Deep Learning to replace or augment model-based seismic inversion?

Introduction

In the last decade, deep learning has led to tremendous advances in computer vision and natural language processing. However, when it comes to scientific disciplines whose phenomena are governed by well-known physical equations, a question arises: should deep learning be used as a substitute to classical model-based inversion or become an integral part of it to complement its outstanding deficiencies? A large body of literature has started to emerge that suggests how the latter approach may be more suitable, especially in the presence of scarce data (e.g., Ongie et al., 2020).

In this work, we consider the well-known problem of *seismic post-stack seismic inversion* and perform a systematic comparative study between a number of deep learning-based inversion methods. We conclude that data-driven inversion requires a (rarely available) extensive coverage of wells and an available background model (used as additional input to the network) to perform favorably against hybrid inversion schemes that rely on Plug-and-Play (PnP) denoisers or Deep Image Prior (DIP) preconditioners. Finally, we suggest that data-driven inversion may be used to kick-start PnP-based seismic inversion providing the best-in-class method in this study.

Theory

Forward problem

The convolutional model lies at the foundation of seismic inversion. The recorded seismic data $d(t)$ can be expressed as the convolution between the reflectivity of the Earth $r(t)$ and the source wavelet $w(t)$:

$$d(t) = w(t) * r(t), \quad r(t) = \frac{i_P(t+\Delta t) - i_P(t)}{i_P(t+\Delta t) + i_P(t)} \approx \frac{1}{2}\frac{\partial i_P(t)}{\partial t} \tag{1}$$

where $i_P(t)$ is the acoustic impedance model and Δt is an infinitesimal time step. Whilst the reflectivity expression is generally nonlinear in acoustic impedance, a logarithmic approximation provides a linear link between acoustic impedance and seismic data. This linearisation holds for small to medium layer contrasts. However, its accuracy reduces at interfaces with large changes in acoustic impedance.

Model-based seismic inversion

Model-based seismic inversion can be performed by solving the following minimization problem,

$$\arg\min_{i_P} \frac{1}{2}\|d(x,t) - G \cdot i_P(x,t)\|_2^2 + \lambda \mathscr{R}(i_P(x,t)) \tag{2}$$

where G is the discretised version of equation 1 and $\mathscr{R}$ is a regularizer used to promote certain features in the model. Equation 2 is solved by means of iterative solvers using a background acoustic impedance model ($i_{P,0}$) as initial guess. We have added a dependency to the spatial coordinate x to remark the point that inversion is performed in a multi-channel fashion and regularization acts across traces.

Data-driven, supervised and semi-supervised seismic inversion

The second approach relies on the supervised learning paradigm: given a limited number of pairs of acoustic impedance profiles and seismic traces $(i_P^{<j>}(t), d^{<j>}(t), j = 1, ..., n)$, a neural network (NN) f_θ is trained to predict the former from the latter (e.g., Das et al., 2019). Moreover, similar to Biswas et al. (2019), for a larger number of seismic traces without matching acoustic impedance $(d^{<k>}(t), k = 1, ..., m)$ (where $m >> n$) an additional *data-consistency* loss can be included such that the operator G is applied to the output of the network and compared with the data:

$$\arg\min_{\theta} \frac{1}{n}\sum_j \|f_\theta(d^{<j>}(t)) - i_P^{<j>}(t)\|_2^2 + \frac{\lambda}{m}\sum_k \|G \cdot f_\theta(d^{<k>}(t)) - d^{<k>}(t)\|_2^2 \tag{3}$$

Finally, the trained model can be applied trace-by-trace to an entire seismic dataset: $\hat{i}_P(t) = f_\theta(d(t))$. To be consistent with the other approaches applied in this work, a 1D-UNet convolutional network is used for this approach.

Hybrid seismic inversion

Two different hybrid approaches are considered, where in both cases a NN f_θ is used to aid the solution of model-based seismic inversion (equation 2):

- Deep Image Prior based seismic inversion: following Lipari et al. (2022), an untrained neural network is used as nonlinear preconditioner in the solution of the post-stack seismic inverse problem. The network is fed with a random noise realization and trained to match the observed seismic data:

$$\arg\min_\theta \|d(x,t) - G \cdot [f_\theta(w(x,t)) + i_{P,0}(x,t)]\|_2^2 \tag{4}$$

where the network f_θ learns to estimate the high-frequency component of the acoustic impedance model. Although not explicitly discussed in Lipari et al. (2022), the background model must be provided like in classical model-based inversion since such information is not contained in the seismic data. In our example, a 2D-UNet with residual blocks is used as deep prior.

- Plug-and-Play (PnP) seismic inversion: PnP priors recently emerged as a powerful replacement to hand-crafted priors in the solution of inverse problems; this is achieved by plugging a denoiser into a convex optimization algorithm, where the denoiser acts in place of the regularizer $\mathscr{R}$ in equation 2. As recently presented in Romero et al. (2022), the following iterations are employed to solve equation 2:

$$\begin{cases} i_P^{k+1} = min_{i_P} \tau \|d(x,t) - G \cdot i_P(x,t)\|_2^2 + \|i_P - i_P^k + \tau y^{k+1}\|_2^2 \\ y^{k+1} = y^k + \mu \bar{i}_P^k - f_\theta((y^k + \mu \bar{i}_P^k)/\mu, \lambda/\mu) \\ \bar{i}_P^{k+1} = 2 i_P^{k+1} - i_P^k, \end{cases} \tag{5}$$

where the denoiser f_θ is chosen to be a pre-trained non-blind denoising neural network, such as the DRUNet network of Zhang et al. (2021). Compared to other learned denoisers, DRUNet introduces an additional input channel containing the standard deviation of the noise to remove: this allows tuning the strength of the denoiser during inference in agreement with the λ/μ parameter in equation 5.

Finally, both hybrid methods can be combined with the previously described supervised learning approach by initializing the acoustic impedance model with the data-driven estimate ($i_{P,0} = \hat{i}_P$) and/or adding a regularization term of the form $\|i_P - \hat{i}_P\|_2^2$. We call this *supervised regularization*.

Numerical Examples

A systematic comparison of the presented methods is performed on a synthetic dataset created from the Marmousi model (Figure 1a) using a Ricker wavelet with a peak frequency of 15 Hz and both the nonlinear (Figure 2a) and linear forms of the reflectivity in equation 1. Their difference is negligible everywhere except at the two high acoustic impedance wedges: we consider both scenarios to assess the impact of relying on a linear modelling operator (G) in both the model-based and hybrid inversion schemes compared to the data-driven approach where the training data can be easily created using the nonlinear reflectivity modelling. Finally, the nonlinear data is also contaminated with band-passed white, Gaussian noise. A summary of the inversion results for the different methods is displayed in Table 1 (supervised methods use all six wells in Figure 1a for training).

Figure 1 focuses on the scenario where the data is modeled using the nonlinear reflectivity equation and contaminated with noise: as far as the model-based inversion is concerned (Figure 1b), Total Variation regularization as described in Ravasi and Birnie (2022) is used. Figures 1c and 1d show the estimated acoustic impedance models from the supervised learning approach using both data and data plus background model as inputs to the network, respectively. Finally, the DIP and PnP approaches are employed without (Figures 1e and 1f) and with (Figures 1g and 1h) the supervised regularization term. Clearly,

Method	**Network/#params**	**Linear**	**Nonlinear**	**NN + noise**
Model-based	-	24.438	22.801	22.693
Supervised	1D-UNet/677k	16.614	16.149	15.803
Supervised w/ background	1D-UNet/677k	23.564	22.830	22.531
Supervised w/ back. & semi-sup. loss	1D-UNet/677k	23.852	22.603	22.718
DIP	2D-UNet/1.99M	25.252	23.738	18.830
PnP	2D-UNet/32.6M	*26.487*	*23.983*	*23.611*
DIP with supervised reg.	2D-UNet/1.99M	24.378	23.712	22.986
PnP with supervised reg.	2D-UNet/32.6M	**26.777**	**24.351**	**24.412**

Table 1: Signal-to-noise ratio for the estimated acoustic impedance models in the linear, nonlinear, and nonlinear+noise data scenarios. Bold: best; Italic: second best; Red: worst.

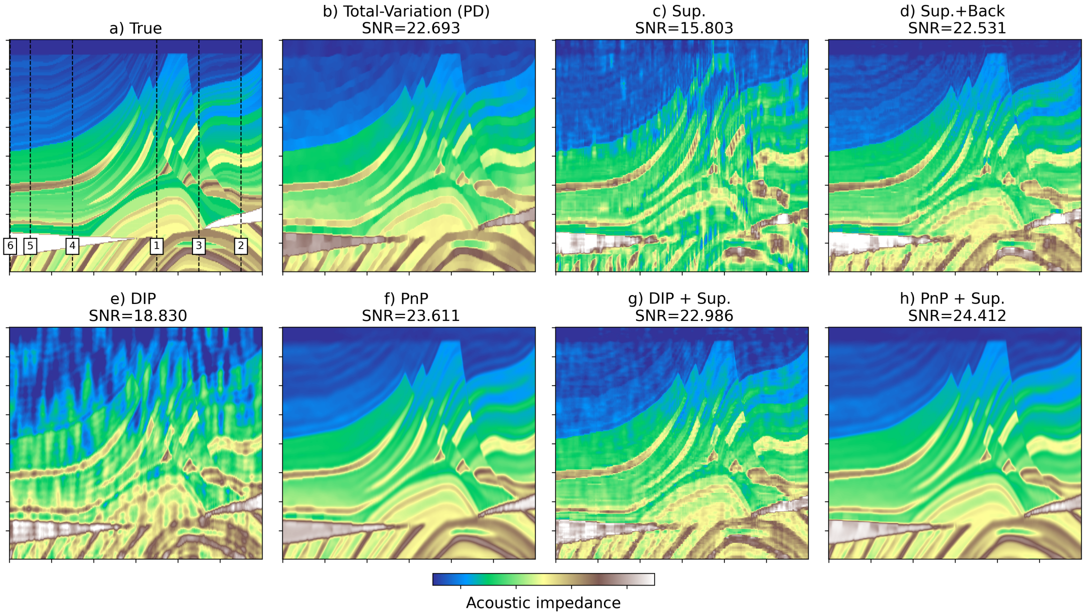

Figure 1: a) True model (with dashed vertical lines representing well locations used to train the supervised models). Estimates for b) TV-regularized inversion, c) supervised, d) supervised with background, e) DIP, f) PnP, g) DIP with supervised initialisation, and h) PnP with supervised initialization.

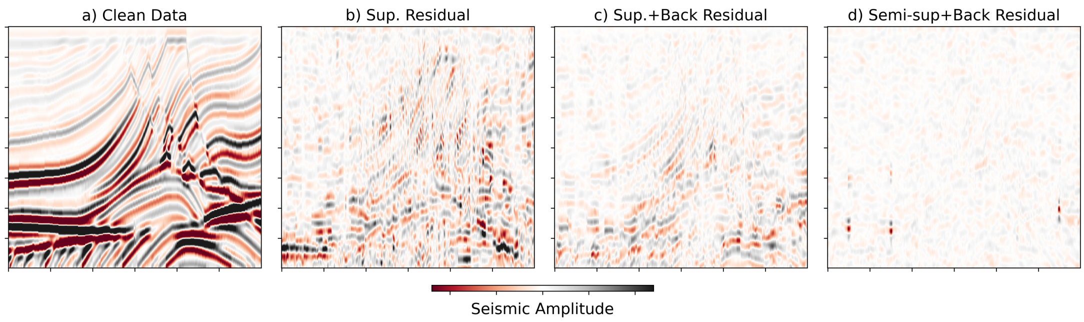

Figure 2: a) Clean nonlinear data and, b-d) residuals for supervised, supervised with background, and semi-supervised methods, respectively.

the supervised learning approach is competitive with other methods only when the background model is used as additional input in the training process. On the other hand, the DIP approach is greatly affected by the presence of noise in the data. Moreover, both the DIP and PnP approaches underestimate the acoustic impedance value of the two wedges; this is partially due the mismatch between the data (nonlinear reflectivity) and the linear modelling operator and can be mitigated by regularizing their inversions with the solution of the supervised method. Figure 2 highlights a major drawback of purely data-driven methods: the difference between the true and predicted data from the estimated acoustic impedance model is fairly large as no direct data-consistency check is performed at the inference stage (see Figures 2b and c). This can be improved by adding the second term in equation 3 with $\lambda = 1$ (Figure 2d): an improved data match does generally lead to a small improvement in the model estimate as shown in Table 1. Finally, the convergence of the hybrid methods is assessed in Figure 3a. As DIP depends on the initialization of the network parameters, we show the mean plus/minus standard deviation for 20 independent runs. In Figure 3b, we also analyse the robustness of the supervised learning approach for an increasing number of wells. Given that the initialization of the network has an impact on the inversion results, all experiments are repeated 20 times. We conclude that when less than six wells are available in the training process, the supervised approach becomes less accurate than the PnP approach.

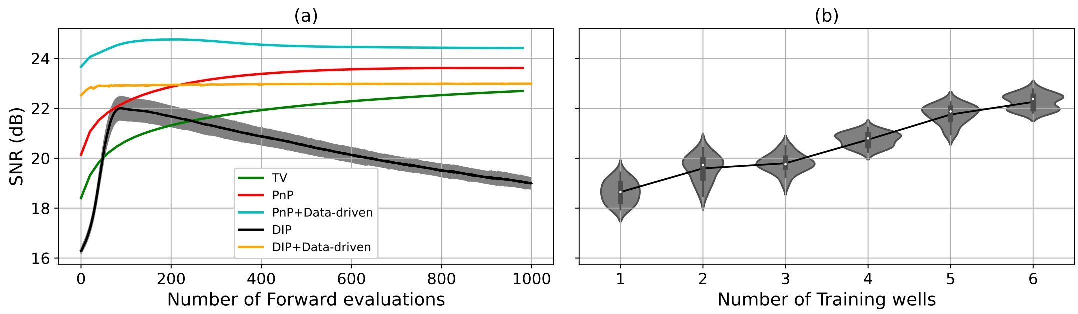

Figure 3: a) SNR as function of operator evaluation for model-based and hybrid approaches, and b) SNR as function of the number of training wells for the supervised learning approach.

Conclusions

After comparing data-driven and hybrid algorithms for seismic post-stack inversion, we conclude that:

- Supervised learning can be performed with a good degree of success only in the presence of a dense well coverage. Our results also highlight the importance of including a background trend in the input dataset such that the network can only focus on learning the deviation from it, and a data-consistency loss for the seismic traces without matching acoustic impedance profiles;
- Hybrid approaches can outperform state-of-the-art model-based methods without requiring training data of any sort. Moreover, PnP shows faster convergence and can naturally handle noisy data, whilst the natural prior induced by DIP is more prone to noise over-fitting;
- Adding a regularization term to both hybrid methods and/or boot-strapping them with the prediction from the supervised learning algorithm can greatly speed up convergence and produce best-in-class results where information from both the available wells and seismic data are integrated in an optimal manner.

Acknowledgements

The authors thank KAUST and the DeepWave Consortium sponsors for supporting this research.

References

[1] Biswas, R., Sen, M. K., and Das, V. [2019] Prestack and poststack inversion using a physics-guided convolutional neural network: Interpretation, 7, no. 3, SE161–SE174.
[2] Das, V., Pollack, A. and Wollner, U. [2019] Convolutional neural network for seismic impedance inversion: Geophysics, 84, no. 6, R869–R880.
[3] Lipari, V., Picetti, F., Bestagini, P., and Tubaro, S. [2021] Post-Stack Inversion with Uncertainty Estimation through Bayesian Deep Image Prior. 82nd EAGE Annual Conference Exhibition. pp. 1-5.
[4] Ongie, G., Jalal, A., Metzler, C. A., Baraniuk, R. G., Dimakis, A. G., and Willett, R. [2020] Deep Learning Techniques for Inverse Problems in Imaging. arXiv.
[5] Ravasi, M., and Birnie, C. [2022] A Joint Inversion-Segmentation approach to Assisted Seismic Interpretation. Geophysical Journal International. pp. 893–912.
[6] Romero, J., Corrales, M., Luiken, N., and Ravasi, M. [2022] Plug and Play Post-Stack Seismic Inversion with CNN-Based Denoisers. Second EAGE Subsurface Intelligence Workshop. pp. 1-5.
[7] Zhang, K., Li, Y., Zuo, W., Zhang, L., Van Gool, L. and Timofte, R. [2021] Plug-and-play image restoration with deep denoiser prior. IEEE Transactions on Pattern Analysis and Machine Intelligence. pp. 6360-6376.