4A

$$x_{t+1} = Ax_t + Bu_t + c$$

$$\overset{?}{=} \bar{A}\bar{x}_t + \bar{B}\bar{u}_t = \begin{pmatrix} A & c \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_t \\ 1 \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u_t$$

$$\overset{?}{=} \begin{bmatrix} A_{11} & \cdots & A_{1n} & c_1 \\ \vdots & \ddots & \vdots & \vdots \\ A_{n1} & \cdots & A_{nn} & c_n \\ 0 & \cdots & 0 & 1 \end{bmatrix}\begin{bmatrix} x_{t1} \\ \vdots \\ x_{tn} \\ 1 \end{bmatrix} + \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{n1} & \cdots & B_{nn} \\ 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} u_{t1} \\ \vdots \\ u_{tn} \end{bmatrix}$$

$$\overset{?}{=} \begin{bmatrix} A_{11} \\ \vdots \\ A_{n1} \\ 0 \end{bmatrix} x_{t1} + \ldots + \begin{bmatrix} A_{1n} \\ \vdots \\ A_{nn} \\ 0 \end{bmatrix} x_{tn} + \begin{bmatrix} c_1 \\ \vdots \\ c_n \\ 1 \end{bmatrix} 1 + \begin{bmatrix} B_{11} \\ \vdots \\ B_{n1} \\ 0 \end{bmatrix} u_{t1} + \ldots + \begin{bmatrix} B_{1n} \\ \vdots \\ B_{nn} \\ 0 \end{bmatrix} u_{tn}$$

$$\overset{?}{=} \begin{bmatrix} A_{11}x_{t1} \\ \vdots \\ A_{n1}x_{t1} \\ 0 \end{bmatrix} + \ldots + \begin{bmatrix} A_{1n}x_{tn} \\ \vdots \\ A_{nn}x_{tn} \\ 0 \end{bmatrix} + \begin{bmatrix} c_1 \\ \vdots \\ c_n \\ 1 \end{bmatrix} + \begin{bmatrix} B_{11}u_{t1} \\ \vdots \\ B_{n1}u_{t1} \\ 0 \end{bmatrix} + \ldots + \begin{bmatrix} B_{1n}u_{tn} \\ \vdots \\ B_{nn}u_{tn} \\ 0 \end{bmatrix}$$

$$\overset{?}{=} \begin{bmatrix} A_{11}x_{t1} + \ldots + A_{1n}x_{tn} + c_1 + B_{11}u_{t1} + \ldots + B_{1n}u_{tn} \\ \vdots \\ A_{n1}x_{t1} + \ldots + A_{nn}x_{tn} + c_n + B_{n1}u_{t1} + \ldots + B_{nn}u_{tn} \\ 1 \end{bmatrix}$$

$$= \begin{pmatrix} x_{t+1} \\ 1 \end{pmatrix} \qquad \text{QED}$$

4B

$$\bar{x}_T^T \bar{Q}_f x_T + \sum_{t=1}^{T-1} \bar{x}_t^T Q \bar{x}_t + \bar{u}_t^T R \bar{u}_t = \dots$$

$$\begin{pmatrix} x_T \\ 1 \end{pmatrix}^T \begin{pmatrix} Q_f & q_f \\ q_f^T & \eta \end{pmatrix} \begin{pmatrix} x_T \\ 1 \end{pmatrix} + \sum_{t=1}^{T-1} \begin{pmatrix} x_t \\ 1 \end{pmatrix}^T \begin{pmatrix} Q & q \\ q^T & \eta \end{pmatrix} \begin{pmatrix} x_t \\ 1 \end{pmatrix} + u_t^T R u_t = \dots$$

$$x_T^T Q_f x_T + 2 q_f^T x_T + \eta$$
$$+ \sum_{t=1}^{T-1} x_t^T Q x_t + 2 q^T x_t + \eta + u_t^T R u_t$$

4C

DISCLAIMER:

I haven't taken a controls course, so this is kind of an educated guess!

Example 1

an affine system is a linear system under the effect of some constant

- this constant can be time varying or non-time varying
- this constant can be interpetted as a bias
- a system under the effect of additive white gaussian noise is affine

Example 2

a simpler example: a helicopter hovering in place is under the constant influence of gravity and thus should be affine?
(although this might not be true depending on the simplicity/complexity of the model)