminish with distance or propagation time gradually. While the exact mathematical description of this effect is unknown (Aki & Richards, 2002), it is known to be frequency-dependent and have exponential energy loss behaviour. Attenuation is also associated with dispersion, leading to gradual changes in signal shape and strength. As a result, the peak energy shifts towards lower frequencies, which can cause the signal to become wider and more distorted over time.

It is crucial to properly account for all attenuation effects from the overburden to gain high-resolution information on the sub-surface. The Full Wavefield Migration (FWM) (Berkhout, 2014b; Davydenko and Verschuur, 2017) and Joint Migration Inversion (JMI) methods (Staal and Verschuur, 2013; Berkhout, 2014c; Verschuur et al., 2016) use the Full Wavefield Modeling (FWMod, Berkhout, 2014a) scheme, which includes geometric spreading, reflectivity, and transmission effects and all multiple scattering by recursively and iteratively propagate wavefields up/down in the subsurface. This modeling engine is optimized for an inversion algorithm that uses the subsurface's propagation velocity and reflectivity models based on measured seismic data. The inversion algorithm uses a gradient descent scheme, where the difference between measured and modeled data is projected onto updates for the reflectivity and propagation velocity models (Staal, 2015). Because of the modular set-up of the FWMod process, where propagation and reflection/transmission are described by wavefield operators, bringing in the Q-effect is relatively simple, by redefining the propagator. Recently, there was a foirst attempt by Alasmri and Verschuur (2019), where they incorporated the impact of the Q effect into both FWM and JMI. They assumed a constant Q-value for the seismic data, integrated the attenuation effects into the full wavefield forward modeling, and compensated for this effect during inversion.

The aim of this study is to incorporate the quality (Q) factor impact into both FWMod and Joint Migration Inversion (JMI) by assuming a function for Q based on the velocity. In addition, we assume that each layer in the model has a distinct Q value and then, account for it during the JMI process. We assume that a function exists that describes the connection between velocity and quality factor. For this preliminary study we assume a deterministic function with few free parameters. However, we envision a process where we rely on machine learning (ML) techniques to obtain the Q function based on velocity, allowing us to apply this method without prior knowledge of the Q function.

## Theory of JMI including Q effects

During the process of estimating propagation operators, the forward propagation of the waveform experiences attenuation, leading to changes in both the shape and amplitude of the wavelet. Not compensating for attenuation will cause the reflector to be affected and result in an inaccurate reflectivity estimation. Including these effects in one-way propagators within the full wavefield modeling engine is easy. To achieve this we follow what Alsmari and Verschuur (2019) did in their work, so consider the attenuation factor to be incorporated into a complex velocity, as shown below:

$$v_c = \frac{\omega}{k} = \frac{\omega}{k_r + i\alpha}. \tag{1}$$

This equation involves the variables $v_c$ which represents complex velocity, ω which represents angular frequency, and k which represents wavenumber. $\alpha$ is defined via the following equation:

$$\alpha = \frac{\pi f}{Qv} = \frac{\omega}{2Qv}, \tag{2}$$

where $Q$and $v$ are the quality factor and the original velocity, respectively. Equation 1 then becomes:

$$v_c = v\frac{\left(1-\frac{i}{2Q}\right)}{\left(1+\frac{1}{(2Q)^2}\right)} = v\left(1+\frac{i}{2Q}\right)^{-1}. \tag{3}$$

Equation 3 combines velocity and quality factor to model wave propagation that considers anelastic attenuation effects (Toksoz and Johnston, 1981; Sheriff and Geldart, 1995; Aki and Richards, 2002).

The JMI process is a data-fitting process, where the objective function is defined as the energy in the data-residual, i.e. the difference of the measured and the modeled seismic records. The inversion parameters are the (migration) propagation velocity model and the seismic reflectivity model. With these two parameters the so-called FWMod scheme (Berkhout, 2014a) can be carried out to regenerate the seismic data from the imaging paramaters. The FWMod employs one-way propagators, usually based on the phase shift operators, e.g. via the phase-shift plus interpolation (PSPI) methodology (Gazdag and Sguazzero, 1984). The involved phase shift operators in the wavenumber domain for 2D

$$W(k_x,\omega) = e^{-jk_z\Delta z};\ k_z = \sqrt{\frac{\omega^2}{v^2} - k_x^2}\ , \tag{4}$$

With $\Delta z$ the extrapolation distance, can be easily extended to include the complex velocity as given by equation 3. This provides the right frequency dependent amplitude and phase effect.

During the inversion process of JMI, in each iteration, the residual will be back-propagated to each depth level to be combined with the forward modeled source field (including all multiple scattering) to achieve either an update for the reflectivity or the propagation velocity (depending on the type of imaging condition that is used after back-propagation). The forward modeling of the source will then include the correct Q-effect. Also, for the determination of the step length – to be applied on the velocity or reflectivity gradient – the linearized effect of the gradient is again forward modeled to be compared with the residual data, in which the Q-effect can also be properly included. Thus, JMI will iteratively adapt model parameters such that forward modeled data, including the Q-effect, fit the measured data.

**Numerical examples**

We incorporated the estimation and compensation of anelastic attenuation effects to enhance JMI's ability to provide higher seismic resolution in complex areas. We achieved that by incorporating prior knowledge of Q in FWMod and during inversion. Even though it's not completely realistic to have a quality factor function based on the velocity, we can still assume a function exists that describes the connection between velocity and quality factor. This assumption may have some merit because there could be a link between velocity, density, and quality factor in the real world. One way to estimate such Q function can be through ML methods. In our new JMI method, the Q value of each grid is updated in every iteration by the assumed Q-velocity relation, which uses the estimated velocity in that iteration as the input. In this example, a 2D lens-shaped model with a low-velocity middle region exhibiting a strong quality factor effect (Q=25), one reflector above with Q=40, the second layers from the top have the Q value of 35, and other reflectors below with higher Q values were used. The frequency ranges from 5 to 40 Hz. The maximum data offset used for inversion is 3000 m, 1500 m in depth, and sources and receivers are equally distributed along the surface. Synthetic data up to the fourth order of multiples are included. Figure 1 and 2 show the result of the standard JMI and our new JMI method, respectively. It is obvious that the accuracy of the estimated velocity and reflectivity is improved in our new method compared to standard JMI. We can see clearly once we do not compensate for $Q$, the attenuation effects imprint on the reflectors and lower the resolution of the image. As a consequence, the reflectors below the lens shape in figure 1 are barely reconstructed with the presence of attenuation in standard JMI.

In a first step towards estimating Q, we examine JMI's sensitivity to changes in the assumed Q-velocity relationship during inversion (while the data remains the same). We, therefore, tested various Q functions, as depicted in figure 3(a). The results for one vertical cross-section of the inverted images are presented in figure 3(b-e). Figure 3(f) shows the maximum obtained reflectivity below the anomaly, showing the best results for the correct Q function. Moving away from this optimal Q function will cause a reduction in the maximum estimated reflectivity. This could be used as metric in  estimation.

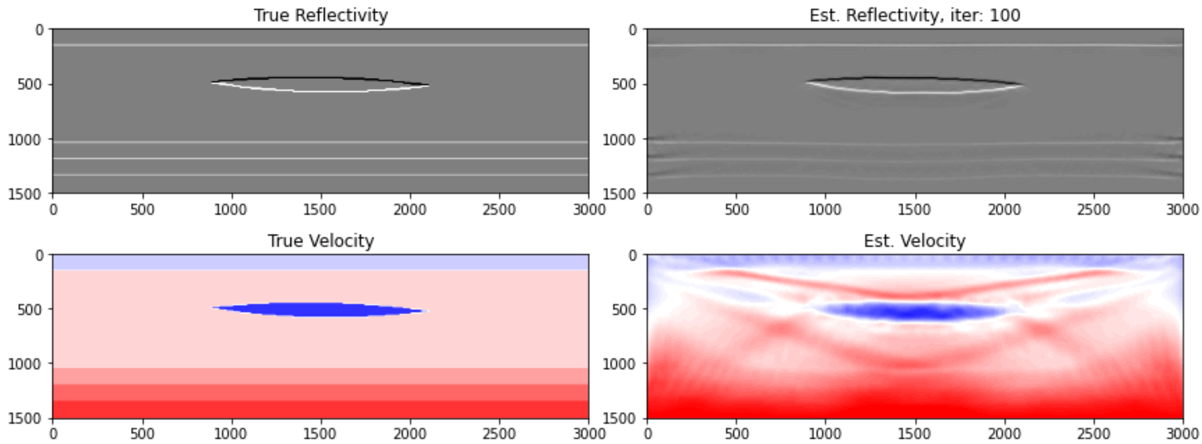


***Figure 1*** *left: True reflectivity and true velocity of the model. right: Estimated reflectivity and velocity model with the standard JMI (without Q compensation), while the data is modeled with Q-effects.*

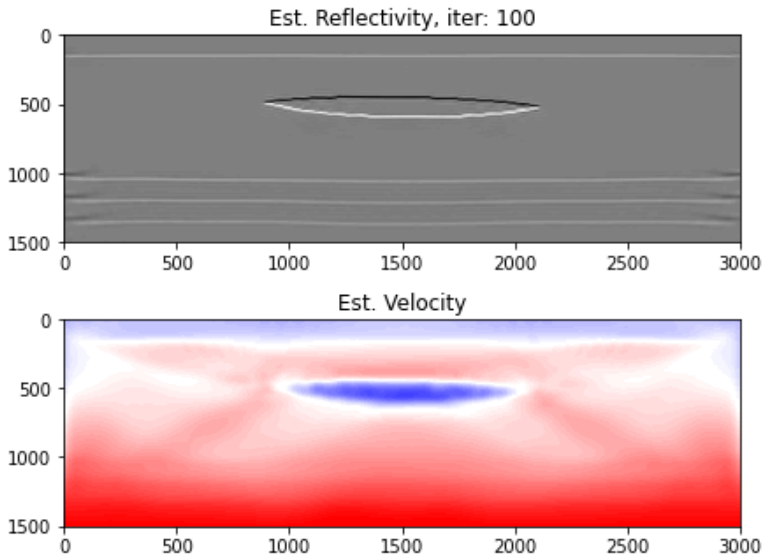


***Figure 2*** *Estimated reflectivity and velocity model with the Q-JMI,*

**Conclusions**

Incorporating the attenuation in seismic imaging and velocity estimation is an important component to optimally address real seismic data. For the JMI process this can be easily done in the forward modeling scheme, as this is built from separate propagation and scattering operators. The propagation operator, based on laterally varying phase shift operators, can easily be modified to include the Q-effect. With numerical examples it was first shown that JMI is able to retrieve a proper image and velocity model, incorporating the correct Q-model. In addition, when varying the Q-model it was demonstrated that the correct Q-model provided the best (strongest) target reflectivity, while deviations in Q (either too low or too high) yield poorer reflectivity estimates. This opens the door for automatic Q-estimation during the JMI process.


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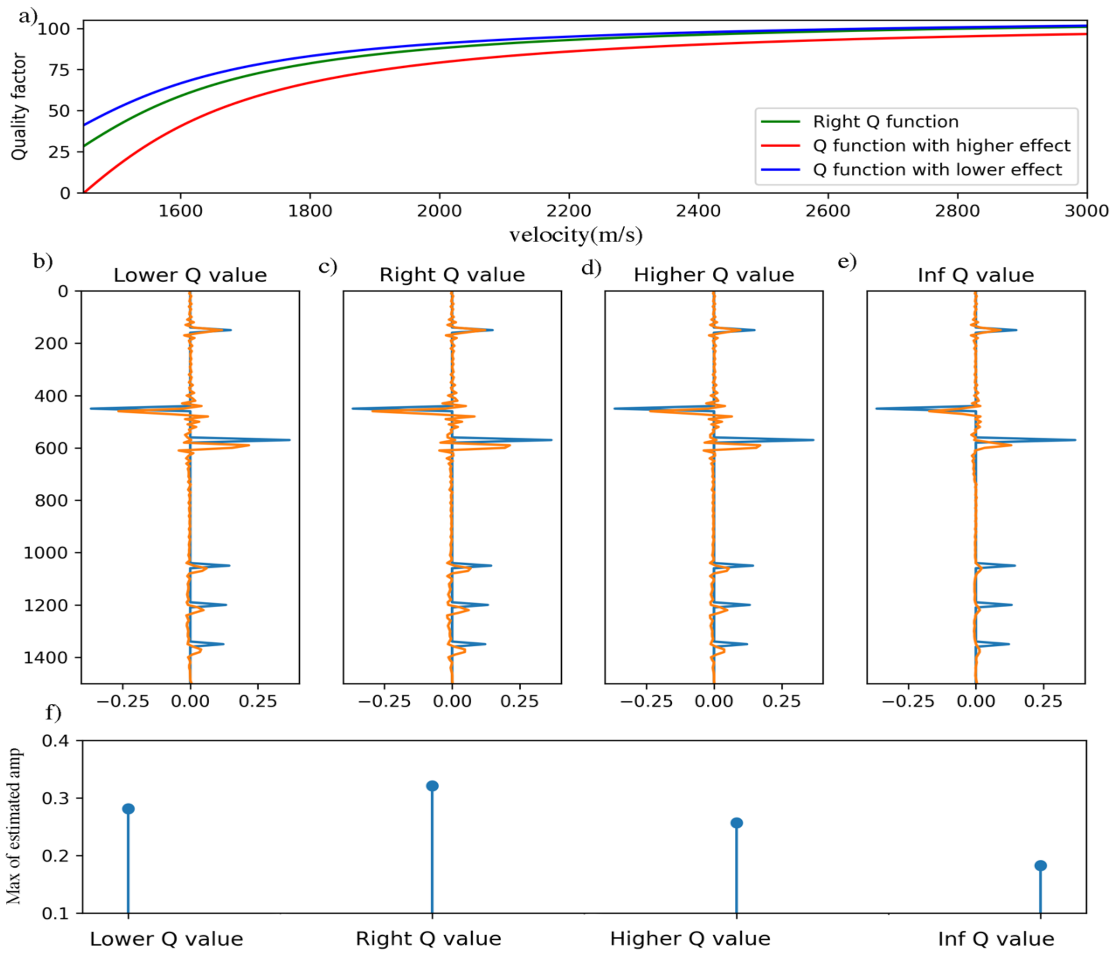


***Figure 3*** *a) Q functions base on velocity. Green curve: right Q model, red/blue curve: Q model with higher/lower effect than the desired Q model. b-e) estimated reflectivity based on different Q functions. f) Estimated maximum amplitude for the recovered target reflectivity values.*

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