

## Exercises 4.17

### Exercise 4.17.1

Let  $f(x_1, x_2) = e^{x_1} \sin(x_2)$ , with  $(x_1, x_2) \in (0, 1) \times (0, \frac{\pi}{2})$ .

- Show that  $f$  is a harmonic function;
- Find  $\|\nabla f\|$ ;
- Show that the equation  $\nabla f = 0$  does not have any solutions;
- Find the maxima and minima for the function  $f$ .

### Exercise 4.17.2

Consider the quadratic function  $Q(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T A \mathbf{x} - b\mathbf{x}$ , with  $A$  nonsingular square matrix of order  $n$ .

- Find the gradient  $\|\nabla Q\|$ ;
- Write the gradient descent iteration;
- Find the Hessian  $H_Q$ ;
- Write the iteration by Newton's formula and compute its limit.

### Exercise 4.17.3

Let  $A$  be a nonsingular square matrix of order  $n$  and  $b \in \mathbb{R}^n$  a given vector. Consider the linear system  $A\mathbf{x} = b$ . The solution can be approximated using the following steps:

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## SOLUTIONS

### Exercise x.y.1 (a)

At vero eos et accusamus et iusto odio dignissimos ducimus qui blanditiis praesentium voluptatum deleniti atque corrupti quos dolores et quas molestias excepturi sint occaecati cupiditate non provident, similique sunt in culpa qui officia deserunt mollitia animi, id est laborum et dolorum fuga.

*Proof.*

$$\begin{aligned}\sigma'''(x) &= \frac{d}{dx} \frac{e^x - e^{2x}}{(1 + e^x)^3} = \frac{(e^x - 2e^{2x})(1 + e^x)^3 - 3(1 + e^x)^2 e^x (e^x - e^{2x})}{(1 + e^x)^6} \\ &= \frac{e^x \{1 - 4e^x + e^{2x}\} (1 + e^x)^2}{(1 + e^x)^6} = \frac{e^x \{1 - 4e^x + e^{2x}\}}{(1 + e^x)^4}\end{aligned}$$

□

### Exercise x.y.2 (a)

Et harum quidem rerum facilis est et expedita distinctio. Nam libero tempore, cum soluta nobis est eligendi optio cumque nihil impedit quo minus id quod maxime placeat facere possimus, omnis voluptas assumenda est, omnis dolor repellendus.

*Proof.* a = a

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