Base manifold (three dimensional)

Metric tensor (cartesian coordinates - norm = False)

$$g = \left[\begin{array}{rrr} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Two dimensioanal submanifold - Unit sphere

Basis not normalised

$$(\theta, \phi) \to (r, \theta, \phi) = [1, \theta, \phi]$$

$$e_{\theta} \cdot e_{\theta} = 1$$

$$e_{\phi} \cdot e_{\phi} = \sin\left(\theta\right)^2$$

$$g = \begin{bmatrix} 1 & 0 \\ 0 & \sin(\theta)^2 \end{bmatrix}$$

$$g.inv = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sin^2(\theta)} \end{bmatrix}$$

Christoffel symbols of the first kind:

$$\Gamma_{1,\alpha,\beta} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\sin{(2\theta)}}{2} \end{bmatrix} \quad \Gamma_{2,\alpha,\beta} = \begin{bmatrix} 0 & \frac{\sin{(2\theta)}}{2} \\ \frac{\sin{(2\theta)}}{2} & 0 \end{bmatrix}$$

Christoffel symbols of the second kind:

$$\Gamma^1_{\alpha,\beta} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\sin{(2\theta)}}{2} \end{bmatrix} \quad \Gamma^2_{\alpha,\beta} = \begin{bmatrix} 0 & \frac{1}{\tan{(\theta)}} \\ \frac{1}{\tan{(\theta)}} & 0 \end{bmatrix}$$

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abla} = oldsymbol{e}_{ heta} rac{\partial}{\partial heta} + oldsymbol{e}_{oldsymbol{\phi}} rac{1}{\sin{(heta)^2}} rac{\partial}{\partial oldsymbol{\phi}}$$

$$\mathbf{
abla} f = \partial_{ heta} f \mathbf{e}_{ heta} + rac{\partial_{oldsymbol{\phi}} f}{\sin{(heta)^2}} \mathbf{e}_{oldsymbol{\phi}}$$

$$F = F^{\theta} \mathbf{e}_{\theta} + F^{\phi} \mathbf{e}_{\phi}$$

$$\mathbf{\nabla}F = \left(\frac{F^{\theta}}{\tan\left(\theta\right)} + \partial_{\theta}F^{\theta} + \partial_{\phi}F^{\phi}\right) + \left(\frac{2F^{\phi}}{\tan\left(\theta\right)} + \partial_{\theta}F^{\phi} - \frac{\partial_{\phi}F^{\theta}}{\sin\left(\theta\right)^{2}}\right)\boldsymbol{e}_{\theta} \wedge \boldsymbol{e}_{\phi}$$

One dimensioanal submanifold

Basis not normalised

$$(\phi) \to (\theta, \phi) = \begin{bmatrix} \frac{\pi}{8}, & \phi \end{bmatrix}$$

$$e_{\phi} \cdot e_{\phi} = -\frac{\sqrt{2}}{4} + \frac{1}{2}$$

$$g = \left[-\frac{\sqrt{2}}{4} + \frac{1}{2} \right]$$

$$\nabla = e_{\phi} \left(2\sqrt{2} + 4 \right) \frac{\partial}{\partial \phi}$$

$$abla h = \left(2\sqrt{2} + 4\right)\partial_{\phi}he_{\phi}$$

$$H = H^{\phi} e_{\phi}$$

$$\nabla H = \partial_{\phi} H^{\phi}$$