Numerical Analysis, MAD 3401: Project 2

Problem 1*. (20 points) A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

Write a program which uses Newton's divided difference to compute the Hermite polynomial to predict the position of the car at $t = 10$s. Output the divided difference table.

*Note: You have the option to do it by hand with a calculator. If so, please write out all the working details.

Problem 2. (10 points) Determine the natural spline S that interpolates the data

$$f(0) = 0, f(1) = 1, f(2) = 2.$$

Problem 3. (15 points) Derive the 3-point midpoint formula by Taylor polynomial

$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6}f'''(\xi).$$

Problem 4. (15 points) Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

$$M = N(h) + K_1h + K_2h^2 + K_3h^3 + \cdots,$$

for some constants K_1, K_2, K_3, $\cdots$. Use the values $N(h)$, $N(h/3)$ and $N(h/9)$ to produce an $O(h^3)$ approximation to M.

Problem 5*. (40 points) Write a program for each of the methods

(1) composite Trapezoidal rule, (2) composite Simpson's rule, (3) composite midpoint rule.

Use your programs to evaluate the integral

$$I = \int_0^2 e^{2x} \sin 3x \, dx.$$

The exact value of this integral is $I = \frac{e^4}{13}(2\sin 6 - 3\cos 6) + \frac{3}{13}$. For each method, compute a series of approximations I_n using $n = 4, 8, 16$, respectively. Output the approximations I_n and the absolute errors $|I - I_n|$. What are your conclusions on the performance of these methods?

*Note: You have the option to do it by hand with a calculator. If so, please write out all the working details.