# Modeling Green Development

## A Rosen-Roback Approach

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# Review

**Research Question**

What characteristics of urban neighborhoods relate to the number of certified green commercial buildings?

Previously,

- Alonso-Muth-Mills Model suggests green development occurs outside the city center
- Even with modifications, still problematic
- Need more than just distance from the city center

# Outline

**Today's Goal**

Create a general equilibrium model that describes how a developer might decide where to build green.

1. Model Environment
2. Specifying the Model
3. Results

# Model Environment

# Model Environment: Among Neighborhoods

Assume a world composed of many distinct neighborhoods

Neighborhood Components:

- Agents: Workers, Firms, One Developer
- Fixed Features: Housing Stock, Commercial Land, "Public Capital"
- Agents and (Tradeable) Goods can move freely between and within Neighborhoods

Key Conditions: All Agents Indifferent Between Neighborhoods (Spatial Equilibrium)

# Model Environment: Within the Neighborhood

**Labor Market**

Actors: Workers, Firms

Choose: Wages, Population

Need: Labor Supply, Labor Demand

**Product Market**

Actors: Firms, World

Choose: Output, No. of Firms

Need: Revenues, Costs

**Commercial Real Estate Market**

Actors: Firms, Developer

Choose: Price & Quantity of Commercial RE, Green/Brown

Need: Supply, Demand, Adoption Condition

Key Conditions: All Agents Optimize in Each Market

# Specifying the Model

# The Worker

$$\max_{H,X} \left\{ U(H,X) = \theta H^{\alpha} X^{1-\alpha} \right\} \quad \text{s.t.} \quad W = p_H H + X$$

| | |
|---|---|
| $H$ | Housing (sq.ft.) |
| $X$ | Composite Good |
| $W$ | Wage |
| $\theta$ | Amenity Index |
| $p_H$ | Price of Housing (per sq.ft.) |
| $p_x$ | Price of Composite Good (Numeraire) |

Endogenous, Exogenous

# The Firm

$$\max_{N,Z} \left\{ \pi(N, Z : d) = A\lambda_d N^{\beta} Z_d^{\gamma} \left(\frac{\bar{K}}{M}\right)^{1-\beta-\gamma} - WN - p_{z_d} Z_d - \kappa \right\}$$

| | |
|---|---|
| $N$ | Number of Workers (Population) |
| $Z$ | Quantity of Commercial Real Estate (sq.ft.) |
| $\bar{K}$ | Neighborhood Capital |
| $M$ | Number of Firms |
| $d$ | Design (Green or Brown) |
| $A$ | Neighborhood Productivity |
| $\lambda$ | Energy Efficiency |
| $p_z$ | Price of Commercial Real Estate |
| $\kappa$ | Fixed Capital Cost |

Endogenous, Exogenous

# The Developer

There is only one developer in a neighborhood, but it will still act competitively

$$\max_{h,d} \left\{ \pi^{\text{Dev}}(h, d : \bar{\ell}) = p_{z_d} h_d \bar{\ell} - c_d h_d^{\delta} \bar{\ell} - p_{\ell} \bar{\ell} \right\}$$

| | |
|---|---|
| $h$ | Height |
| $\bar{\ell}$ | Commercial Land |
| $d$ | Design (Green or Brown), $d \in \{g, b\}$ |
| $c$ | Material Cost |
| $p_{\ell}$ | Price of Commercial Land |
| $\delta$ | Height Friction Parameter, $\delta > 1$ |

Endogenous, Exogenous

# Derivation Overview

1. Optimization Conditions
   - First-Order Conditions
   - Market Clearing: Labor Market, Commercial Real Estate Market

2. Spatial Equilibrium Conditions
   - Agents must be indifferent between where they are and anywhere else
   - Workers – Uniform Utility
   - Firms and the Developer – Zero Profits

3. Five equations in five unknowns ($N$, $W$, $Z_d$, $p_{z_d}$, $M$)

# Results

*Conditions and Comparative Statics*

# Design

A developer will choose the design that maximizes the price it can afford to pay for the land. This leads to the result:

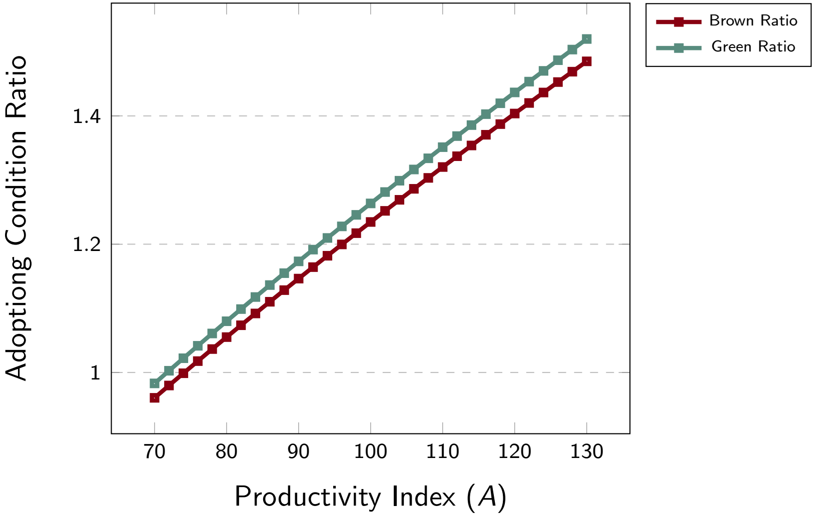
**The Adoption Condition**

The developer will choose to build green if and only if

$$\frac{p_{z_g}^{\delta}}{c_g} > \frac{p_{z_b}^{\delta}}{c_b}$$

where $c_g$ and $c_b$ are exogenous and $\delta > 1$.

# Do Neighborhood Characteristics Matter?



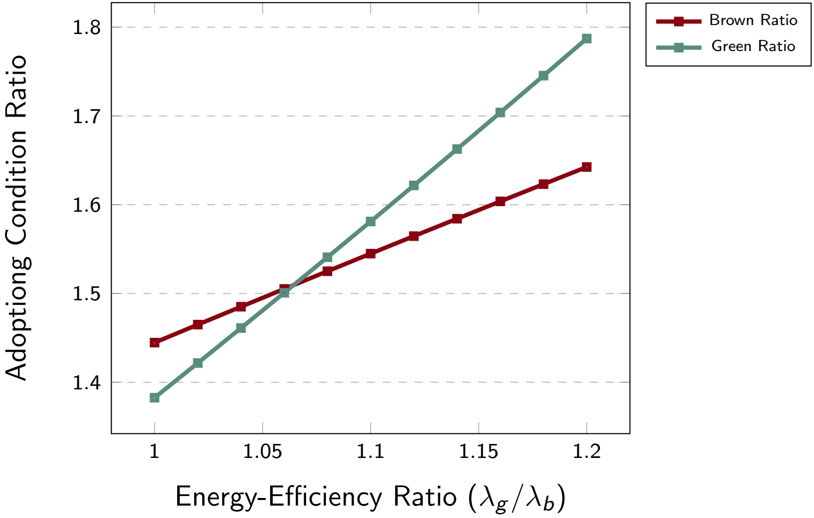
# The Problematic Premium

The premium occurs proportionally

$$\frac{p_{z_g}}{p_{z_b}} = \left(\frac{\lambda_g}{\lambda_b}\right)^{\phi_1} \left(\frac{c_g}{c_b}\right)^{\phi_2}$$

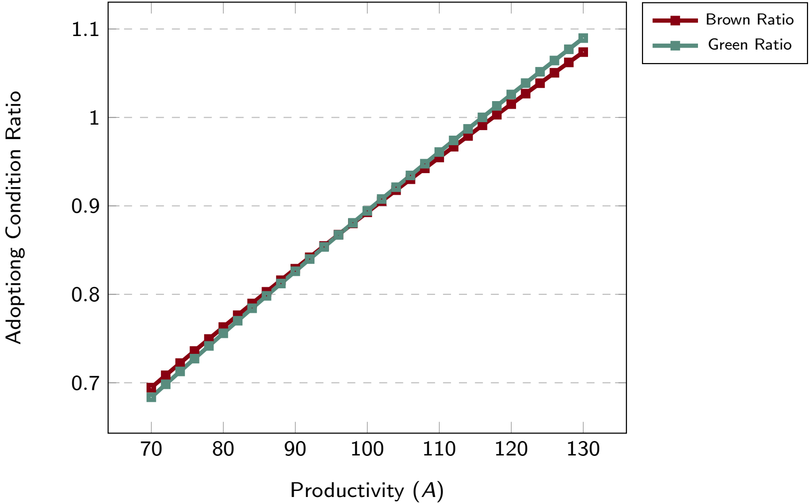
With the adoption condition, implies there are only two ways to change the design decision: $\lambda$ and $c$.

# City Characteristics Matter



# Making $\lambda$ an Exponent

$$\max_{N,Z}\left\{\pi(N,Z:d) = A^{\lambda_d} N^{\beta} Z_d^{\gamma}\left(\frac{\bar{K}}{M}\right)^{1-\beta-\gamma} - WN - p_{z_d} Z_d - \kappa\right\}$$



# Next Steps

Continue to refine model:

- Get clear interaction between area characteristics and the adoption decision
  - Give workers preferences over working for green firms
  - Non-proportional $\lambda$
- Too deterministic and homogenous
- What variables are actually useful and observable?

Collect data to address previous concerns, and for estimation

# References

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**Glaeser, Edward Ludwig**, *Cities, agglomeration, and spatial equilibrium*, Oxford University Press, 2008.

**Roback, Jennifer**, "Wages, rents, and the quality of life," *Journal of political Economy*, 1982, *90* (6), 1257–1278.

**Rosen, Sherwin**, "Wage-based indexes of urban quality of life," *Current issues in urban economics*, 1979, pp. 74–104.

# Appendix

*Some Additional Math*

# System of Equations

Labor Supply and Spatial Equilibrium for the Worker

$$NW^{\frac{\alpha-1}{\alpha}} = \left(\frac{\theta}{\bar{V}}\right)^{\frac{1}{\alpha}} \bar{H}(1-\alpha)^{1-\alpha}$$

Labor Demand (given $p_{z_d}$)

$$N\left(W^{1-\gamma} p_{z_d}^{\gamma}\right)^{\frac{1}{1-\beta-\gamma}} = \left(A\lambda_d \beta^{1-\gamma}\gamma^{\gamma}\right)^{\frac{1}{1-\beta-\gamma}} \bar{K}$$

Commercial Real Estate Supply

$$Zp_{z_d}^{\frac{-1}{\delta-1}} = (\delta c_d)^{\frac{-1}{\delta-1}} \bar{\ell}_c$$

Commercial Real Estate Demand (given $W$)

$$Z\left(W^{\beta} p_{z_d}^{1-\beta}\right)^{\frac{1}{1-\beta-\gamma}} = \left(A\lambda_d \beta^{\beta}\gamma^{1-\beta}\right)^{\frac{1}{1-\beta-\gamma}} \bar{K}$$

Zero-Profit condition for the Firm (Spatial Equilibrium)

$$\left(W^{\beta} p_{z_d}^{\gamma}\right)^{\frac{1}{1-\beta-\gamma}} M = \left(\frac{\Phi\bar{K}}{\kappa}\right)(A\lambda_d)^{\frac{1}{1-\beta-\gamma}}$$