Exercise 1.9.1

A factory has n suppliers that produce quantities $x_1 \dots x_n$ per day. The factory is connected with suppliers by a system of roads, which can be at variable capacities $c_1 \dots c_n$, so that the factory is supplied daily the amount $x = c_1 x_1 + \cdots + c_n x_n$.
(a) Given that the factory production process starts when the supply reaches the critical daily level b, write a formula for the daily factory revenue.
(b) Formulate the problem as a learning problem.

Exercise 1.9.2

A number of finantial institutions, each having a wealth x_i, deposit amounts of money in a fund, at some adjustable rates of deposit w_i, so the money in the fund is given by $x = x_1 w_1 + \cdots + x_n w_n$. The fund is set up to function as in the following: as long a sthe fund has less than a certain reserve fund M, the fund manager does not invest. Only the money exceeding the reserve found M is invested. Let $k = e^{rt}$, where r and t denote the investment rate of return and time of investment, respectively.
(a) Find the formula for the investment.
(b) Formulate the problem as a learning problem.

Exercise 1.9.3

(a) Given a continous function $f : [0,1] \to \mathbb{R}$, find a linear function $L(x) = ax + b$ with $L(0) = f(0)$ and such that $\frac{1}{2}\int_0^1 (L(x) - f(x))^2 dx$ is minimized.
(b) Given a continous function $f : [0,1] \times [0,1] \to \mathbb{R}$, find a linear function $L(x,y) = ax + by + c$ with $L(0,0) = f(0,0)$ and such that the error $\frac{1}{2}\int_{[0,1]^2} (L(x,y) - f(x,y))^2 dx$ is minimized.

Exercise 1.9.4

For any compact $K \subset \mathbb{R}^n$ we associate the symetric matrix $\rho_{ij} = \displaystyle\int_K x_i x_j dx_1 \dots dx_n$

The invertiblity of the matrix (ρ_{ij}) depends both on the shape of K and the dimension n.
(a)Show that if $n = 2$ then $det(\rho_{ij}) \neq 0$ for any compact $K \subset \mathbb{R}^2$.
(b)Asume $K = [0,1]^n$. Show that $det(\rho_{ij}) \neq 0$, for any $n \geq 1$.

SOLUTIONS: