

Base manifold (three dimensional)

Metric tensor (cartesian coordinates - norm = False)

$$g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two dimensionanal submanifold - Unit sphere

Basis not normalised

$$(\theta, \phi) \rightarrow (r, \theta, \phi) = [1, \quad \theta, \quad \phi]$$

$$e_\theta \cdot e_\theta = 1$$

$$e_\phi \cdot e_\phi = \sin(\theta)^2$$

$$g = \begin{bmatrix} 1 & 0 \\ 0 & \sin(\theta)^2 \end{bmatrix}$$

$$g_inv = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sin^2(\theta)} \end{bmatrix}$$

Christoffel symbols of the first kind:

$$\Gamma_{1,\alpha,\beta} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\sin(2\theta)}{2} \end{bmatrix} \quad \Gamma_{2,\alpha,\beta} = \begin{bmatrix} 0 & \frac{\sin(2\theta)}{2} \\ \frac{\sin(2\theta)}{2} & 0 \end{bmatrix}$$

$$\nabla = e_\theta \frac{\partial}{\partial \theta} + e_\phi \frac{1}{\sin(\theta)^2} \frac{\partial}{\partial \phi}$$

$$\nabla f = \partial_\theta f e_\theta + \frac{\partial_\phi f}{\sin(\theta)^2} e_\phi$$

$$F = F^\theta e_\theta + F^\phi e_\phi$$

$$\nabla F = \left(\frac{F^\theta}{\tan(\theta)} + \partial_\phi F^\phi + \partial_\theta F^\theta \right) + \left(\frac{2F^\phi}{\tan(\theta)} + \partial_\theta F^\phi - \frac{\partial_\phi F^\theta}{\sin(\theta)^2} \right) e_\theta \wedge e_\phi$$

One dimensionanal submanifold

Basis not normalised

$$(\phi) \rightarrow (\theta, \phi) = \left[\frac{\pi}{8}, \quad \phi \right]$$

$$e_\phi \cdot e_\phi = -\frac{\sqrt{2}}{4} + \frac{1}{2}$$

$$g = \left[-\frac{\sqrt{2}}{4} + \frac{1}{2} \right]$$

$$\nabla = e_\phi \left(2\sqrt{2} + 4 \right) \frac{\partial}{\partial \phi}$$

$$\nabla h = \left(2\sqrt{2} + 4 \right) \partial_\phi h e_\phi$$

$$H = H^\phi e_\phi$$

$$\nabla H = \partial_\phi H^\phi$$