Numerical Analysis, MAD 3401: Project 1

Project rules:

- All work must be independent. No collaboration is allowed. You can only get help from published books and free public online resources such as wiki, but make sure to quote and make references. No paid online resources are allowed.
- You can use any programming language to write your code. For programming problems, please show your computer-generated results with your interpretation, along with your code.
- Project is due 2/14 noon. Late project will lose 10% for each day it is late. Project later than 2/21 noon will get a zero.
- Print or handwriting or combination is fine. You can also email me your project in PDF and source code (either in PDF or original format) to weiwang1@fiu.edu.

Problem 1. (20 points) To solve $x^3 - 2x + 1 = 0$, the following two fixed-point problems are proposed:

$$x = g_1(x) = \frac{1}{2}(x^3 + 1), \quad x = g_2(x) = \frac{2}{x} - \frac{1}{x^2}, \quad p_0 = 0.5$$

(a) (10 points) Write a fixed-point program to show which methods work appropriate. Use $|p_n - p_{n-1}| < 0.01$ as a measure of error in the iteration. Output all of the iterations.

(b) (10 points) Use Fixed-point theorem 2.4 to show whether the two methods converge on [0.1,0.7].

Problem 2. (20 points) (a) Derive the formula of Newton's method. Use the example of finding the root of $f(x) = x^2 - 4$ to show how you obtain p_1 and p_2 graphically if $p_0 = 1$.

(b) Derive the formula of Secant's method. Use the example of finding the root of $f(x) = x^2 - 4$ to show how you obtain p_2 and p_3 graphically if $p_0 = 0$ and $p_1 = 4$.

Problem 3. (40 points) Write a program for each of the following root-finding methods:

(1) Bisection method, (2) Newton's method, (3) Secant method.

Use your programs to find approximations of the roots $p = 0$ of the following equations. Use $|p_n - p_{n-1}| < 10^{-4}$ as a measure of error in the iteration. Output all of the iterations. Choose appropriate initial guesses or interval by yourself.

(a) $x^3 + x^2 + 2x = 0$
(b) $e^x - x - 1 = 0$

Problem 3 (part 2). (10 points) For the same equation, what are your conclusions on the performances of different methods? For the same method, what are your conclusion on its performances on different equations?

Problem 4. (10 points) For your Bisection method in Problem 3(a), use the Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-4}.