Week 2

* สั่งงาน Assignment 1

Penguin.

01046725 DIGITAL SIGNAL PROCESSING

Lecture #1: Introduction

Semester 2/2566

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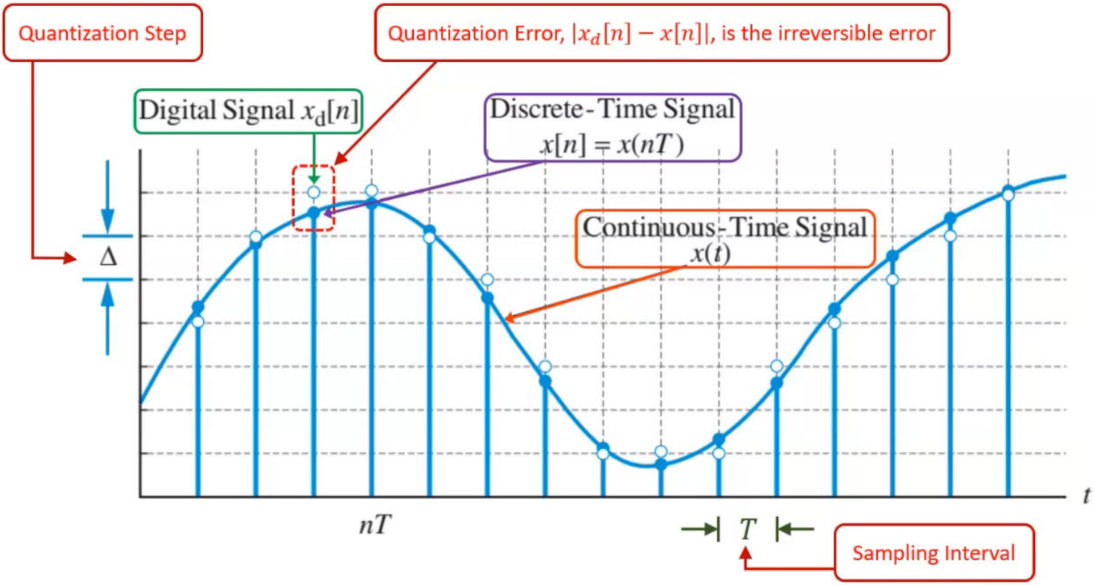
Irreversible Information Loss in Quantization Process



How does the signal sampling work?

$$f(sample^{-1}) = \frac{F(sec^{-1})}{F_s(sample/sec)} \quad \Longleftrightarrow \quad \omega(rad/sample) = \frac{\Omega(rad/sec)}{F_s(sample/sec)}$$

From this relation, $f(sample^{-1})$ and $\omega(rad/sample)$ are called relative frequencies or normalized frequencies.

The sampling process converts unlimited range of analog frequency, $F(sec^{-1})$ or $\Omega(rad/sec)$ to limited range of discrete frequency, $f(sample^{-1})$ or $\omega(rad/sample)$.

$$\Omega = 2\pi F$$
$$-\infty < \Omega(rad/sec) < \infty$$
$$-\infty < F(sec^{-1}) < \infty$$

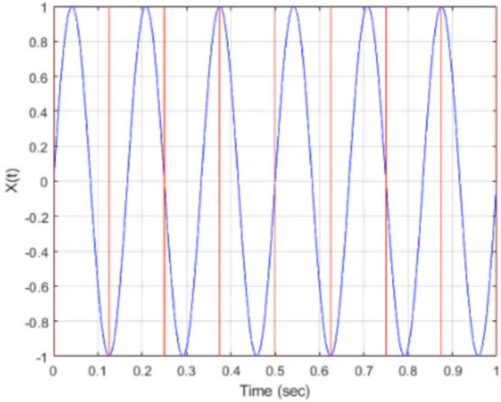
$$\xrightarrow{f = F/F_s} \qquad \xleftarrow[F = f.F_s]{}$$

$$\omega = 2\pi f$$
$$-\pi \leq \omega(rad/sample) \leq \pi$$
$$-\frac{1}{2} \leq f(sample^{-1}) \leq \frac{1}{2}$$

MATLAB Code Example of Signal Conversion

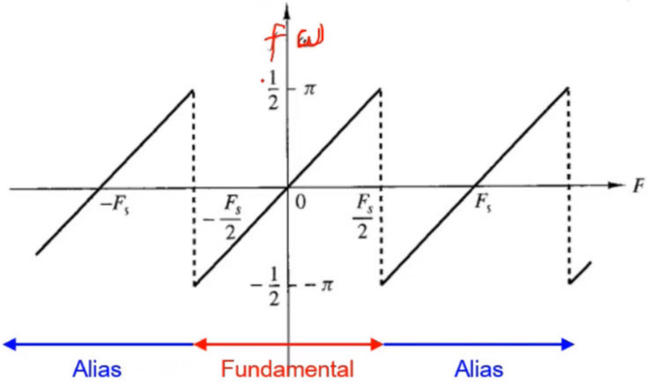
```matlab
t=linspace(0,1,1001);   % Time Scale in second (Continuous Time)
F=6;                    % Continuous Frequency (1/sec)
xt=sin(2*pi*F*t);       % Continuous Time Sinusoidal signal

disp(['The continuous signal frequency, F is ',num2str(F),'Hz']);
figure(1);clf;          % Display Original Continuous Time Signal
A=axes;
plot(t,xt,'b');
grid on
xlabel("Time (sec)");
ylabel("X(t)");

FS=8;                   % Sampling Frequency (Samples/Sec)
T=1/FS;                 % Sampling Period (Sec/Sample)
N=[0:T:1];
YL=get(A,'YLim');
for a=1:length(N)
    line(N(a)*[1 1],YL,'Color','r');  % Sampling Points
end
disp(['The Sampling frequency, FS is ',num2str(FS),'samples/sec']);
```

The continuous signal frequency, F is 6Hz



The Sampling frequency, FS is 8samples/sec

Continuous-Time Frequency ⇔ Discrete-Time Frequency



Relationship between Continuous-Time Frequency, F or Ω
and Discrete-Time Frequency, f or ω
through the sampling frequency, F_s

Example of Signal Conversion at $F_s = 2F_{max}$

$$x(t) = 5\sin(300\pi t) = 5\sin(2\pi.150t) \Rightarrow F = 150\text{Hz}$$

If $F_s = 300\text{Hz}$ (sample/sec) $\Rightarrow T = \frac{1}{300}$ (sec/sample)

$$F_s = 300 = 2F$$

$$x(nT) = 5\sin(2\pi.150nT) = 5\sin\left(2\pi.\frac{150}{300}n\right) \Rightarrow x[n] = 5\sin(\pi n)$$

Restore $x(t)$ from $x[n]$ at $F_s = 300\text{Hz}$ (sample/sec)

$$x[n] = 5\sin(\pi n) = 5\sin\left(2\pi.\frac{1}{2}n\right) \Rightarrow f = \frac{1}{2}\ (\text{sample}^{-1})$$

$$x[n] = 5\sin(\pi n) = 5.(0) = \{0,0,0,\ldots\}$$

$x(t)$ can NOT be restored from $x[n] = 0$

Note that $x(t)$ can perfectly be restored from $x[n]$ when $F_s > 2F_{max}$

MATLAB Code Example of Signal Conversion

```
figure(2);clf;              % Display Discrete Time Signal
n=[0:length(N)-1];          % Time Scale in Sample (Discrete Time)
nT=n*T;
xn=sin(2*pi*F*nT);          % Discrete Time Sinusoidal signal
A=axes;
stem(n,xn,"filled");
grid on
xlabel("Time (sample)");
ylabel("X[n]");

f=F/FS;                     % Relative Frequency "f"
if abs(f)>0.5               % Correcting relative frequency "f" (Alias Effect)
    disp(['The original relative frequency, f is ',num2str(f)]);
    if f<0
        while abs(f)>0.5
            f=f+1;
        end
    else
        while abs(f)>0.5
            f=f-1;
        end
    end
    disp(['The corrected relative frequency, f is ',num2str(f)]);
end
```