* Determinant

→ A determinant is a square matrix

EX $\begin{vmatrix} a & b \\ c & D \end{vmatrix}$ is 2x2 determinant

EX $\begin{vmatrix} 1 & 3 \\ 4 & 7 \end{vmatrix}$, $\begin{vmatrix} ax & by \\ cz & dv \end{vmatrix}$, $\begin{vmatrix} 3x+a & 2y+b \\ 4z+3 & 7 \end{vmatrix}$

$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ is 3x3 determinant

→ Row

↓ ~~Columb~~ column.

⇒ $i \times j$ ⇒ In this way matrix is denoted

where i = Row

j = column

If $i = j$, it is determinant

If $i \neq j$, it is not determinant

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$\Rightarrow$ value of determinant (for 2x2 determinant)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ?$$

$$= \cancel{(a \times d)} \quad [a \times d] - [b \times c]$$

$$= ad - bc$$

EX $\begin{vmatrix} 3 & 2 \\ 4 & 9 \end{vmatrix} = [3 \times 9] - [2 \times 4]$

$$= \cancel{24 - 8} \; 27 - 8$$

$$= 19$$

Ex $\begin{vmatrix} -3 & 5 \\ -4 & 6 \end{vmatrix} = [(-3) \times 6] - [5 \times (-4)]$

$$= (-18) - (-20)$$

$$= (-18) + 20$$

$$= 2$$

Ex $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

$$= [\cos\theta \times \cos\theta] - [\sin\theta \times (-\sin\theta)]$$

$$= \cos^2\theta - (-\sin^2\theta)$$

$$= \cos^2\theta + \sin^2\theta = 1$$

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EX $\begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} = (1\times 0) - (-1\times 0)$
$= 0$

EX $\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} = [0\times(-1)] - [1\times 0]$
$= 0$

so from above 2 examples, we can conclude the If in a determinant,

in any one row or any one column, if all values are zero then,

$\therefore$ value of determinant is zero

EX $\begin{vmatrix} x & 4 \\ 1 & 2 \end{vmatrix} = 6$, find x

$\therefore [x\times 2] - [4\times 1] = 6$

$\therefore 2x - 4 = 6$
$\therefore 2x = 10$
$\therefore x = 5$

EX $\begin{vmatrix} x & -5 \\ 3 & -2 \end{vmatrix} = 5$, find x

$\therefore [x\times(-2)] - [(-5)\times 3]$
$\therefore -2x - (-15) = 5$
$\therefore -2x + 15 = 5$

$\therefore \cancel{+}2x = \cancel{+}10$

$\therefore x = \frac{10}{2} = 5$

Ex $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 6$; find $\begin{vmatrix} 3a & 3b \\ c & d \end{vmatrix}$

$\therefore a \times d - b \times c = 6$

$\therefore ad - bc = 6$ —①

$\therefore \begin{vmatrix} 3a & 3b \\ c & d \end{vmatrix} = 3ad - 3bc$

$= 3(ad - bc)$

Now putting eq ①

$\therefore 3(6) = 18$

⇒ short method

$\begin{vmatrix} 3a & 3b \\ c & d \end{vmatrix}$

taking 3 common

$3\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\therefore 3 \times 6 = 18$

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EX $\begin{vmatrix} a & -b \\ c & -d \end{vmatrix} = 8$, find. $\begin{vmatrix} 7a & b \\ 7c & d \end{vmatrix}$

$\therefore \quad a(-d) - (-b)c = 8$

$\therefore -ad - (-bc) = 8$

$\therefore -ad + bc = 8 \quad (1)$

$\therefore$ ~~$bc - ad = 8 \quad (1)$~~

$\therefore \begin{vmatrix} 7a & b \\ 7c & d \end{vmatrix} = 7ad - 7bc$

$= 7(ad - bc)$

$\therefore -7(-ad + bc)$

putting eq (1)

$\therefore -7(8)$

$= (-56)$.

~~⇒ Short method~~

EX $\begin{vmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{5}{2} & 1 \end{vmatrix} = ?$

$\therefore \frac{3}{2} \times 1 - \frac{1}{2} \times \frac{5}{2}$

$= \frac{3}{2} - \frac{5}{4} = \frac{3 \times 2 - 5}{4} = \frac{6-5}{4}$

$= \frac{1}{4}$

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Ex $\begin{vmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{vmatrix} = ?$

$\therefore \tan\theta \times \tan\theta - \sec\theta \times \sec\theta$

$\tan^2\theta - \sec^2\theta = 1$

$\begin{vmatrix} x-1 & x \\ x & x+1 \end{vmatrix} = ?$

$\therefore (x-1)(x+1) - x^2$

$\therefore x(x+1) - 1(x+1) - x^2$

$\therefore \cancel{x^2} + \cancel{x} - \cancel{x} - 1 - \cancel{x^2}$

$= -1$

Ex $\begin{vmatrix} x+3 & 4 \\ x-2 & 5 \end{vmatrix} = 13$, find x

$\therefore (x+3)5 - 4(x-2) = 13$

$\therefore 5x + 15 - (4x - 8) = 13$

$\therefore 5x + 15 - 4x + 8 = 13$

$\therefore x + 23 = 13$

$\therefore x = -10$

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⇒ value of determinant (for 3x3 determinant)

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = ?$$

for example only

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\therefore \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a[exi - fxh] - b[dxi - fxg] + c[dxh - exg]$$

$$\cancel{= a[exi - fg]}$$

$$\cancel{= a[exi -}$$

$$= aei - afh - bdi + bfg + cdh - ceg$$

EX $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$$= 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= 1[5\times9 - 6\times8] - 2[4\times9 - 6\times7] + 3[4\times8 - 5\times7]$$

$$= 1[-3] - 2[-6] + 3(-3) = 0$$

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Note: if all the values of 3x3 matrix are in sequence then value is zero

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{vmatrix} = 0$$

$$\begin{vmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{vmatrix} = 0$$

Ex $\begin{vmatrix} 3 & 1 & 3 \\ 2 & 4 & 0 \\ 1 & 1 & 2 \end{vmatrix}$

$$= 3\begin{vmatrix} 4 & 0 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix}$$

$$= 3[8-0] - 1(4-0) + 3(2-4)$$

$$\cancel{= 3(8) - (1)(4) + 3(-2)}$$

$$= 3(8) - 1(4) + 3(-2) = 14$$

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Ex $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ -2 & 1 & 4 \end{vmatrix} = ?$

$\cancel{\therefore 1 \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} - 0 + 1 \begin{vmatrix} 0 & 3 \\ -2 & 1 \end{vmatrix}}$

$\cancel{\therefore 1(4-3) - 0 + 1(0+6)}$

$\cancel{1(1) + 1(6) = 7}$

$1 \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} - 0 + 1 \begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix}$

$\therefore 1(4-3) - 0 + 1(0+2)$

$\therefore 1(1) + 1(2) = 3$

Ex $\begin{vmatrix} -2 & 1 & 4 \\ 0 & 9 & 3 \\ -2 & 1 & 4 \end{vmatrix} = ?$

$(-2) \begin{vmatrix} 9 & 3 \\ 1 & 4 \end{vmatrix} - (1) \begin{vmatrix} 0 & 3 \\ (-2) & 4 \end{vmatrix} + 4 \begin{vmatrix} 0 & 9 \\ (-2) & 1 \end{vmatrix}$

$\therefore (-2)[9 \times 4 - 3 \times 1] - (1)[0 \times 4 - ((-2) \times 3)] + 4[0 \times 1 - ((-2) \times 9)]$

$\therefore (-2)[36 - 3] - 1[0 - (-6)] + 4[0 - (-18)]$

$-2[33] - 1[6] + 4[18]$

$\therefore -66 - 6 + 72$

$\therefore 0$

Note: In a 3×3 determinant
If any two rows or any two columns are equal, then value of determinant is zero

EX $\begin{vmatrix} 3 & 1 & 3 \\ 2 & -4 & 2 \\ 1 & -1 & 1 \end{vmatrix}$

here column 1 = column 3

$\therefore$ ans = 0

EX $\begin{vmatrix} 3a & b & cx+y \\ 2 & -4 & 2 \\ 3a & b & cx+y \end{vmatrix}$

here Row 1 = Row 3

$\therefore$ ans = 0

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EX $\begin{vmatrix} x-1 & 2 & 1 \\ x & 1 & x+1 \\ 1 & 1 & 0 \end{vmatrix} = 0$, find x

$\therefore \; x-1 \begin{vmatrix} 1 & x+1 \\ 1 & 0 \end{vmatrix} - 2 \begin{vmatrix} x & x+1 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} x & 1 \\ 1 & 1 \end{vmatrix}$

$\therefore \; \cancel{x-1}$

$\therefore (x-1)[0-(x+1)] - 2[0-(x+1)] + 1[x-1] = 0$

$\therefore (x-1)[-x-1] - 2[-x-1] + 1[x-1] = 0$

$\therefore -x(x-1) - 1(x-1) - 2(-x) - 2(-1) + x - 1 = 0$

$\therefore -x^2 + x - \cancel{x} + \cancel{1} + 2x + 2 + \cancel{x} - \cancel{1} = 0$

$\therefore -x^2 + x + 2x + 2 = 0$

$\therefore -x^2 + 2x + x + 2 = 0 \Rightarrow -x^2 + 3x + 2 = 0$

$-x^2 + 3x + 2 = 0$

using quadratic formula

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\therefore$ here: $a = -1$

$b = 3$

$c = 2$

$$\therefore x = \frac{-3 \pm \sqrt{3^2 - 4(-1)(2)}}{2 \times (-1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 8}}{-2}$$

$$x = \frac{-3 \pm \sqrt{17}}{-2}$$

$$x = \frac{-3 + \sqrt{17}}{-2} \quad \text{or} \quad x = \frac{-3 - \sqrt{17}}{-2}$$

$\therefore$ ~~$x \approx 3.56$ or $x \approx$~~

$$\therefore x = \frac{3 - \sqrt{17}}{2} \quad \text{or} \quad x = \frac{3 + \sqrt{17}}{2}$$

$\therefore x \approx -0.56$ or $x \approx 3.56$

Ex If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 25$, find $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g & h & i \end{vmatrix} = ?$

$\therefore$ Now taking -3 common

$$\therefore \begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g & h & i \end{vmatrix} = -3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\therefore -3(25) = \boxed{-75}$$

* Matrices

⇒ concept

Arrangement of numbers in rows and column enclosed by square brackets is called as matrix or matrices

⇒ ~~$A = [a_{ij}]_{1\times t}$~~

⇒ $A = [a_{ij}]_{i\times j}$ ⇒ i = row, j = column

Ex $A = \begin{bmatrix} 1 & 2 & 4 \\ 7 & 6 & 9 \\ 10 & 2 & 0 \end{bmatrix}_{3\times 3}$

$B = \begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix}_{2\times 2}$

$D = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 5 & 9 \end{bmatrix}_{3\times 2}$

$E = \begin{bmatrix} 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix}_{2\times 3}$

⇒ Suffix notation of matrix

Ex $A = [a_{ij}]_{3\times3}$

Suffix notation of A is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Ex $D = [d_{ij}]_{2\times2}$

Suffix notation of D is

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

Ex $C = [c_{ij}]_{3\times2}$

Suffix notation of C is

$$\cancel{\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}}$$

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

⇒ diagonal elements

elements having same number of row and same number of column are diagonal elements

EX A $[a_{ij}]_{3\times3}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

a_{11}, a_{22}, a_{33}, are diagonal elements

EX A $[a_{ij}]_{2\times2}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

a_{11}, a_{22} ⇒ diagonal elements

⇒ Types of matrix

1. Diagonal matrix

• A square matrix in which all the elements other than diagonal elements, are zero

Ex $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$, $\begin{bmatrix} 9 & 0 \\ 0 & 7 \end{bmatrix}$

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2. Scalar matrix

- A diagonal matrix, in which all diagonal elements are same

Ex $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

3. Identity matrix or unit matrix

- A diagonal matrix, in which all diagonal elements are 1

Ex $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ denoted by 'I'

4. Null matrix or zero matrix

- A matrix in which all the elements are zero

Ex $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

~~Also~~ denoted by $A = [0]$

(Not is syllabus)

~~5. Sig~~ * But necessary to understand *

⇒ Transpose of matrix

Ex $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$

~~then A' or A^T ⇒ (denoted by)~~

Transpose of $A = A'$ or A^T (denoted by)

$$A^T = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Ex: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Ex $A = \begin{bmatrix} 1 & 3 & 5 \\ 4 & 6 & 9 \\ 7 & 2 & 1 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 6 & 2 \\ 5 & 9 & 1 \end{bmatrix}$$

Ex $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

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* In Syllabus *

⇒ Determinant of matrix

EX $A = \begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}$

mode of matrix $A = |A|$, is determinant of matrix A

$\therefore \quad |A| = \begin{vmatrix} 1 & 3 \\ 5 & 6 \end{vmatrix}$

$|A| = [1 \times 6] - [3 \times 5]$

$|A| = (-9)$

---

5. Singular matrix

• A matrix whose determinant is zero is called singular matrix

EX $A = \begin{bmatrix} 2 & 3 \\ 12 & 18 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 \\ 12 & 18 \end{vmatrix} = [2 \times 18] - [3 \times 12] = 0$

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6. Non singular matrix

- A matrix whose determinant is not zero is called non singular matrix

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* Not is Syllabus but necessary to understand *

7. Symmetric matrix

- A matrix whose transpose is same is called symmetric matrix

$$A = A^T$$

8. Skew symmetric matrix

- A matrix whose transpose is negative

~~Ex~~ $\cancel{A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}}$ $\cancel{A^T}$ $A^T = -A$

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⇒ Algebra of matrix

⇒ Addition of matrix

condition: order of ~~matrix~~ ~~s~~ matrices ~~shol~~ should be same

EX $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$ $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}_{2\times2}$

$\therefore A+B = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}_{2\times2}$

~~Ex A=~~

⇒ Subtraction of matrix

Condition: order of matrices ~~shol~~ should be same

EX $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$ $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}_{2\times2}$

$A-B = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}_{2\times2}$

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⇒ Scalar multiplication of matrix

EX $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

let α be a non zero scalar

$\therefore \alpha A = \alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix}$

Examples:

EX $A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 & -1 \\ 2 & 5 & 7 \end{bmatrix}$

find (i) $A+B$
(ii) $A-B$
(iii) $3A-4B$

(i) $A+B = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 3 & -1 \\ 2 & 5 & 7 \end{bmatrix}$

$A+B = \begin{bmatrix} 1+4 & 2+3 & 5+(-1) \\ (-1)+2 & 3+5 & 8+7 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 4 \\ 1 & 8 & 15 \end{bmatrix}$

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(ii) $A - B = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 3 & -1 \\ 2 & 5 & 7 \end{bmatrix}$

$$A - B = \begin{bmatrix} 1-4 & 2-3 & 5-(-1) \\ (-1)-2 & 3-5 & 8-7 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -3 & -1 & 6 \\ -3 & -2 & 1 \end{bmatrix}$$

(iii) $3A - 4B$

$$3A = 3\begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 15 \\ -3 & 9 & 24 \end{bmatrix}$$

$$4B = 4\begin{bmatrix} 4 & 3 & -1 \\ 2 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 16 & 12 & -4 \\ 8 & 20 & 28 \end{bmatrix}$$

$$\therefore 3A - 4B = \begin{bmatrix} 3 & 6 & 15 \\ -3 & 9 & 24 \end{bmatrix} - \begin{bmatrix} 16 & 12 & -4 \\ 8 & 20 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 3-16 & 6-12 & 15-(-4) \\ (-3)-8 & 9-20 & 24-28 \end{bmatrix}$$

$$3A - 4B = \begin{bmatrix} -13 & -6 & 19 \\ -11 & -11 & -4 \end{bmatrix}$$

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⇒ commutative property of matrix

- $A+B = B+A$

⇒ Associative property of matrix

- $A+(B+C) = (A+B)+C$

- $3(A+B) = 3A+3B$

Ex $A = \begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & -2 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 5 & -6 \\ 3 & 1 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 5 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

find matrix 'X' such that; $3A+2B-X=C$

$\therefore \quad 3A = 3\begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -3 & 6 & -9 \\ 0 & 3 & -6 \end{bmatrix}$

$$2B = 2\begin{bmatrix} 4 & 5 & -6 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 10 & -12 \\ 6 & 2 & 4 \end{bmatrix}$$

$$\therefore \quad 3A + 2B - X = C$$

$$\therefore \quad \begin{bmatrix} -3 & 6 & -9 \\ 0 & 3 & -6 \end{bmatrix} + \begin{bmatrix} 8 & 10 & -12 \\ 6 & 2 & 4 \end{bmatrix} - X = \begin{bmatrix} 5 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$$\therefore \quad 3A + 2B - C = X$$

$$\therefore \quad \begin{bmatrix} -3 & 6 & -9 \\ 0 & 3 & -6 \end{bmatrix} + \begin{bmatrix} 8 & 10 & -12 \\ 6 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix} = X$$

$$\therefore \quad X = \begin{bmatrix} (-3)+8-5 & 6+10-(-1) & (-9)+(-12)-(-1) \\ 0+6-2 & 3+2-3 & (-6)+4-(-1) \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 17 & -20 \\ 4 & 2 & -1 \end{bmatrix}$$

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⇒ Product of two matrix

Condition: ~~colu No~~ Number of column of first matrix should be equal to number

Condition: Number of columns of first matrix should be equal to number of rows of second matrix.

and the order of resultant matrix will be
~~Number of rows of 1st matrix X Number of row~~

rows of 1st matrix X columns of 2nd matrix

Ex $A = [a_{ij}]_{3\times 2}$, $B = [b_{ij}]_{2\times 3}$, $C = [c_{ij}]_{2\times 2}$

multiplication	possibility of multiplication	order of Resultant matrix
A · B	Possible	3 x 3
B · A	Possible	2 x 2
A · C	Possible	3 x 2
C · B	Possible	2 x 3
B · C	Not possible	–
C · A	Not possible	–

Ex $A\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}_{2\times2}$, $B\begin{bmatrix}3 & 6\\5 & 9\end{bmatrix}_{2\times2}$

$$\therefore \quad A\cdot B = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\cdot\begin{bmatrix}3 & 6\\5 & 9\end{bmatrix}$$

$$A\cdot B = \begin{bmatrix}1\times3+2\times5 & 1\times6+2\times9\\3\times3+4\times5 & 3\times6+4\times9\end{bmatrix}$$

$$A\cdot B = \begin{bmatrix}3+10 & 6+18\\9+20 & 18+36\end{bmatrix} = \begin{bmatrix}13 & 24\\29 & 54\end{bmatrix}_{2\times2}$$

Similarly

$$B\cdot A = \begin{bmatrix}3 & 6\\5 & 9\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

$$B\cdot A = \begin{bmatrix}3\times1+6\times3 & 3\times2+6\times4\\5\times1+9\times3 & 5\times2+9\times4\end{bmatrix}$$

$$B\cdot A = \begin{bmatrix}3+18 & 6+24\\5+27 & 10+36\end{bmatrix} = \begin{bmatrix}21 & 30\\32 & 46\end{bmatrix}_{2\times2}$$

Note : $A \cdot B \neq B \cdot A$, $A \cdot I = I \cdot A$ (I = identity matrix)

Ex

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$$

$$A \cdot B = \begin{bmatrix} 1\times1+2\times4+3\times1 & 1\times2+2\times5+3\times1 & 1\times3+2\times7+3\times1 \\ 4\times1+5\times4+6\times1 & 4\times2+5\times5+6\times1 & 4\times3+5\times7+6\times1 \\ 7\times1+8\times4+9\times1 & 7\times2+8\times5+9\times1 & 7\times3+8\times7+9\times1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1+8+3 & 2+10+3 & 3+14+3 \\ 4+20+6 & 8+25+6 & 12+35+6 \\ 7+32+9 & 14+40+9 & 21+\cancel{40}\,56+9 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 12 & 15 & 20 \\ 30 & 39 & 53 \\ 48 & 63 & 86 \end{bmatrix}_{3\times3}$$

Similarly

$$B \cdot A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$$

$$B \cdot A = \begin{bmatrix} 1\times1+2\times4+3\times7 & 1\times2+2\times5+3\times8 & 1\times3+2\times6+3\times9 \\ 4\times1+5\times4+7\times7 & 4\times2+5\times5+7\times8 & 4\times3+5\times6+7\times9 \\ 1\times1+1\times4+1\times7 & 1\times2+1\times5+1\times8 & 1\times3+1\times6+1\times9 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 1+8+21 & 2+10+24 & 3+12+27 \\ 4+20+49 & 8+25+56 & 12+30+63 \\ 1+4+7 & 2+5+8 & 3+6+9 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 30 & 36 & 42 \\ 73 & 89 & 105 \\ 12 & 15 & 18 \end{bmatrix}_{3\times3}$$

EX $\quad A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}_{2\times3}, \quad B = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}_{3\times1}$

$$A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}_{2\times3} \cdot \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}_{3\times1}$$

$$A \cdot B = \begin{bmatrix} 1\times1+2\times5+3\times6 \\ 5\times1+6\times5+7\times6 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1+10+18 \\ 5+30+42 \end{bmatrix} = \begin{bmatrix} 29 \\ 77 \end{bmatrix}_{2\times1}$$

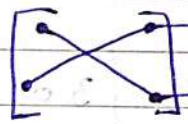
$B \cdot A$ = Not possible

⇒ ~~cofactor of matrix 2x2~~

⇒ Adjoint of matrix of order 2x2

Ex $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$Adj.A = \begin{bmatrix} d & b \\ -c & a \end{bmatrix}$$



Sign change

Element change

Ex $A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$

$$Adj.A = \begin{bmatrix} -7 & -3 \\ 5 & 1 \end{bmatrix}$$

Ex $A = \begin{bmatrix} -1 & -6 \\ 7 & 9 \end{bmatrix}$

$Adj.A = \begin{bmatrix} 9 & 6 \\ -7 & -1 \end{bmatrix}$

⇒ Inverse of matrix of order 2x2

Condition : Matrix should be non singular

Ex $A = [a_{ij}]$

$|A| \neq 0$

~~thus~~ $\therefore A^{-1}$ is possible

Ex $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

If $|A| \neq 0$

then $A^{-1} = \dfrac{Adj.A}{|A|}$

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Ex $A = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix}$

$\therefore |A| = [1 \times 5] - [3 \times 7] = -16$

$|A| \neq 0$ $\therefore$ given matrix is non singular

$\therefore A^{-1}$ is possible

$\therefore$ Adj. $A = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{\text{Adj. } A}{|A|}$

$\therefore A^{-1} = \dfrac{1}{-16} \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} \frac{5}{-16} & \frac{3}{16} \\ \frac{7}{16} & \frac{1}{-16} \end{bmatrix}$

Ex $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$

$\therefore |A| = [6 \times 1] - [5 \times 2] = (-4)$

$|A| \neq 0$ $\therefore A^{-1}$ is possible

$$\therefore \quad \text{Adj. } A = \begin{bmatrix} 1 & -5 \\ -2 & 6 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{Adj. } A}{|A|} = \frac{1}{-4}\begin{bmatrix} 1 & -5 \\ -2 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{4} & \frac{5}{4} \\ \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

⇒ Solution of simultaneous linear equations of two variables

EX $\quad 5x + y = 13$
$\quad\quad 3x + 2y = 5$

putting ~~both~~ eq. in matrix form

$$\underbrace{\begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 13 \\ 5 \end{bmatrix}}_{B}$$

$$\therefore \quad A \cdot X = B$$

$$\therefore \quad X = A^{-1} \cdot B$$

$$\therefore \quad \cancel{|A|} = |A| = [5 \times 2] - [1 \times 3] = 7$$

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$$\therefore \quad Adj.A = \begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{Adj.A}{|A|} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$$

$$\therefore \quad X = A^{-1} \cdot B$$

$$\therefore \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2\times13+(-1)\times5 \\ (-3)\times13+5\times5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 21 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21/7 \\ -14/7 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\therefore \quad x = 3$$
$$y = (-2)$$