# Integration formulas

Working rules of integration:

1. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
2. $\int k\,f(x)\,dx = k\int f(x)\,dx$, Where k is a non-zero constant.
3. $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b)+c$ , Where a and b are constants and $a \neq 0$ .

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c,\ n \neq -1,\ n \in R$
2. $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \log|x|+c,\ x \in R^+$
3. $\int 1\,dx = x+c$
4. $\int k\,dx = kx+c$ , where k is constant
5. $\int e^x\,dx = e^x+c$
6. $\int e^{kx}\,dx = \frac{1}{k}e^{kx}+c$
7. $\int a^x\,dx = \frac{a^x}{\log a}+c$
8. $\int \sin x\,dx = -\cos x+c$
9. $\int \sin kx\,dx = -\frac{1}{k}\cos kx+c$
10. $\int \cos x\,dx = \sin x+c$
11. $\int \cos kx\,dx = \frac{1}{k}\sin kx+c$
12. $\int \sec^2 x\,dx = \tan x+c$
13. $\int \cos ec^2 x\,dx = -\cot x+c$
14. $\int \sec x\tan x\,dx = \sec x+c$
15. $\int \cos ecx\cot x\,dx = -\operatorname{cosec} x+c$
16. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c,\ x^2>a^2$
17. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c,\ x^2<a^2$
18. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
19. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|+c$
20. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|+c$
21. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c$
22. $\int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$
23. $\int \tan x\,dx = \log|\sec x|+c$
24. $\int \cot x\,dx = \log|\sin x|+c$
25. $\int \sec x\,dx = \log|\sec x+\tan x|+c$

    $= \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+c$
26. $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x-\cot x|+c$

    $= \log\left|\tan\left(\frac{x}{2}\right)\right|+c$
27. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+$

    $\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$
28. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-$

    $\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$
29. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$
30. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx-b\cos bx]+c$

31. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right] + c$

32. $\int e^{x}\left[f(x) + f'(x)\right]dx = e^{x} f(x) + c$

**Integration by the method of substitution:**

1. $\int \frac{f'(x)}{f(x)}dx = \log\left|f(x)\right| + c$
2. $\int \left[f(x)\right]^{n} f'(x)\,dx = \frac{\left[f(x)\right]^{n+1}}{n+1} + c,\ n \neq -1$
3. If $\int f(x)dx = F(x)$, then $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c$

**Integration by parts**

1. $\int u\,v\,dx = u\int v\,dx - \int\left[\frac{d}{dx}(u)\int v\,dx\right]dx$
   1) Logarithmic form (L) (e.g. log x)
   2) Inverse trigonometric form (I) (e.g. $\sin^{-1}x, \tan^{-1}x, .....$ )
   3) Algebraic form (A) (e.g. $1 = x^0,\ x,\ x^2, .....$ )
   4) Trigonometric form (T) (e.g. $\sin x, \tan x, \cos x, ...$)
   5) Exponential form (E) (e.g. $e^x, e^{2x}$ .... )
   6) $a^x, a^{2x}, .....$

Note: a) Use LIATE rule. (Between two functions (in integration by parts method) which comes first in above order is considered as u and another function considered as v.)

b) When only one function is given for integration then take 1 as another function.

**Definite Integration**

1. $\int_a^a f(x)dx = 0$
2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. If $a < b < c$ , then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
4. If $f$ is continuous on $[0,a]$, then $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
5. If $f$ is continuous and even on $[-a,a]$, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$
6. If $f$ is continuous and odd on $[-a,a]$, then $\int_{-a}^a f(x)dx = 0$
7. If $f$ is continuous on $[a,b]$, then $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

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# * Integration

$\int x\,dx = \frac{x^2}{2} + c$

($C$ = constant)

1 $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$, $n \in R^+$ or $n \neq -1$

2 $\int 1\,dx = x + c$, ~~$\int e^x dx = e^x + c$~~ $\int e^x\,dx = e^x + c$

3 $\int \frac{1}{x}\,dx = \log|x| + c$, $x \in R - \{0\}$

$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + c$

4 $\int \cos x\,dx = \sin x + c$, $x \in R$

5 $\int \sin x\,dx = -\cos x + c$, $x \in R$

6 $\int \sec^2 x\,dx = \tan x + c$, $x \in R - \{(2k+1)\frac{\pi}{2} / k \in z\}$

7 $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

8 $\int \sec x \tan x\,dx = \sec x + c$

9 $\int \cot x \operatorname{cosec} x = -\operatorname{cosec} x + c$

10 $\int a^x\,dx = \frac{a^x}{\log a} + c$

11 $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

12 $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

13 $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right| + c$

14 $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$

Let $f(x)$ be a function

$\therefore f'(x)$ is its derivative

$\therefore \frac{d}{dx} f(x) = f'(x)$

$\therefore \int f'(x) = f(x)$

Integration of derivative of a function ~~give~~ is equal to function itself

Ex $\frac{d}{dx}\sin x = \cos x$

& $\int \cos x = \sin x + c$

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15 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

16 $\int \frac{1}{|x|\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$

* Note : working rules are same

⇒ Examples

1 $\int 5x^3 - \frac{1}{x} + \cos x - e^x \; dx$

$5\int x^3 - \int \frac{1}{x} + \int \cos x - \int e^x$

$5\frac{x^4}{4} - \log x + \sin x - e^x + c$

2 $\int \left(x + \frac{1}{x}\right)^2 dx$

$\therefore \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$

$\therefore \int x^2 + \int 2 + \int \frac{1}{x^2}\,dx$

$\therefore \frac{x^3}{3} + 2x + \int x^{-2}\,dx$

$\therefore \frac{x^3}{3} + 2x + \frac{x}{-2+1}$

$\therefore \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}$

$\therefore \frac{x^3}{3} + 2x + \frac{1}{-x}$

simplified

$\therefore \frac{x^3}{3} + 2x - \frac{1}{x} + c \Rightarrow \frac{x^4 + 6x^2 - 3}{3x} + c$

3 $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

$= \int \left(x + 2 + \frac{1}{x}\right) dx$

$= \int x + \int 2 + \int \frac{1}{x} \, dx$

$= \frac{x^2}{2} + 2x + \log x$

4 $\int \frac{2x^2 - 3x - 11}{x} \, dx$

$\therefore \int \left(2x - 3 - \frac{11}{x}\right) dx$

$\therefore 2\int x - \int 3 - 11\int \frac{1}{x} \, dx$

$\therefore \frac{\cancel{2}x^2}{\cancel{2}} - 3x - 11 \log x + c$

$\therefore x^2 - 3x - 11 \log x + c$

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5 $\int \frac{3x^2+2x+5}{x}\,dx$

$\int \left(3x + 2 + \frac{5}{x}\right) dx$

$\therefore \quad \frac{3x^2}{2} + 2x + 5\log x + C$

6 $\int \frac{x^3+5x^2+4x+1}{x^2}\,dx$

$\therefore \quad \int \left(x^2 + 5 + \frac{4}{x} + \frac{1}{x^2}\right) dx$

$\therefore \quad \frac{x^3}{3} + 5x + 4\log x + \left(-\frac{1}{x}\right) + C$

$\therefore \quad \frac{x^3}{3} + 5x + 4\log x - \frac{1}{x} + C$

7 $\int \frac{1}{x^2+25}\,dx$

$\therefore \quad \int \frac{1}{x^2+5^2}\,dx$

$= \quad \frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C \qquad \left(\because \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C\right)$

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8: $\int \frac{1}{\sqrt{x^2-4}}\, dx$

$\therefore \int \frac{1}{\sqrt{x^2-2^2}}\, dx$

$\therefore \log\left|x+\sqrt{x^2-2^2}\right| + c$

$\left(\because \int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \log\left|x+\sqrt{x^2 \pm a^2}\right| + c\right)$

9 $\int \frac{1}{\sqrt{4-x^2}}\, dx$

$\therefore \int \frac{1}{\sqrt{2^2-x^2}}\, dx$

$\therefore \sin^{-1}\left(\frac{x}{2}\right) + c$ $\left(\because \int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right)$

10 $\int \frac{x^2}{x^2-9}\, dx$

$\therefore \int \cancel{x^2+9-9}$

$\therefore \int \frac{x^2-9+9}{x^2-9}\, dx$

$\therefore \int 1\, dx + \int \frac{9}{x^2-9}\, dx$

$\therefore \int 1\, dx + 9\int \frac{1}{x^2-9}\, dx$

$\therefore x + 9\int \frac{1}{x^2-3^2}\, dx$

$\therefore \cancel{x + 9\left(\frac{1}{3}\log\left(\frac{x}{3}\right)\right) + c}$

$\therefore \cancel{x + 3\log\frac{x}{3}}$

$\therefore \cancel{x + 9\left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right] + c}$

$\therefore \cancel{x + 3\tan^{-1}\left(\frac{x}{3}\right) + c}$

$x + 9\left[\frac{1}{2(3)}\log\left|\frac{x-3}{x+3}\right|\right] + c$

$\therefore x + \frac{3}{2}\log\left|\frac{x-3}{x+3}\right| + c.$

11 $\int \frac{x^2+5x+6}{x^2+2x}\,dx$

$\therefore \cancel{\int \frac{(x-3)(x-2)}{x^2+2x}\,dx}$

$\therefore \int \frac{(x+3)\cancel{(x+2)}}{x\cancel{(x+2)}}\,dx$

$\therefore \int \frac{x+3}{x}\,dx$

$\therefore \int \left(1+\frac{3}{x}\right)dx$

$\therefore x + 3\log x + c$

12 $\int \frac{x^4+x^2+1}{x^2+1}\, dx$

$\therefore \int \frac{x^4+x^2+1+x^2-x^2}{x^2+1}\, dx$

$\therefore \int \frac{x^4+2x^2+1-x^2}{x^2+1}\, dx$

$\therefore \int \frac{(x^2+1)^2-x^2}{x^2+1}\, dx$

$\therefore \int x^2+1\, dx - \int \frac{x^2}{x^2+1}\, dx$

$\therefore \frac{x^3}{3}+x - \int \frac{x^2+1-1}{x^2+1}\, dx$

$\therefore \frac{x^3}{3}+x - \int 1 - \frac{1}{x^2+1}\, dx$

$\therefore \frac{x^3}{3}+x - \left[x - \frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right] + C$

$\therefore \frac{x^3}{3}+\cancel{x}-\cancel{x}+\tan^{-1}x+C$

$\therefore \frac{x^3}{3}+\tan^{-1}x+C$

13 $\int \frac{\sec^2 x}{\operatorname{cosec} x} \, dx$

$\therefore \int \frac{1/\cos^2 x}{1/\sin x} \, dx$

$\therefore \int \frac{\sin x}{\cos^2 x} \, dx$

$\therefore \int \tan x \cdot \frac{1}{\cos x} \, dx$

$\therefore \int \tan x \cdot \sec x \, dx$

$\therefore \sec x + C$

14 $\int \tan^2 x \, dx$

Now as

$1 + \tan^2 x = \sec^2 x$

$\therefore \tan^2 x = \sec^2 x - 1$

$\therefore \int \sec^2 x - 1 \, dx$

$\therefore \tan x - x + C$

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15 $\int \frac{2+3\sin x}{\cos^2 x}\,dx$

$\therefore \int \frac{2}{\cos^2 x} + \int 3\frac{\sin x}{\cos^2 x}\,dx$

$\therefore \int 2\sec^2 x + \int 3\tan x \sec x\,dx$

$\therefore 2\tan x + 3\sec x + C$

16 $\int \frac{5+2\sin x}{\cos^2 x}\,dx$

$\therefore \int \frac{5}{\cos^2 x} + \frac{2\sin x}{\cos^2 x}\,dx$

$\therefore \int 5\sec^2 x + 2\tan x \sec x\,dx$

$\therefore 5\tan x + 2\sec x + C$

17 $\int \frac{4+3\cos x}{\sin^2 x}\,dx$

$\therefore \int 4\operatorname{cosec}^2 x + 3\cot x \operatorname{cosec} x\,dx$

$\therefore 4(-\cot x) + 3(-\operatorname{cosec} x) + C$

$\therefore -4\cot x - 3\operatorname{cosec} x + C$

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18 $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \times \cos^2 x} dx$

$\therefore \int \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} dx$

$\therefore \int \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} dx$

$\therefore \int \tan x \sec x + \cot x \operatorname{cosec} x \, dx$

$\therefore \sec x - \operatorname{cosec} x + C$

19 $\int \sec^2 x \cdot \operatorname{cosec}^2 x \, dx$

$\therefore \int \frac{1}{\cos^2 x \cdot \sin^2 x} dx$

$\therefore \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \cdot \sin^2 x} dx \quad (\because \sin^2 x + \cos^2 x = 1)$

$\therefore \int \frac{\sin^2 x}{\cos^2 x \cdot \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \cdot \sin^2 x} dx$

$\therefore \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx$

$\therefore \int \sec^2 x + \operatorname{cosec}^2 x \, dx$

$\therefore \tan x - \cot x + c$

20 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} \, dx$

$\therefore \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \times \sin^2 x}$ $\left(\because \cos 2x = \cos^2 x - \sin^2 x\right)$

$\therefore \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} \, dx$

$\therefore \int \operatorname{cosec}^2 x - \sec^2 x \, dx$

$\therefore \cancel{-\cot^2 x} \; -\cot x - \tan x + c$

21 $\int \sqrt{1 + \sin 2x} \, dx$

$\int \sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} \, dx$ $\left(\because \sin 2x = 2 \sin x \cos x, \ \sin^2 x + \cos^2 x = 1\right)$

$\therefore \int \sqrt{(\sin x + \cos x)^2} \, dx$ $\left(\because (a+b)^2 = a^2 + b^2 + 2ab\right)$

$\therefore \int \sin x + \cos x \, dx$

$\therefore \cancel{\cos x + \sin x + c}$

$\therefore -\cos x + \sin x + c$

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⇒ ~~Itegration~~ Integration by method of substitution

EX If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$, then find the expression of $f(x)$.

→ $\int f'(x) = f(x)$

$\therefore \int 4x^2 + 6x - 3 \, dx$

$\therefore 4\frac{x^3}{3} + 6\frac{x^2}{2} - 3x + C$

$\therefore \frac{4}{3}x^3 + 3x^2 - 3x + C = f(x)$

$\therefore f(1) = \frac{4}{3}(1)^3 + 3(1)^2 - 3(1) + C$

$\therefore 2 = \frac{4}{3} + 3 - 3 + C \qquad (\because f(1) = 2)$

$\therefore 2 - \frac{4}{3} = C$

$\therefore C = \frac{2}{3}$

$\therefore f(x) = \frac{4}{3}x^3 + 3x^2 - 3x + \frac{2}{3}$

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Ex If $\frac{dy}{dx} = 4x^2+6x-1$ and $y=5$ when $x=2$, then represent $y$ as a function of $x$.

$\rightarrow \frac{d}{dx}(y) = y'$ ( $y'$ = differentiation of $y$
or
$\frac{dy}{dx}$ = differentiation of $y$ )

$\therefore \int y' = y$

$\therefore \int 4x^2+6x-1 = y$

$\therefore 4\frac{x^3}{3}+6\frac{x^2}{2} - x = y$

$\therefore \frac{4}{3}x^3+3x^2-x+c = y$

$\therefore \frac{4}{3}(2)^3+3(2)^2-2+c = 5$

$\therefore \frac{32}{3}+12-2+c = 5$

$\therefore \frac{32}{3}+c = -5$

$\therefore c = -5-\frac{32}{3}$

$\therefore c = -\frac{47}{3}$

$\therefore$ ~~$y = \frac{4}{3}x^3+3x^2-x-\frac{37}{3}$~~

$y = \frac{4}{3}x^3+3x^2-x-\frac{47}{3}$

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$\Rightarrow$ formulas (Method of substitution)

1 $$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C}$$

Ex $\int \cot x\,dx$

$\therefore \int \frac{\cos x}{\sin x}\,dx$

now $\frac{d}{dx}\sin x = \cos x$

$\therefore \int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \log|\sin x| + C$

$\therefore \int \cot x = \log|\sin x| + C$

~~2 $\int [f(x)]^n\,dx =$~~

2 $$\boxed{\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)}$$

Ex $\int \sin^2 x \cdot \cos x\,dx$

so here $\frac{d}{dx}\sin x = \cos x$

$\therefore \int \sin^2 x \cdot \cos x\,dx = \frac{(\sin x)^{2+1}}{2+1} + C$

$= \frac{\sin^3 x}{3} + C$

3 $\boxed{\text{If } \int f(x)\,dx = F(x), \text{ then } \int f(ax+b)\,dx = \frac{1}{a}F(ax+b)+c}$

Ex $\int \frac{1}{x} = \log x + c$

$\therefore \int \frac{1}{ax+b} = \frac{1}{a} \log |ax+b| + c$

Ex $\int \sin x = -\cos x + c$

$\therefore \cancel{\int \sin(ax+b) = \frac{-1}{-a}\cos(ax+b) + c}$

$\therefore \int \sin(ax+b) = \frac{1}{a}\left[-\cos(ax+b)\right] + c$

$\therefore \int \sin(ax+b) = -\frac{1}{a}\cos(ax+b) + c$

4 $\int \tan x\,dx = \log|\sec x| + c = -\log|\cos x| + c$

5 $\int \cot x\,dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$

6 $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$

7 $\int \sec x\,dx = \log|\sec x + \tan x| + c$

=> Examples

1 $\int \frac{dx}{9+4x^2} dx$

$\therefore \int \frac{dx}{3^2+(2x)^2}$

$\therefore$ applying $\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

$\therefore \frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right) \times \frac{1}{2} + c$ ($\because$ see formula 3)

$\therefore \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + c$

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2 $\int \frac{dx}{16-25x^2}$

$\therefore \int \frac{dx}{4^2-(5x)^2}$

applying $\int \frac{1}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{x+a}{x-a}\right| + C$

$\therefore \frac{1}{2(4)} \log\left|\frac{5x+4}{5x-4}\right| \times \frac{1}{5} + C$

$\therefore \frac{1}{40} \log\left|\frac{5x+4}{5x-4}\right| + C$

3 $\int \frac{1}{\sqrt{16-9x^2}} dx$

$\therefore \int \frac{1}{\sqrt{4^2-(3x)^2}} dx$

$\therefore$ applying $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

$\therefore \sin^{-1}\left(\frac{3x}{4}\right) \times \frac{1}{3} + C$

$\therefore \frac{1}{3} \sin^{-1}\left(\frac{3x}{4}\right) + C$

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4 $\int \frac{3x^2-2x}{x+4}\,dx$

$$\int \frac{3x^2-2x+14x-14x}{x+4}\,dx$$

$$\therefore \int \frac{3x^2+12x-14x}{x+4}\,dx$$

$$\therefore \int \frac{3x(x+4)-14x}{x+4}\,dx$$

$$\therefore \int \frac{3x(x+4)-14x-56+56}{x+4}\,dx$$

$$\therefore \int \frac{3x(x+4)-14(x+4)+56}{x+4}\,dx$$

$$\therefore \int \frac{(3x-14)(x+4)+56}{x+4}\,dx$$

$$\therefore \int \left(3x-14+\frac{56}{x+4}\right)dx$$

$$\therefore \frac{3x^2}{2}-14x+56\left[\frac{1}{1}\log|x+4|+c\right]$$

$$\therefore \frac{3}{2}x^2-14x+56\log|x+4|+c$$

Some IMP trigonometric formulas

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$

$\sin(x+y) - \sin(x-y) = 2 \cos x \sin y$

$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cancel{\cos x \cos y}\ \cos x \cos y + \sin x \sin y$

$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$

$\cos(x+y) - \cos(x-y) = -2 \sin x \sin y$

$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

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5 $\int \sin 5x \sin 6x \, dx$

$\therefore \int \frac{-2}{-2} \sin 5x \sin 6x \, dx$

$\therefore -\frac{1}{2} \int -2 \sin 5x \sin 6x \, dx$

$\therefore -\frac{1}{2} \int \cos(5x+6x) - \cos(5x-6x) \, dx$

$\therefore -\frac{1}{2} \int \cancel{\frac{\sin 11x}{11} - \sin}$

$\therefore -\frac{1}{2} \int \cos(11x) - \cos(-x) \, dx$

$\therefore -\frac{1}{2} \int \cos(11x) - \cos x \, dx \qquad (\because \cos(-\theta) = \cos\theta)$

$\therefore -\frac{1}{2} \left[ \frac{\sin 11x}{11} - \sin x \right] + C$

6 $\int \sin 5x \sin 3x \, dx$

$\therefore \int \frac{-2}{-2} \sin 5x \sin 3x \, dx$

$\therefore -\frac{1}{2} \int -2 \sin 5x \sin 3x \, dx$

$\therefore -\frac{1}{2} \int \cos(5x+3x) - \cos(5x-3x) \, dx$

$\therefore -\frac{1}{2} \int \cos 8x - \cos 2x \, dx$

$\therefore -\frac{1}{2} \left[ \frac{\sin 8x}{8} - \frac{\sin 2x}{2} \right] + C$

$\cancel{\therefore -\frac{\sin 8x}{16}}$

7 $\int \cos 5x \sin 2x \, dx$

$\therefore \int \frac{2}{2} \cos 5x \sin 2x \, dx$

$\therefore \frac{1}{2} \int 2 \cos 5x \sin 2x \, dx$

$\therefore \frac{1}{2} \int \sin(5x+2x) - \sin(5x-2x) \, dx$

$\therefore \frac{1}{2} \int \sin 7x - \sin 3x \, dx$

$\therefore \frac{1}{2} \left[ -\frac{\cos 7x}{7} - \left( -\frac{\cos 3x}{3} \right) \right] + C$

$\therefore \frac{1}{2}\left[\frac{\cos 3x}{3} - \frac{\cos 7x}{7}\right] + c$

8

~~...~~

8 $\int (2x+1)\sqrt{x^2+x+9}\ dx$

$\therefore \int \sqrt{x^2+x+9}\ (2x+1)\ dx$

$\therefore \int (x^2+x+9)^{\frac{1}{2}} (2x+1)\ dx$

Now $\frac{d}{dx}(x^2+x+9) = 2x+1$

$\therefore \int (x^2+x+9)^{\frac{1}{2}} (2x+1)\ dx = \frac{(x^2+x+9)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$ ($\because$ see formula 2)

$\therefore \frac{(x^2+x+9)^{\frac{3}{2}}}{3/2} + c = \frac{2}{3}(x^2+x+9)^{3/2} + c$

9 $\int \frac{x+2}{\sqrt{x^2+4x+3}}\,dx$

$\therefore \int (x+2)\left(\sqrt{x^2+4x+3}\right)^{-1} dx$

$\therefore \frac{1}{2}\int 2(x+2)\,(x^2+4x+3)^{\frac{-1}{2}}\,dx$

$\therefore \frac{1}{2}\int (2x+4)\,(x^2+4x+3)^{\frac{-1}{2}}\,dx$

$\therefore$ Now $\frac{d}{dx}(x^2+4x+3) = 2x+4$

$\therefore I = \frac{1}{2}\frac{[x^2+4x+3]^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$

$= \cancel{\frac{1}{2}}\frac{(x^2+4x+3)^{1/2}}{\cancel{1/2}} + c = \sqrt{x^2+4x+3} + c$

10 $\int \frac{\log x}{x}\, dx$

$\therefore \int \frac{1}{x}(\log x)\, dx$

$\therefore$ Now $\frac{d}{dx}(\log x) = \frac{1}{x}$

$\therefore I = \frac{(\log x)^2}{2} + C$

11 $\int \cos x \sqrt{\sin x}\, dx$

$\int \cos x (\sin x)^{\frac{1}{2}}\, dx$

$I = \frac{(\sin x)^{\frac{3}{2}}}{3/2} + C = \frac{2}{3}(\sin x)^{\frac{3}{2}} + C$

12 $\int \sin^5 x \cos x\, dx$

$\therefore = \frac{(\sin x)^{5+1}}{5+1} + C$

$= \frac{(\sin x)^6}{6}$

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13 $\int \tan^3 x \, dx$

$\therefore \int \tan^2 x \cdot \tan x \, dx$

$\therefore \int (\sec^2 x - 1) \cdot \tan x \, dx$

$\therefore \int \sec^2 x \cdot \tan x - \tan x \, dx$

$\therefore \int \sec^2 x \cdot \tan x - \int \tan x$

$\therefore \cancel{\sec} \frac{(\tan x)^2}{2} - \log(\sec x) + C \qquad \left(\because \frac{d}{dx}(\tan x) = \sec^2 x\right)$

14 $\int \frac{2x+3}{x^2+3x-2} \, dx$

Now here $\frac{d}{dx}(x^2+3x-2) = 2x+3$

$\therefore$ applying $\int \frac{f'(x)}{f(x)} \, dx = \log|f(x)| + C$

$\therefore I = \log|x^2+3x-2| + C$

15 $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$

here $\frac{d}{dx}(e^x + e^{-x}) = e^x + (-e^{-x}) = e^x - e^{-x}$

$\therefore I = \log|e^x + e^{-x}| + C$

16 $\int \frac{1}{1+e^{-x}}\,dx$

$$= \int \frac{1}{1+\frac{1}{e^x}}\,dx$$

$$= \int \frac{1}{\frac{e^x+1}{e^x}}\,dx$$

$$= \int \frac{e^x}{e^x+1}\,dx = \log|e^x+1| + c$$

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17 $\int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

here $\frac{d}{dx}(\cos x + \sin x) = -\sin x + \cos x$

$\therefore I = \log|\cos x + \sin x| + c$

18 $\int \frac{\sin x \cos x}{1 + \sin^2 x} dx$

here $\frac{d}{dx}(1 + \sin^2 x) = 0 + 2\sin x \cos x$

$\therefore \frac{1}{2}\int \frac{2\sin x \cos x}{1 + \sin^2 x} dx$

$= \frac{1}{2}\log|1 + \sin^2 x| + c$

=> Examples

18 $\int e^{x^2} \cdot x \, dx$

Let $x^2 = t$ eq ①

$\therefore \frac{d}{dx} x^2 = \frac{d}{dx}(t)$

$\therefore 2x = \frac{dt}{dx}$

$\therefore x\,dx = \frac{dt}{2}$ ②

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$\therefore$ putting eq ① and ②

$\therefore \int e^t \frac{dt}{2}$

$\therefore \int \frac{e^t}{2} dt$

$\therefore \frac{1}{2} \int e^t dt$

$\therefore = \frac{1}{2} e^t + C$

$\therefore \frac{1}{2} e^{x^2} + C$

2 $\int \frac{x^2}{1+x^6} dx$

Let $x^3 = t$

$\therefore 3x^2 = \frac{dt}{dx} \Rightarrow x^2 dx = \frac{dt}{3}$

$\therefore$ ~~$\int \frac{t}{1+t^3}$~~ $\int \frac{x^2 dx}{1+x^6}$

$\therefore \int \frac{1}{1+t^2} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{1}{1+t^2} dt$

$$= \frac{1}{3} \int \frac{1}{1^2+t^2} \, dt$$

$$= \frac{1}{3} \times \frac{1}{1} \tan^{-1}\left(\frac{t}{1}\right) + C \qquad \left(\because \int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right)$$

$$\cancel{= \frac{1}{3} \log \left| \frac{1+t}{1-t} \right| + c}$$

$$\therefore \frac{1}{3} \tan^{-1} x^3 + C$$

3 $$\int \frac{x^5}{1+x^{12}} \, dx$$

$$\therefore \text{Let } x^6 = t$$

$$\therefore 6x^5 = \frac{dt}{dx}$$

$$\therefore x^5 dx = \frac{dt}{6}$$

$$\therefore \int \frac{x^5 dx}{1+x^{12}} \, dx = \int \frac{dt/6}{1+t^2}$$

$$= \frac{1}{6} \int \frac{1}{1+t^2} \, dx$$

$$= \frac{1 \times 1}{6 \quad 1} \tan^{-1}\left(\frac{t}{1}\right) + C$$

$$= \frac{1}{6} \tan^{-1} x^6 + C$$

4 $\int e^{x} \sin e^{x} \, dx$

$\therefore$ Let $e^{x} = t$

$\therefore e^{x} = \frac{dt}{dx}$

$\therefore e^{x} dx = dt$

$\therefore \int (\sin e^{x}) \, e^{x} dx$

$\therefore \int \sin t \, dt$

$= -\cos t + c$

$= -\cos e^{x} + c$

5 $\int e^{\sin x} \cos x \, dx$

Let $\sin x = t$

$\therefore \cos x = \frac{dt}{dx}$

$\therefore \cos x \, dx = dt$

$\therefore \int e^{t} dt$

$= e^{t} + c$

$= e^{\sin x} + c$

6 $\int e^{\tan x} \sec^2 x \, dx$

$\therefore \tan x = t$

$\therefore \sec^2 x = \frac{dt}{dx}$

$\therefore \sec^2 x \, dx = dt$

$\therefore \int e^t \, dt$

$= e^t + c$

$= e^{\tan x} + c$

$\boxed{\text{Note : } \int e^{f(x)} \cdot f'(x) \, dx = e^{f(x)} + c}$

$\boxed{\text{Note : } \int e^x [f(x) + f'(x)] \, dx = e^x \cdot f(x) + c}$

7 $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

$\therefore \int e^{\tan^{-1}x} \times \frac{1}{1+x^2}\,dx$

Now applying $\int e^{f(x)} \cdot f'(x) = e^{f(x)} + c$

$\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right)$

$\therefore I = e^{\tan^{-1}x} + c$

8 $\int \frac{\cos(\log x)}{x}\,dx$

Let $\log x = t$

$\therefore \frac{d}{dx}(\log x) = \frac{dt}{dx}$

$\therefore \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{dx}{x} = dt$

$\therefore \int \cos t \times \frac{dx}{x}$

$\therefore \int \cos t \, dt$

$= \sin t + C$

$\therefore \sin(\log x) + C$

9 $\int \frac{\sin(\log x)}{x} dx$

$\therefore \log x = t$

$\therefore \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{dx}{x} = dt$

$\therefore \int \sin t \, dt$

$= -\cos t + C$

$= -\cos(\log x) + C$

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10 $\int \frac{(1+x)e^x}{\cos^2(xe^x)}\, dx$

Let $xe^x = t$

$\therefore \frac{d}{dx}(xe^x) = \frac{dt}{dx}$

$\therefore x(e^x)' + e^x(x)' = \frac{dt}{dx}$

$\therefore xe^x + e^x = \frac{dt}{dx}$

$\therefore e^x(x+1) = \frac{dt}{dx}$

$\therefore e^x(x+1)\, dx = dt$

$\therefore \int \frac{dt}{\cos^2 t}$

$\therefore \int \sec^2 t \, dt$

$= \tan t + C$

$\therefore \tan(xe^x) + C$

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11 $\int \frac{(1+x)e^{x}}{\sin^2(xe^{x})}\,dx$

$\therefore$ Let $xe^{x} = t$

$\therefore xe^{x} + e^{x} = \frac{dt}{dx}$

$\therefore e^{x}(x+1)\,dx = dt$

$\therefore \int \frac{dt}{\sin^2 t}$

$\therefore \int \operatorname{cosec}^2 t\,dt$

$= -\cot t + C$

$= -\cot(xe^{x}) + C$

12 $\int x^3 \tan^5(x^4)\,\sec^2(x^4)\,dx$

$\therefore x^4 = t$

$\therefore 4x^3 = \frac{dt}{dx} \Rightarrow 4x^3\,dx = dt$

$\therefore \frac{1}{4}\int 4x^3 \tan^5(x^4)\,\sec^2(x^4)\,dx$

$\therefore \frac{1}{4}\int \tan^5(x^4)\,\sec^2(x^4)\,4x^3\,dx$

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$\therefore \frac{1}{4}\int \tan^5 t \cdot \sec^2 t \, dt$

Now $(\tan t)' = \sec^2 t$

$\therefore$ applying $\int (f(x))^n \cdot f'(x) \, dx = \frac{(f(x))^{n+1}}{n+1} + C$

$\therefore \frac{1}{4}\left[\frac{(\tan t)^{5+1}}{5+1}\right] + C$

$\therefore \frac{1}{4} \frac{\tan^6 t}{6} + C$

$\therefore \frac{1}{24} \tan^6 t + C$

$\therefore \frac{1}{24} \tan^6(x^4) + C$

13 $$\int \frac{x^2 \tan^{-1}(x^3)}{1+x^6}\, dx$$

$$\text{Let } x^3 = t$$

$$\therefore 3x^2 = \frac{dt}{dx} \Rightarrow 3x^2 dx = dt$$

$$\therefore \frac{1}{3}\int \frac{\tan^{-1} t}{1+\cancel{x^6}}$$

$$\therefore \frac{1}{3}\int \frac{\tan^{-1}(t)}{1+t^2} \times 3x^2 dx$$

$$\therefore \frac{1}{3}\int \tan^{-1}(t) \times \frac{1}{1+t^2}\, dt$$

$$\therefore \frac{1}{3}\,\frac{(\tan^{-1} t)^2}{2} + c$$

$$\therefore \frac{1}{6}\left(\tan^{-1}(x^3)\right)^2 + c$$

⇒ LIATE Rule (Integration by part)

1 L - Logarithmic function : $\log x, \ldots$

2 I - Inverse function: $\tan^{-1}, \sec^{-1}x, \sin^{-1}x, \ldots$

3 A - Arithmatic function: $x^2+x, x^3, x^2, x^6, 1, 2, 3, \ldots$

4 T - Trigonomatric function: $\sin x, \cos x, \tan x, \ldots$

5 E - Exponential function : $e^x, e^{2x}, \ldots$

6 $a^x, a^{2x}, \ldots$

Ex $\int x e^x dx$

Arithmetic function

exponential function

$\therefore$ $1^{st}$ function $= x$

$2^{nd}$ function $= e^x$

Ex $\int e^x \log x \, dx$

exponential function

logarithmic function

$\therefore$ $1^{st}$ function = $\log x$
$2^{nd}$ function = $e^x$

Ex $\int x \tan^{-1} x$

Arithmatic function

Inverse function

$\therefore$ $1^{st}$ function = $\tan^{-1} x$
$2^{nd}$ function = $x$

→ $1^{st}$ function is denoted by '$u$'
→ $2^{nd}$ function is denoted by '$v$'

$$\cancel{u\int v\,dx - \int\left(\frac{du}{dx} \times \int v\,dx\right)dx}$$

$$\boxed{u\int v\,dx - \int\left(\frac{du}{dx} \times \int v\,dx\right)dx}$$

⇒ Examples

1 $\int x e^{x} \, dx$

$u = x$
$v = e^{x}$

$\therefore \quad u \int v \, dx - \int \left( \frac{du}{dx} \times \int v \, dx \right) dx$

$\therefore \quad x \int e^{x} dx - \int \left( (x)' \times \int e^{x} dx \right) dx$

$\therefore \quad x e^{x} - \int (1 \times e^{x}) \, dx$

$x e^{x} - e^{x} + c$

$\therefore \quad e^{x}(x - 1) + c$

2 $\int x^2 e^{x} \, dx$

$u = x^2$
$v = e^{x}$

$\therefore \quad x^2 \int e^{x} dx - \int \left( (x^2)' \times \int e^{x} dx \right) dx$

$\therefore \quad x^2 e^{x} - \int (2x \times e^{x}) \, dx$

$\therefore \quad x^2 e^{x} - 2 \int \underset{u}{x} \, \underset{v}{e^{x}} \, dx$

$\therefore\ x^2e^x - 2\left[x\int e^x dx - \int\left((x)'\times\int e^x dx\right)dx\right]$

$x^2e^x - 2\left[xe^x - \int(1\times e^x)dx\right]$

$x^2e^x - 2\left[xe^x - e^x\right] + c$

$\therefore\ x^2e^x - 2xe^x + 2e^x + c$

$e^x(x^2 - 2x + 2) + c$

3 $\int x e^{3x}\,dx$

$\therefore\ x\int e^{3x}dx - \int\left((x)'\times\int e^{3x}dx\right)dx$

$\therefore\ x\times\frac{1}{3}e^{3x} - \int\left(1\times\frac{1}{3}e^{3x}\right)dx$

$\therefore\ \frac{xe^{3x}}{3} - \int\frac{e^{3x}}{3}dx$

$\therefore\ \frac{1}{3}\left[xe^{3x} - \int e^{3x}dx\right]$

$\therefore\ \frac{1}{3}\left[xe^{3x} - \frac{1}{3}e^{3x}\right] + c$

$\therefore\ \cancel{\frac{e^{3x}}{3}\left[x - \frac{1}{9}\right] + c} = \frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + c$

4 $\int x^2 e^{mx}$

$\therefore x^2 \int e^{mx} dx - \int \left( (x^2)' \times \int e^{mx} dx \right) dx$

$\therefore \frac{x^2 e^{mx}}{m} - \int \left( 2x \times \frac{e^{mx}}{m} \right) dx$

$\therefore \frac{x^2 e^{mx}}{m} - \frac{2}{m} \int x e^{mx} dx$

$\therefore \frac{x^2 e^{mx}}{m} - \frac{2}{m} \left[ x \int e^{mx} - \int \left( (x)' \times \int e^{mx} dx \right) dx \right]$

$$\frac{x^2 e^{mx}}{m} - \frac{2}{m}\left[\frac{x e^{mx}}{m} - \int \frac{e^{mx}}{m}\,dx\right]$$

$$\therefore \frac{x^2 e^{mx}}{m} - \frac{2}{m}\left[\frac{x e^{mx}}{m} - \frac{e^{mx}}{m^2}\right] + C$$

5 $\int x \cos x \, dx$

$$\therefore \quad x\int \cos x\,dx - \int\left((x)' \times \int \cos x\,dx\right)dx$$

$$\therefore x \sin x - \int \sin x \, dx$$

$$x \sin x - (-\cos x) + C$$

$$\therefore \quad x \sin x + \cos x + C$$

6 $\int x \sin x$

$$\therefore x\int \sin x\,dx - \int\left((x)' \times \int \sin x\,dx\right)dx$$

$$\therefore -x \cos x - \int -\cos x \, dx$$

$$\therefore -x\cos x - (-\sin x) + C$$

$$\therefore \quad \sin x - x\cos x + C$$

7 $\int x^2 \sin x$

$\therefore x^2 \int \sin x\,dx - \int \left( (x^2)' \times \int \sin x\,dx \right) dx$

$\therefore -x^2 \cos x - \int -2x \cos x\,dx$

$\therefore -x^2 \cos x + 2\int x \cos x\,dx$

$\therefore -x^2 \cos x + 2\left[ x \int \cos x - \int \left( (x)' \times \int \cos x\,dx \right) dx \right]$

$\therefore -x^2 \cos x + 2\left[ x \sin x - \int \sin x\,dx \right]$

$-x^2 \cos x + 2\left[ x \sin x + \cos x \right] + C$

8 $\int x \log x$

$\therefore \cancel{x \int \log x\,dx - \int \left( (x)' \times \int \log x\,dx \right)} dx$

$\therefore \cancel{x}$ $\log x \int x\,dx - \int \left( (\log x)' \times \int x\,dx \right) dx$

$\therefore \log x \times \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2}\,dx$

$\therefore \frac{x^2 \log x}{2} - \frac{1}{2}\int x\,dx$

$\therefore \frac{x^2 \log x}{2} - \frac{1}{4}x^2 + C = \frac{x^2}{2}\left[ \log x - \frac{1}{2} \right] + C$

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ADURIT

9 $\int \log x \, dx$

$\therefore \int \log x \times 1 \, dx$

$\therefore u = \log x$

$v = 1$

$\therefore \frac{du}{dx} = \frac{1}{x}$

$\cancel{\frac{dv}{dx}} = \int v \, dx = x$

$\therefore u \int v \, dx - \int \left( (u)' \times \int v \, dx \right) dx$

$\therefore \log x \times x - \int \left( \frac{1}{x} \times x \right) dx$

$\therefore x \log x - \int 1 \, dx$

$x \log x - x + C$

$\therefore x(\log x - 1) + C$

10 $\int \cos^{-1} x \, dx$

$\int \cos^{-1} x \times 1 \, dx$

$\therefore u = \cos^{-1} x$ $\frac{du}{dx} =$ [illegible]

$v = 1$

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\int v\,dx = x$$

$$\therefore\ u\int v\,dx - \int\left((u)' \times \int v\,dx\right)dx$$

$$\therefore\ \cancel{\cos^{-1}x \times x - \theta \cos \int \frac{-1}{\sqrt{1-x^2}}}$$

$$\cos^{-1}x\, x - \int\left(\frac{-1}{\sqrt{1-x^2}} \times x\right)dx$$

$$\therefore\ x\cos^{-1}x - \int \frac{-x}{\sqrt{1-x^2}}\,dx$$

Let $1-x^2 = t$

$\therefore -2x = \frac{dt}{dx}$

$\therefore -2x\,dx = dt$

$\therefore x\cos^{-1}x - \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$

$\therefore x\cos^{-1}x - \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$

$\therefore x\cos^{-1}x - \frac{1}{2}\int 1 \times (\sqrt{t})^{-1}\,dt$

$\therefore x\cos^{-1}x - \frac{1}{2}\int 1 \times (t)^{-1/2}\,dt$

Now $\frac{d}{dt}(t) = 1$

$\therefore x\cos^{-1}x - \frac{1}{2}\left[\frac{(t)^{-1/2+1}}{-\frac{1}{2}+1}\right] + C$

$\therefore x\cos^{-1}x - \cancel{\frac{1}{2}}\left[\frac{(t)^{1/2}}{\cancel{1/2}}\right] + C$

$\therefore x\cos^{-1}x - \sqrt{t} + C$

$\therefore x\cos^{-1}x - \sqrt{1-x^2} + C$

11 $\int x \tan^{-1} x \, dx$

$u = \tan^{-1} x$

$v = x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$\int v \, dx = \frac{x^2}{2}$

$\therefore \; u \int v \, dx - \int \left( (u)' \times \int v \, dx \right) dx$

$\therefore \; \tan^{-1} x \left( \frac{x^2}{2} \right) - \int \left( \frac{1}{1+x^2} \times \frac{x^2}{2} \right) dx$

$\therefore \; \cancel{\frac{x^2 \tan^{-1} x}{2}} - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx$

$\frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx$

$\frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{1+x^2-1}{1+x^2} \, dx$

$\therefore \; \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int 1 - \frac{1}{1+x^2} \, dx$

$$\frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left[x - \frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right]$$

$$\therefore \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left[x - \tan^{-1}x\right] + C$$

~~$x^2\tan^{-1}x - x + \tan^{-1}$~~

⇒ formulas

1 $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$

2 $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + C$

3 $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + C$

4 $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

5 $\int e^{ax}\sin(bx+k)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+k) - b\cos(bx+k)\right] + C$

6 $\int e^{ax}\cos(bx+k)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+k) + b\sin(bx+k)\right] + C$

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⇒ Examples

1 $\int e^{x}\left(\frac{1}{x}-\frac{1}{x^2}\right) dx$

$\therefore$ here $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$\therefore$ applying $\int e^{x}(f(x) + f'(x)) dx = e^{x} f(x) + C$

$\therefore I = e^{x}\left(\frac{1}{x}\right) + C$

2 $\int (\sin x + \cos x) e^{x} dx$

$= e^{x} \sin x + C$

3 $\int x(x+2)e^{x} dx$

$\therefore \int (x^2+2x)e^{x} dx$

$= e^{x} \cdot x^2 + C$

4 $\int (\log(\sin x) + \cot x) e^{x} dx$

$\therefore \frac{d}{dx}(\log(\sin x)) = \frac{1}{\sin x} \times \frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$

$\therefore I = e^{x} \log(\sin x) + C$

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5 $\int \frac{x}{(x+1)^2} e^{x} dx$

$\therefore \int \left[\frac{x+1-1}{(x+1)^2}\right] e^{x} dx$

$\therefore \int \left[\frac{x+1}{(x+1)^2} - \frac{1}{(x+1)^2}\right] e^{x} dx$

$\therefore \int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] e^{x} dx$

$\therefore$ Now $\frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{d}{dx}(x+1)^{-1}$

$= -(x+1)^{-2} = \frac{-1}{(x+1)^2}$

$\therefore I = \left(\frac{-1}{(x+1)^2}\right) e^{x} + C$

6 $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

$\therefore \int e^x \left(\frac{1+\sin\left(2\frac{x}{2}\right)}{1+\cos\left(2\frac{x}{2}\right)}\right) dx$

$\therefore \int e^x \left(\frac{1+2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos\left(\frac{x}{2}\right)^2}\right) dx$ $\quad$ $\left(\because \sin 2x = 2\sin x \cos x\right.$
$\left.\cos 2x = 2\cos^2 x - 1\right)$

$\therefore \int e^x \left(\frac{1}{2}\sec^2\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)\right) dx$

Now $\frac{d}{dx}\left(\tan\frac{x}{2}\right) = \sec^2 x \frac{d}{dx}\left(\frac{x}{2}\right)$

$= \sec^2 x \cdot \frac{1}{2}$

$\therefore I = e^x \cdot \tan\left(\frac{x}{2}\right) + C$

7 $\int x^3 e^x dx$

$\int e^x (x^3) dx$

$\int e^x (x^3 + 3x^2 - 3x^2) dx$ $\quad$ $\left(\because (x^3)' = 3x^2\right)$

$\int e^x (x^3 + 3x^2 - 3x^2 + 6x - 6x) dx$ $\quad$ $\left(\because (3x^2)' = 6x\right)$

$\int e^x (x^3 + 3x^2 - 3x^2 + 6x - 6x + 6 - 6) dx$ $\quad$ $\left(\because (6x)' = 6\right)$

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$$\int \left(e^x(x^3+3x^2) + e^x(-3x^2-6x) + e^x(6x+6)\right) dx$$

$$\therefore I = e^x x^3 + e^x(-3x^2) + e^x 6x + C$$

$$= x^3 e^x - 3x^2 e^x + 6x e^x + C$$

⇒ quadratic equation — only for understanding —

$\cancel{ax^2 + 2ab + by^2}$

middle term

$$a^2 + 2ab + y^2 = (a+b)^2$$

$$a^2 - 2ab + y^2 = (a-b)^2$$

1st Term — middle Term — Last Term

~~Last Term~~

$$\text{Last Term (L.T.)} = \frac{(\text{Middle Term (M.T.)})^2}{4 \times (\text{first term (F.T.)})}$$

$$L.T. = \frac{(M.T.)^2}{4 \times F.T.}$$

⇒ Examples

1 $\int \sqrt{4x^2+4x-15}\, dx$

here

$FT = 4x^2$

$MT = 4x$

$LT = \frac{(MT)^2}{4 \times FT} = \frac{(4x)^2}{4 \times 4x^2} = \frac{16x^2}{16x^2} = 1$

$\therefore \int \sqrt{4x^2+4x+1-1-15}\, dx$

$\therefore \int \sqrt{(2x+1)^2-16}\, dx$

$\therefore \int \sqrt{(2x+1)^2-4^2}\, dx$

$\therefore$ Now applying $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + C$

$\therefore \frac{1}{2}\left[\frac{2x+1}{2}\left(\sqrt{(2x+1)^2-4^2}\right) - \frac{4^2}{2}\left(\log|2x+1+\sqrt{(2x+1)^2-4^2}|\right)\right] + C$

$\therefore \frac{2x+1}{4}\sqrt{4x^2+4x-15} - 4\left(\log|2x+1+\sqrt{4x^2+4x-15}|\right) + C$

2 $\int \sqrt{8-2x-x^2}\, dx$

$\therefore \int \sqrt{8-(2x+x^2)}\, dx$

here

$F.T = x^2$

$M.T = 2x$

$L.T. = \frac{(M.T)^2}{4 \times F.T.} = \frac{(2x)^2}{4 \times x^2} = 1$

$\therefore \int \sqrt{8-(2x+x^2+1-1)}\, dx$

$\therefore \int \sqrt{8-(x+1)^2+1}\, dx$

$\therefore \int \sqrt{9-(x+1)^2}\, dx$

$\int \sqrt{3^2-(x+1)^2}\,dx$

$\therefore$ applying $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$

$\therefore \frac{x+1}{2}\left(\sqrt{3^2-(x+1)^2}\right)+\frac{3^2}{2}\sin^{-1}\left(\frac{x+1}{3}\right)+C$

$\therefore \frac{x+1}{2}\left(\sqrt{8-2x-x^2}\right)+\frac{9}{2}\sin^{-1}\left(\frac{x+1}{3}\right)+C$

3 $\int \sqrt{(x-3)(7-x)}\,dx$

$\therefore \int \sqrt{7x-x^2-21+3x}\,dx$

$\therefore \int \sqrt{-x^2+10x-21}\,dx$

$\therefore \cancel{\int \sqrt{-(x^2-10x)-21}\,dx}$

$\int \sqrt{-(x^2-10x+21)}\,dx$

$\therefore$ LT $= \frac{(10x)^2}{4\times x^2} = 25$

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$\therefore \int \sqrt{-(x^2-10x+25-25+21)}\ dx$

$\therefore \int \sqrt{-((x+5)^2-4)}\ dx$

$\therefore \int \sqrt{2^2-(x+5)^2}\ dx$

$\therefore I = \frac{x+5}{2}\left(\sqrt{4-(x+5)^2}\right) + \frac{2^2}{2}\sin^{-1}\left(\frac{x+5}{2}\right) + C$

$\therefore I = \frac{x+5}{2}\left(\sqrt{4-(x+5)^2}\right) + 2\sin^{-1}\left(\frac{x+5}{2}\right) + C$

or

$\frac{x+5}{2}\left(\sqrt{(x-3)(7-x)}\right) + 2\sin^{-1}\left(\frac{x+5}{2}\right) + C$

4 $\int 2x\sqrt{x^4+1}\ dx$

Let $x^2 = t$

$\therefore 2x = \frac{dt}{dx}$ $\therefore 2x\,dx = dt$

$\therefore \int \left(\sqrt{(t)^2+1}\right) 2x\,dx$

$$\therefore \int \sqrt{t^2+1}\, dt$$

$$= \frac{t}{2}\left(\sqrt{t^2+1}\right) + \frac{t^2}{2} \log\left| t + \sqrt{t^2+1} \right| + C$$

$$= \frac{x^2}{2}\left(\sqrt{x^4+1}\right) + \frac{x^4}{2} \log\left| x^2 + \sqrt{x^4+1} \right| + C$$

⇒ Definite Integral: ~~(no + c)~~ (no + c)

Examples

1 $\int_{2}^{5} (x^2 + 2x + 1)\, dx$

$\therefore \cancel{\left[\frac{x^3}{3}\right.} \left[\frac{x^3}{3} + 2\frac{x^2}{2} + x\right]_2^5$

$\therefore \left[\frac{x^3}{3} + x^2 + x\right]_2^5$

$\therefore \left[\frac{5^3}{3} + 5^2 + 5\right] - \left[\frac{2^3}{3} + 2^2 + 2\right]$

$\left[\frac{125}{3} + 25 + 5\right] - \left[\frac{8}{3} + 4 + 2\right]$

$\left[\frac{125 + 90}{3}\right] - \left[\frac{8 + 18}{3}\right]$

$\therefore \left[\frac{215}{3}\right] - \left[\frac{26}{3}\right]$

$\therefore \frac{189}{3} = 63$

2 $\int_1^3 (2x^2+5x+1)\,dx$

$$\left[\frac{2x^3}{3}+\frac{5x^2}{2}+x\right]_1^3$$

$$\therefore \left[\frac{2(3)^3}{3}+\frac{5(3)^2}{2}+3\right]-\left[\frac{2}{3}+\frac{5}{2}+1\right]$$

$\therefore\ 18 + \frac{45}{2} + 3 - \frac{2}{3} - \frac{5}{2} - 1$

$\therefore\ \frac{40}{2} - \frac{2}{3} + 20$

$\therefore\ 20 + 20 - \frac{2}{3}$

$= 40 - \frac{2}{3}$

$= \frac{118}{3}$

Note:

$a^3 + b^3 = (a+b)(a^2 - 2ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + 2ab + b^2)$

3 $\int_{-1}^{1} \frac{x^3 - 8}{x-2}\, dx$

$\int_{-1}^{1} \cancel{x^{3}-8}\ \frac{x^3 - 2^3}{x-2}\, dx$

$\therefore\ \int_{-1}^{1} \frac{\cancel{(x-2)}(x^2 + 2x + 2^2)}{\cancel{x-2}}\, dx$

$\therefore\ \int_{-1}^{1} x^2 + 2x + 4\ dx$

$\therefore\ \left[ \frac{x^3}{3} + x^2 + 4x \right]_{-1}^{1}$

$\therefore\ \frac{1}{3} + 1 + 4 - \left[ -\frac{1}{3} + 1 - 4 \right]$

$\frac{1}{3} + 5 + \frac{1}{3} + 3$

$\frac{2}{3} + 8 = \frac{26}{3}$

4 $\int_0^1 \frac{2}{1+x^2}\, dx$

$2\left[\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right]_0^1$

$\therefore 2\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$2\left[45 - 0\right]$ or $2\left[\frac{\pi}{4} - 0\right]$

$= 90$ or $\frac{\pi}{2}$

5 $\int_0^1 \frac{x}{x+1}\, dx$

$\int_0^1 \frac{x+1-1}{x+1}\, dx$

$\int_0^1 1 - \frac{1}{x+1}\, dx$

$\left[x - \log(x+1)\right]_0^1$

$= 0$

$1 - \log 2 - \left[0 - \log 1\right] \Rightarrow \underline{1 - \log 2}$

5 $\int_{-4}^{-3} \frac{x}{7+x} \, dx$

$\therefore \int_{-4}^{-3} \frac{x+7-7}{7+x} \, dx$

$\therefore \int_{-4}^{-3} 1 - \frac{7}{x+7} \, dx$

$\therefore \left[ x - 7\log(x+7) \right]_{-4}^{-3}$

$= -3 - 7\log 4 - \left[ -4 - 7\log 3 \right]$

$\therefore -3 - 7\log 4 + 4 + 7\log 3$

$1 - 7(\log 4 + \log 3)$

6 $\int_{0}^{\pi/4} \cos^2 x \, dx$

$\therefore \int_{0}^{\pi/4} \frac{\cos 2x + 1}{2} \, dx \quad (\because \cos 2x = 2\cos^2 x - 1)$

$\therefore \frac{1}{2} \left[ \frac{1}{2} (\sin 2x) + x \right]_{0}^{\pi/4}$

$\left[ \frac{1}{4} \sin 2x + \frac{x}{2} \right]_{0}^{\pi/4}$

$\frac{1}{4}\sin\frac{\pi}{2} + \frac{\pi}{6} - \left[\frac{1}{4}\sin 0 + 0\right]$

$\frac{1}{4}(1) + 30 - [0+0]$

$\therefore \frac{121}{4}^{\circ}$

7 $\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx = \int_0^1 \tan^{-1}x \, \frac{1}{1+x^2}\,dx$

$\therefore$ ~~$\frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \tan^{-1}x$~~

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$\therefore I = \left[\frac{(\tan^{-1}x)^2}{2}\right]_0^1$

$\therefore \frac{(\tan^{-1}1)^2}{2} - \frac{(\tan^{-1}0)^2}{2}$

$\frac{(\pi/4)^2}{2} = \frac{(45)^2}{2} - 0 = \frac{2025}{2}^{\circ} = \frac{\pi^2}{32}$

8 $\int_1^e \frac{(\log x)^2}{x} dx$

$\therefore \int_1^e (\log x)^2 \times \frac{1}{x} \, dx$

$\therefore \left[\frac{(\log x)^3}{3}\right]_1^e$

$\frac{(\log e)^3}{3} - \frac{(\log 1)^3}{3}$

$= \frac{(1)^3}{3} - \frac{(0)^3}{3} = \frac{1}{3}$

9 $\int_1^e \log x \, dx = \int_1^e 1 \times \log x \, dx$

$u = \log x$ , $\frac{du}{dx} = \frac{1}{x}$

$v = 1$ $\qquad \int v \, dx = x$

$\therefore u \int v \, dx - \int \left((u)' \times \int v \, dx\right) dx$

$\therefore \cancel{\log x \int}$ $\log x \cdot x - \int \left(\frac{1}{x} \times x\right) dx$

$\left[x \log x - x\right]_1^e$

$e \log e - e - [\log 1 - 1]$

$\therefore \quad e - e - [0 - 1]$

$0 + 1 = 1$

10 $\int_0^1 x \tan^{-1} x \, dx$

$\therefore \quad u = \tan^{-1} x$

$v = x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$\int v \, dx = \frac{x^2}{2}$

$\therefore u \int v \, dx - \int ((u)' \times \int v \, dx) \, dx$

$\tan^{-1} x \times \frac{x^2}{2} - \int \frac{1}{1+x^2} \times \frac{x^2}{2} \, dx$

$\therefore \quad \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} \, dx$

$\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int 1 - \frac{1}{x^2+1} \, dx$

$\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} [x - \tan^{-1} x]$

$$\therefore \left[\frac{x^2 \tan^{-1}x - x + \tan^{-1}x}{2}\right]_0^1$$

$$\therefore \frac{1^2 \tan^{-1} 1 - 1 + \tan^{-1} 1}{2} - \left[\frac{0 \tan^{-1} 0 - 0 + \tan^{-1} 0}{2}\right]$$

$$\therefore \cancel{45 = 1} \ \frac{\pi/4 - 1 + \pi/4}{2} - \left[\frac{0 - 0 + 0}{2}\right]$$

$$\therefore \frac{\pi/2 - 1}{2}$$

$$\therefore \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4} \text{ or } \frac{43^{\circ}}{4}$$

11 $\int_0^2 \frac{x^2}{1+x^3} dx$

$$\therefore \frac{1}{3}\int_0^2 \frac{3x^2}{1+x^3} dx$$

$\left(\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c\right)$

$$\therefore \frac{1}{3}\left[\log(x^3+1)\right]_0^2$$

$$\therefore \frac{1}{3}\left[\overset{\log 9}{\cancel{\log 8}} - \log 1\right] = \frac{\cancel{\log 8}}{\cancel{3}} \quad \frac{\log 9}{3}$$

12 $$\int_0^{\pi/3} \frac{\sin x}{3+4\cos x}\,dx$$

$$\therefore \frac{-1}{4}\int_0^{\pi/3} \frac{-4\sin x}{3+4\cos x}\,dx$$

$$\because \frac{d}{dx}(3+4\cos x) = -4\sin x$$

$$\therefore I = -\frac{1}{4}\Big[\log(3+4\cos x)\Big]_0^{\pi/3}$$

$$\therefore = -\frac{1}{4}\left[\log\left(3+4\cos\frac{\pi}{3}\right) - \log(3+4\cos 0)\right]$$

$$= -\frac{1}{4}\left[\log\left(3+4\left(\frac{1}{2}\right)\right) - \log(3+4(1))\right]$$

$$= -\frac{1}{4}\left[\log 5 - \log 7\right]$$

or

$$\frac{-1}{4}\log\left(\frac{5}{7}\right)$$

13 $\int_0^1 x e^x \, dx$

$u = x$
$v = e^x$

$\frac{dv}{dx} = \cancel{\frac{x^2}{2}}\ 1$

$\int v\,dx = e^x$

$u \int v\,dx - \int \left((u)' \times \int v\,dx\right) dx$

$\therefore \cancel{x \times e^x - \int \frac{x^2}{2} \times e^x d}$

$\cancel{x e^x - \frac{1}{2} \int x^2 e^x \, dx}$

$\cancel{\therefore x e^x - \frac{1}{2} \left[ x^2 e^x - \right.}$

$x e^x - \int 1 \times e^x \, dx$

$\left[ x e^x - e^x \right]_0^1$

$\therefore 1e^1 - e^1 - \left[ 0 - e^0 \right]$

$e - e - [0 - 1]$

$= \underline{1}$

⇒ Properties of Definite Integral

1 $\int_a^a f(x)\,dx = 0$

2 $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx = \int_a^b f(a+b-x)\,dx$

3 $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

4 If f is even $(f(-x) = f(x))$,

then, $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

5 If f is odd $(f(-x) = -f(x))$,

then, $\int_{-a}^{a} f(x)\,dx = 0$

6 If $f(2a-x) = f(x)$, then $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$

7 If $f(2a-x) = -f(x)$, then $\int_0^{2a} f(x)\,dx = 0$

8 $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

9 $\int_0^a f(x)\,dx = 0$, if $f(a-x) = -f(x)$, odd function

⇒ Examples

1 $\int_{-2}^{2} x^3 \, dx$

$\therefore f(x) = x^3$

$f(-x) = (-x)^3 = -x^3$

$\therefore f(-x) = -f(x)$

$\therefore f(x)$ is odd function

$\therefore \int_{-2}^{2} x^3 \, dx = 0$

2 $\int_{-\pi}^{\pi} \sin^3 x \, dx$

$\therefore$ here $f(x) = \sin^3 x$

$\therefore f(-x) = \sin^3(-x) = -\sin^3 x$

$\therefore f(x) = -f(x)$

$\therefore \int_{-\pi}^{\pi} \sin^3 x = 0$

or

we, know that $\sin x$ is odd function

$\therefore \int_{-\pi}^{\pi} \sin^3 x \, dx = 0$

3 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x$

$f(x) = x \cos x$

$f(-x) = (-x) \cos(-x) = -x \cos x$

$\therefore f(-x) = -f(x)$ , odd function

$\therefore \int_{-\pi/2}^{\pi/2} x \cos x = 0$

4 $\int_{-2}^{2} x^5(1-x^2)^{\frac{3}{2}} \, dx$

$f(x) = x^5(1-x^2)^{\frac{3}{2}}$

$\therefore f(-x) = (-x)^5(1-(-x)^2)^{\frac{3}{2}}$

$= -x(1-x^2)^{\frac{3}{2}}$

$\therefore f(-x) = -f(x)$ , odd function

$\therefore \int_{-2}^{2} x^5(1-x^2)^{\frac{3}{2}} \, dx = 0$

5 $\int_{-1}^{1} \sin^3 x \cos^4 x \, dx$

$\therefore f(x) = \sin^3 x \cos x$

$\therefore f(-x) = \sin^3(-x) \cos(-x)$

$= -\sin^3 x \cos x$

$\therefore f(-x) = -f(x)$ , $\therefore I = 0$

6 $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$

$\therefore f(-x) = \log\left(\frac{2-(-x)}{2+(-x)}\right) = \log\left(\frac{2+x}{2-x}\right)$

$= \log\left(\left(\frac{2-x}{2+x}\right)^{-1}\right)$

$= -\log\left(\frac{2-x}{2+x}\right)$

$\therefore f(-x) = -f(x)$, odd function

$\therefore I = 0$

7 $\int_{-1}^{1} (x^2+1)\, dx$

$f(-x) = (-x)^2+1 = x^2+1$

$\therefore f(-x) = f(x)$, even function

$\therefore \int_{-1}^{1} (x^2+1)\, dx = 2\int_{0}^{1} (x^2+1)\, dx$

$= 2\left[\frac{x^3}{3}+x\right]_0^1$

$$= 2\left[\frac{1}{3}+1\right]$$

$$= \frac{8}{3}$$

8 $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

$$f(x) = \frac{1}{1+x^2}$$

$$f(-x) = \frac{1}{1+(-x)^2} = \frac{1}{1+x^2}$$

$$\therefore f(-x) = f(x)$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2}\,dx = 2\int_{0}^{1} \frac{1}{1+x^2}\,dx$$

$$= 2\left[\tan^{-1} x\right]_0^1$$

$$\therefore 2\left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$2\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{2}$$

9 $\int_{0}^{2\pi} \sin^5 x \cos^4 x \, dx$

$\therefore f(x) = \sin^5 x \cos^4 x$

$\cancel{f(x)}$ $f(2\pi - x) = \sin^5(2\pi - x) \cos^4(2\pi - x)$

$= -\sin^5 x \cos^4 x$

$\therefore \cancel{f(x) =} f(2\pi - x) = -f(x)$, $\therefore I = 0$

10 $\int_0^{\frac{\pi}{2}} \log \tan x \, dx$

$\cancel{f(x)} = f\left(\frac{\pi}{2} - x\right) = \log\left(\tan\left(\frac{\pi}{2} - x\right)\right)$

$= \log(\cot x)$

$= \log\left(\left(\frac{1}{\cot x}\right)^{-1}\right)$

$= \log\left((\tan x)^{-1}\right)$

$= -\log \tan x$

$\therefore \cancel{f(x)}\ f\left(\frac{\pi}{2} - x\right) = -f(x)$

$\therefore \int_0^{\pi/2} \log \tan x \, dx = 0$

11 $\int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} \, dx$

$\therefore f\left(\frac{\pi}{2} - x\right) = \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)}$

$= \frac{\sin x - \cos x}{1 + \cos x \sin x} = -\left(\frac{\cos x + \sin x}{1 + \cos x \sin x}\right)$

$\cancel{= x +} \sin x \quad \therefore f\left(\frac{\pi}{2} - x\right) = -f(x)$

$\therefore I = 0$

12 $\int_0^1 x^2 (1-x)^{\frac{3}{2}} dx$

$$= \int_0^1 (1-x)^2 (1-(1-x))^{\frac{3}{2}} dx$$

$$= \int_0^1 (1-x)^2 (x)^{\frac{3}{2}} dx$$

$$= \int_0^1 (1-2x+x^2)\, x^{\frac{3}{2}}\, dx$$

$$= \int_0^1 \left(x^{\frac{3}{2}} - 2x^{1+\frac{3}{2}} + x^{2+\frac{3}{2}}\right) dx$$

$$= \int_0^1 \left(x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + x^{\frac{7}{2}}\right) dx$$

$$\left[\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 2\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1}\right]_0^1$$

$$\left[\frac{x^{\frac{5}{2}}}{5/2} - \frac{2x^{7/2}}{7/2} + \frac{x^{9/2}}{9/2}\right]_0^1$$

$$\therefore \left[\frac{2x^{5/2}}{5} - \frac{4x^{7/2}}{7} + \frac{2x^{9/2}}{9}\right]_0^1$$

$$= \frac{2}{5} - \frac{4}{7} + \frac{2}{9} - [0]$$

$$= \frac{2\times7\times9 - 4\times5\times9 + 2\times5\times7}{5\times7\times9}$$

$$= \frac{126-180+70}{315} = \frac{376}{315}$$

13 $$\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$$

$$= \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)+\cos(\frac{\pi}{2}-x)}\,dx$$

$$= \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\,dx$$

$$\therefore I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\, dx = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\, dx$$

$\therefore$

$$I + I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\, dx + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\, dx$$

$$\therefore 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\cos x + \sin x}\, dx$$

$$\therefore 2I = \int_0^{\pi/2} 1\, dx$$

$$\therefore I = \frac{1}{2}\left[x\right]_0^{\pi/2}$$

$$\therefore I = \frac{1}{2}\,\frac{\pi}{2} = \frac{\pi}{4}$$

14 $\int_0^{\pi/2} \frac{\sec x}{\sec x + \operatorname{cosec} x} dx$

$$I = \int_0^{\pi/2} \frac{\sec(\frac{\pi}{2} - x)}{\sec(\frac{\pi}{2} - x) + \operatorname{cosec}(\frac{\pi}{2} - x)} dx$$

$$I = \int_0^{\pi/2} \frac{\operatorname{cosec} x}{\operatorname{cosec} x + \sec x} dx$$

$$\therefore 2I = \int_0^{\pi/2} \frac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x + \sec x} dx$$

$$I = \frac{1}{2} \int_0^{\pi/2} 1 \, dx$$

$$\therefore I = \frac{1}{2} [x]_0^{\pi/2} = \frac{\pi}{4}$$

15 $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx$

$$I = \frac{1}{2} \int_0^{\pi/2} 1 \, dx$$

$$\therefore I = \frac{\pi}{4}$$

16 $\displaystyle\int_0^{\pi/2} \frac{1}{1+\tan x}\,dx$

$$\int_0^{\pi/2} \frac{1}{1+\tan\left(\frac{\pi}{2}-x\right)}\,dx$$

$$\therefore \int_0^{\pi/2} \frac{1}{1+\cot x}\,dx$$

$$\therefore \int_0^{\pi/2} \frac{1}{1+\frac{1}{\tan x}}\,dx$$

$$\therefore \int_0^{\pi/2} \frac{1}{\frac{1+\tan x}{\tan x}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\tan x}{1+\tan x}\,dx$$

$$\cancel{\therefore f\left(\frac{\pi}{2}-x\right) = f(x)}$$

$$\therefore 2I = \int_0^{\pi/2} \frac{1}{1+\tan x}\,dx + \int_0^{\pi/2} \frac{\tan x}{1+\tan x}\,dx$$

$$2I = \int_0^{\pi/2} \frac{1+\tan x}{1+\tan x}\,dx$$

$$\therefore 2I = \int_0^{\pi/2} 1\,dx$$

$$\therefore I = \frac{1}{2}[x]_0^{\pi/2} = \frac{\pi}{4}$$

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17 $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

$\therefore \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)}+\sqrt{\cos(\frac{\pi}{2}-x)}}\,dx$

$\therefore \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$

$\therefore 2I = \int_0^{\pi/2} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$

$\therefore 2I = \int_0^{\pi/2} 1\,dx$

$\therefore I = \frac{\pi}{4}$

18 $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$

$\therefore \int_0^{\pi/2} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)}+\sqrt{\sin(\frac{\pi}{2}-x)}}\,dx$

$\therefore \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

$$\therefore 2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

$$\therefore 2I = \int_0^{\pi/2} 1\, dx$$

$$\therefore I = \frac{1}{2}\left[x\right]_0^{\pi/2}$$

$$I = \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$$

19 $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\, dx$

$$\therefore \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}\, dx$$

$$\therefore \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\, dx$$

$$\therefore \int_0^{\pi/2} \frac{1}{1+\sqrt{\left(\frac{1}{\tan x}\right)}}\, dx$$

$$\therefore \int_0^{\pi/2} \frac{1}{\frac{\sqrt{\tan x} + \sqrt{1}}{\sqrt{\tan x}}}\, dx$$

$$\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}\,dx$$

$$\therefore 2I = \int_0^{\pi/2} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$$

$$\therefore 2I = \int_0^{\pi/2} 1\,dx$$

$$\therefore I = \frac{1}{2}\left[x\right]_0^{\pi/2} = \frac{\pi}{4}$$

20 $$\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}\,dx$$

$$2I = \int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}\,dx + \int_0^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$$

$$\therefore 2I = \int_0^{\pi/2} 1\,dx$$

$$\therefore I = \frac{\pi}{4}$$

21 $\displaystyle\int_0^{\pi/2} \left(\frac{\pi x}{2} - x^2\right) \cos 2x \, dx$

$$\therefore \int_0^{\pi/2} \left[\frac{\pi}{2}\left(\frac{\pi}{2} - x\right) - \left(\frac{\pi}{2} - x\right)^2\right] \cos\left(2\left(\frac{\pi}{2} - x\right)\right) dx$$

$$\therefore \int_0^{\pi/2} \left[\frac{\pi^2}{4} - \frac{\pi x}{2} - \frac{\pi^2}{4} + \pi x - x^2\right] \cos(\pi - 2x) \, dx$$

$$\therefore \int_0^{\pi/2} \left(\pi x - \frac{\pi}{2} x - x^2\right)(-\cos 2x) \, dx$$

$$\therefore -\int_0^{\pi/2} \left(\frac{\pi x}{2} - x^2\right) \cos 2x \, dx$$

$$\therefore f\left(\frac{\pi}{2} - x\right) = -f(x)$$

$$\therefore I = 0$$

22 $\displaystyle\int_0^{\pi/4} \log(1 + \tan x) \, dx$

$$\therefore \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$$

$$\therefore \int_0^{\pi/4} \log\left(1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \cdot \tan x}\right) dx$$

$\cancel{\because \tan \alpha - \beta = \tan}$

$\left(\because \tan \alpha - \beta = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}\right)$

$$\therefore \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$$

$$\therefore \int_0^{\pi/4} \log\left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x}\right) dx$$

$$\therefore \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$$

$$\therefore \int_0^{\pi/4} \log 2 - \int_0^{\pi/4} \log(1 + \tan x)\, dx$$

$$\therefore I = \int_0^{\pi/4} \log 2 - I$$

$$\therefore 2I = \int_0^{\pi/4} \log 2 \, dx \qquad (\log 2 \text{ is constant})$$

$$\therefore 2I = \log 2 \int_0^{\pi/4} 1\, dx$$

$$\therefore 2I = \log 2 \left[x\right]_0^{\pi/4}$$

$$2I = \log 2\left(\frac{\pi}{4}\right)$$

$$\therefore I = \frac{\pi}{8}\cdot\log 2$$