

Exercises 1.9

Exercise 1.9.1

A factory has n suppliers that produce quantities $x_1 \dots x_n$ per day. The factory is connected with suppliers by a system of roads, which can be at variable capacities $c_1 \dots c_n$, so that the factory is supplied daily the amount $x = c_1x_1 + \dots + c_nx_n$.

- Given that the factory production process starts when the supply reaches the critical daily level b , write a formula for the daily factory revenue.
- Formulate the problem as a learning problem.

Exercise 1.9.2

A number of financial institutions, each having a wealth x_i , deposit amounts of money in a fund, at some adjustable rates of deposit w_i , so the money in the fund is given by $x = x_1w_1 + \dots + x_nw_n$. The fund is set up to function as in the following: as long as the fund has less than a certain reserve fund M , the fund manager does not invest. Only the money exceeding the reserve fund M is invested. Let $k = e^{rt}$, where r and t denote the investment rate of return and time of investment, respectively.

- Find the formula for the investment.
- Formulate the problem as a learning problem.

Exercise 1.9.3

- Given a continuous function $f : [0, 1] \rightarrow \mathbb{R}$, find a linear function $L(x) = ax + b$ with $L(0) = f(0)$ and such that $\frac{1}{2} \int_0^1 (L(x) - f(x))^2 dx$ is minimized.
- Given a continuous function $f : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$, find a linear function $L(x, y) = ax + by + c$ with $L(0, 0) = f(0, 0)$ and such that the error $\frac{1}{2} \int_{[0, 1]^2} (L(x, y) - f(x, y))^2 dx$ is minimized.

Exercise 1.9.4

For any compact $K \subset \mathbb{R}^n$ we associate the symmetric matrix $\rho_{ij} = \int_K x_i x_j dx_1 \dots dx_n$

Find the formula for the investment. The invertibility of the matrix (ρ_{ij}) depends both on the shape of K and the dimension n .

- Show that if $n = 2$ then $\det(\rho_{ij}) \neq 0$ for any compact $K \subset \mathbb{R}^2$.
- Assume $K = [0, 1]^n$. Show that $\det(\rho_{ij}) \neq 0$, for any $n \geq 1$.

SOLUTIONS