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Plug and Play Post-Stack Seismic Inversion with CNN-Based Denoisers

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Summary

Seismic inversion is the prime method to estimate subsurface properties from seismic data. However, such inversion is a notoriously ill-posed inverse problem due to the band-limited and noisy nature of the data. Consequently, the data misfit term must be augmented with appropriate regularization that incorporates prior information about the sought-after solution. Conventionally, model-based regularization terms are problem-dependent and hand-crafted; this can limit the modeling capability of the inverse problem. Recently, a new framework has emerged under the name of Plug-and-Play (PnP) regularization, which suggests reinterpreting the effect of the regularizer as a denoising problem. Convolutional neural networks-based denoisers are state-of-the-art methods for image denoising: their adoption in the PnP framework has led to algorithms with improved capabilities over classical regularization in computer vision and medical imaging applications. In this work, we present a comparison between standard model-based and data-driven regularization techniques in post-stack seismic inversion and give some insights into the optimization and denoiser-related parameters tuning. The results on synthetic seismic data indicate that PnP regularization using a bias-free CNN-based denoiser with an additional noise map as input can outperform standard model-based methods.

Introduction

Post-stack seismic inversion is a notoriously ill-posed inverse problem due to the band-limited and noisy nature of seismic data. Therefore, a meaningful solution is obtained by augmenting the data-misfit term with an appropriate regularization that incorporates prior information about the underlying model. In practice, this requires solving a variational problem of the form

$$\arg\min_x \frac{1}{2}\|Gx-d\|_2^2+\mathscr{R}(x), \tag{1}$$

where $G = w \star \partial/\partial t$ is the modelling operator, w is the seismic wavelet, $\star$ denotes the convolution operator, x is the unknown geological model, d is the post-stack seismic data, and $\mathscr{R}$ is a user-defined regularization. One of the most commonly used regularization terms in seismic inversion is the non-smooth Total-Variation (TV) regularization due to its ability to capture the layered structure of the subsurface. An algorithm that naturally deals with non-smooth regularization terms is the Primal-Dual (PD) algorithm (Chambolle and Pock, 2011). This algorithm typically consist of a gradient-based minimization step, followed by a denoising step due to the presence of a non-smooth regularization. Recently, a new framework has emerged under the name of Plug-and-Play (PnP) regularization (Venkatakrishnan et al., 2013), where a generic denoiser of choice can be used to perform such denoising step. This approach is appealing in that it could allow the use of Convolutional Neural Networks (CNN), which have recently shown great denoising capabilities.

In this work, we conduct a systematic comparison between classical, model-based regularization and PnP for post-stack seismic inversion on the 2D Marmousi model. We use two CNN-based denoisers in the PnP algorithm: a pre-trained DnCNN (Zhang et al., 2017) and a pre-trained DRUNet (Zhang et al., 2021). We compare the convergence of the deep denoisers and evaluate their effect through iterations.

Theory

The Primal-Dual algorithm: Equation 1 can be seen as a special case of the optimization problem

$$\min_x f(Kx)+g(x), \tag{2}$$

where g is smooth and convex and f is non-smooth but convex, with

$$f(Kx)=\mathscr{R}(x), \quad g(x)=\frac{1}{2}\|Gx-d\|_2^2.$$

Because of the non-smoothness of f, this composite optimization problem can not be solved using gradient-based methods. Variable splitting algorithms can instead naturally handle the non-smoothness of f. A prominent example of such a kind is the Primal-Dual algorithm (Chambolle and Pock, 2011), a solver with proven $O(1/N)$ convergence in finite dimensions that converges to the saddle point through the alternate minimization of the primal (x) and dual (y) variables:

$$\begin{aligned} y_{k+1} &= \operatorname{prox}_{\sigma f^\star}(y_k+\sigma K\hat{x}_k) \\ x_{k+1} &= \operatorname{prox}_{\tau g}(x_k-\tau K^H y_{k+1}) \\ \hat{x}_{k+1} &= x_{k+1}+\theta(x_{k+1}-x_k), \end{aligned} \tag{3}$$

where $\theta \in [0,1]$, σ and τ represent the step lengths for the two subgradients and convergence is guaranteed when $\sigma\tau L^2 < 1$, where $L^2 = \|K\|_2^2$. The proximal operators ('prox' in Equation 3) often have closed-form solutions which make the PD algorithm a very efficient method to solve non-smooth optimization problems. The proximal operator of a generic function $f(x)$ is given by

$$\operatorname{prox}_{\sigma f}(x)=\arg\min_y\left(\frac{1}{2\sigma}\|x-y\|_2^2+f(y)\right), \tag{4}$$

and the proximal operator of its convex conjugate $f^*(x)$ is obtained via the Moreau identity:

$$\operatorname{prox}_{\sigma f^\star}(x)=x-\sigma\operatorname{prox}_{f/\sigma}(x/\sigma). \tag{5}$$

From a statistical point of view, Equation 4 can be understood as a non-blind Gaussian denoiser in a maximum a-posteriori probability (MAP) sense, where σ corresponds to the variance of Gaussian noise, y plays the role of the unknown clean vector and x is the known noisy vector (i.e., data term). In that sense, we can use any general denoiser of choice to replace the proximal operator of $f(x)$ in the Moreau identity (Equation 5) yielding a modified version of the PD algorithm of Equation 3:

$$\begin{aligned} y_{k+1} &= y_k + \sigma \hat{x}_k - \text{Denoiser}(y_k + \sigma \hat{x}_k, \sigma) \\ x_{k+1} &= \text{prox}_{\tau g}(x_k + \tau \hat{y}_{k+1}) \\ \hat{x}_{k+1} &= x_{k+1} + \theta(x_{k+1} - x_k) \end{aligned} \tag{6}$$

where $K = I$ $(L = 1)$. The idea of replacing the dual update with a denoiser in variable-splitting algorithms is referred to as the Plug-and-Play framework (PnP) (Venkatakrishnan et al., 2013) and Equation 6 corresponds to its PD implementation (Meinhardt et al., 2017). Note that the proximal operator depends on the parameter σ which can be considered as the noise level. This suggests that the denoiser replacing the proximal operator should also depend on the noise level. Denoisers that use the noise level are called *non-blind denoisers* and denoisers that do not take the noise level as an input are called *blind denoisers*. As blind denoisers do not adapt to a different value of σ, non-blind denoisers are therefore preferred in the Plug-and-Play framework.

CNN-based denoisers: Interpreting the y-update of the PD algorithm as a denoising step offers the possibility to choose among a wide variety of denoisers. In this work, we focus on CNN-based denoisers: more specifically we compare a Denoising Convolutional Neural Network (DnCNN) pre-trained on natural images with a noise level of $\sigma = 0.1$ (Zhang et al., 2017) and a DRUNet pre-trained on a larger set of natural images with noise levels in the range $(0, 2)$ (Zhang et al., 2021). DnCNNs uses the residual learning approach (Figure 1), meaning that for a given clean and noisy image pair $y = x + \varepsilon$, the network is trained to reproduce the noise ε instead of the underlying signal x:

$$\mathscr{L}(\theta) = \frac{1}{2N} \sum_{i=1}^{N} \|\text{DnCNN}(y_i; \theta) - (y_i - x_i)\|_F^2 \tag{7}$$

DnCNN is known to be an efficient alternative for image denoising problems and has shown to outperform statistical-based denoising techniques (Zhang et al., 2017). However, the main drawback of blind denoisers like DnCNN is that their performance quickly degrades when testing on images with higher noise levels than the one used in the training stage. This generalization issue has been recently addressed by removing the bias term from all convolutional and batch normalization layers (Mohan et al., 2019).

As shown in Equation 4, the denoiser used within PnP is by definition a non-blind Gaussian denoiser and therefore a noise level σ must be specified as input of the network. In that sense, although blind-Gaussian denoisers like bias-free DnCNN can be used in PnP, they tend to be suboptimal as they may denoise more or less than the level required by the solver, in our case PD. To create a non-blind denoiser as required in Equation 4, Zhang et al. (2021) propose the use of the Fast and Flexible Denoising convolutional neural Network (FFDNet), which adds an extra input channel representing the noise level map σ of the input sample y. This extra input noise map allows a single network to operate under different noise levels and controls the trade-off between noise reduction and detail preservation. In addition to FFDNet, Zhang et al. (2021) leverages the effectiveness of UNet for image-to-image translation and the bigger modeling capacity of ResNet, which are all combined to form the so-called DRUNet (Figure 1). The DRUNet architecture, contrary to DnCNN, aims to predict the clean image by optimizing the loss function:

$$\mathscr{L}(\theta) = \sum_{i=1}^{N} \|\text{DRUNet}(y_i, \sigma_i; \theta) - x_i\|_1 \tag{8}$$

Finally, when using these deep denoisers in the PnP framework, the input image is scaled between 0 and 1, and the output values of the denoising step are returned to their original input range.

Results

The proposed PnP method is here compared to L_2 and TV regularized seismic inversion applied on synthetic seismic image (Figures 2a) computed using the post-stack modelling operator (G) on the Marmousi model (Figure 2b). The seismic image is generated using a Ricker wavelet with a peak frequency of 15 Hz. Moreover, to mimic noise commonly observed in seismic field data, band-passed noise is created by filtering Gaussian noise along both the vertical and horizontal axes. We assess the quality of the reconstruction through the signal-to-noise ratio (SNR). Figure 2c shows the result for an L_2-spatially regularization inversion. As expected, this inversion is unable to retrieve the high-frequency components of the subsurface, yielding a low-resolution image with a SNR of 48.2 dB. TV-regularization, on

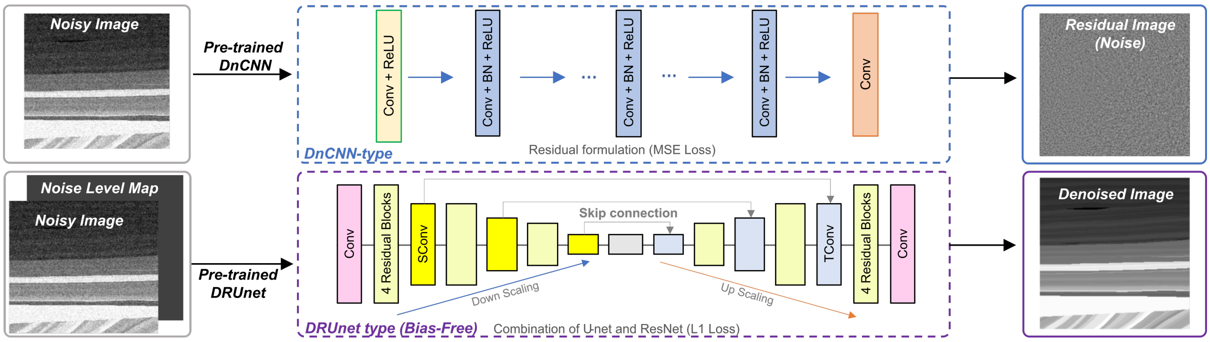

Figure 1 *Pipeline and architecture of the networks used as denoising step.*

the other hand, is known to retrieve high-resolution model estimates (e.g., Ravasi and Birnie (2021)), as confirmed by our result in Figure 2d and an overall SNR of 53.2 dB. The result of PnP-DnCNN is shown in Figure 2e. With a SNR of 50 dB, PnP-DnCNN outperforms L_2-regularization but is worse than TV-regularization. Visually, the effect of the DnCNN is similar to that of L_2-regularization, where the high-frequency details are not properly retrieved. Figure 2f shows the result of PnP-DRUNet, which exhibits a considerable improvement over the other methods with a SNR value of 57.2 dB. PnP-DRUNet produces a higher resolution model than TV-PD, with the main differences arising in the shallower layers and the high-impedance wedge. In the TV-PD result, the shallow layers look slightly noisy and the amplitude of the wedge is not well recovered.

As a way to further assess the performance of the different methods, Figure 3 shows the convergence of the x and y updates of the TV-regularized and PnP with deep denoisers. PnP-DRUNet is the method with the highest convergence rate and the final lowest error values. Although PnP-DnCNN has a similar convergence to PnP-DRUNet in the first few iterations, it stops at a certain error value higher than PnP-DnCNN. Finally, TV has the lowest convergence rate in the primal update but after 30 iterations it gets lower error values than PnP-DnCNN. Lastly, Figure 4 shows the progression of x_k and y_k and the noise that is removed by the denoiser in PnP-DRUNet. One can see that both the structure and the amplitude of the removed noise change over the iterations. In the first iteration, the noise looks rather structured and quite strong, whilst the noise becomes much more incoherent and weaker as the iterations progress.

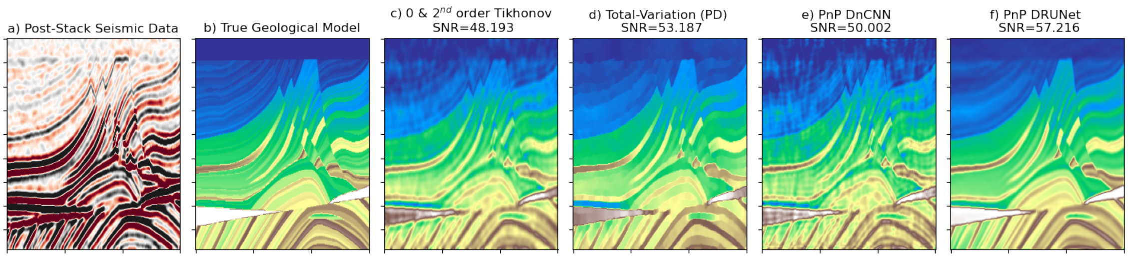

Figure 2 *a) Noisy seismic data, b) True geological model, c) O and 2^{nd} Tikhonov regularized inversion, d) TV-Regularized inversion, e) PnP with a pre-trained DnCNN, f) PnP with pre-trained DRUNet.*

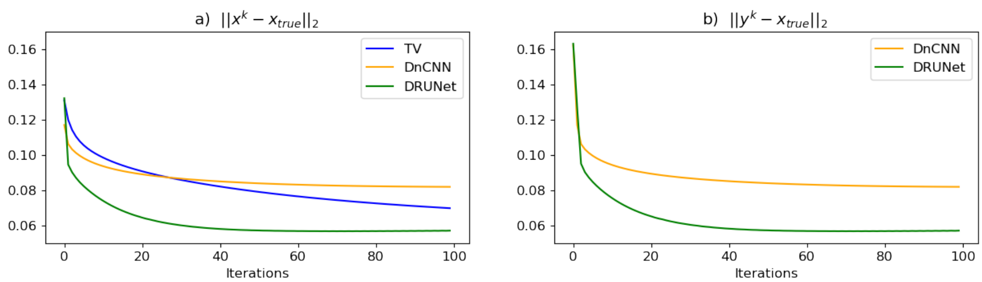

Figure 3 *x(left) and y(right) error convergence curves for TV and deep denoiser regularizations.*

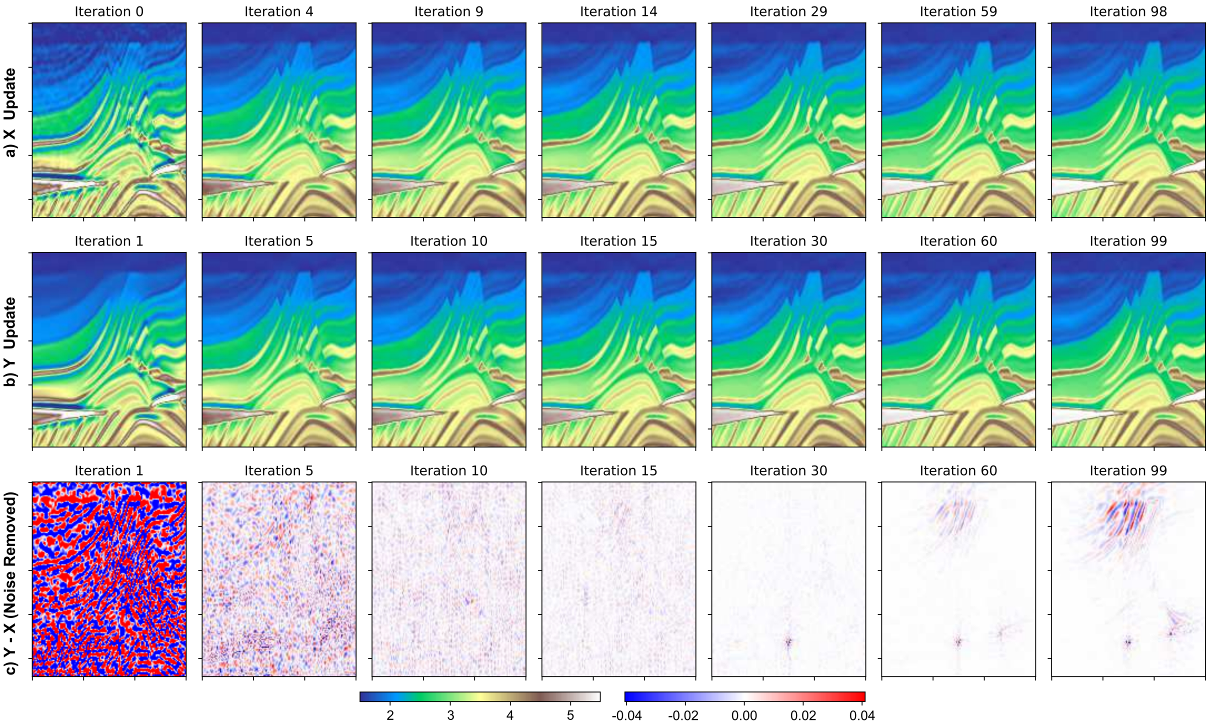

Figure 4 *Inversion and denoising intermediate steps in PnP-DRUNet. a) x estimate retrieved after the inversion step. b) y estimate obtained after removing the noise. c) Noise removed.*

Conclusions

We have presented the PnP framework with CNN-based denoisers as a novel strategy for post-stack seismic inversion. Our numerical results suggest that the PD algorithm coupled with the DRUNet denoiser can significantly outperform classical L2- or TV-regularized seismic inversion. The superiority of DRUNet over other CNN denoisers can be attributed to two of its design choices: 1. a bias-free neural network and 2. an extra channel representing the noise level. PnP seismic inversion is also competitive from a computational point of view: models pre-trained on natural images can be in fact used directly to denoise geological images and, therefore, expensive re-training of the network to adapt to different geological settings may not be necessary.

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