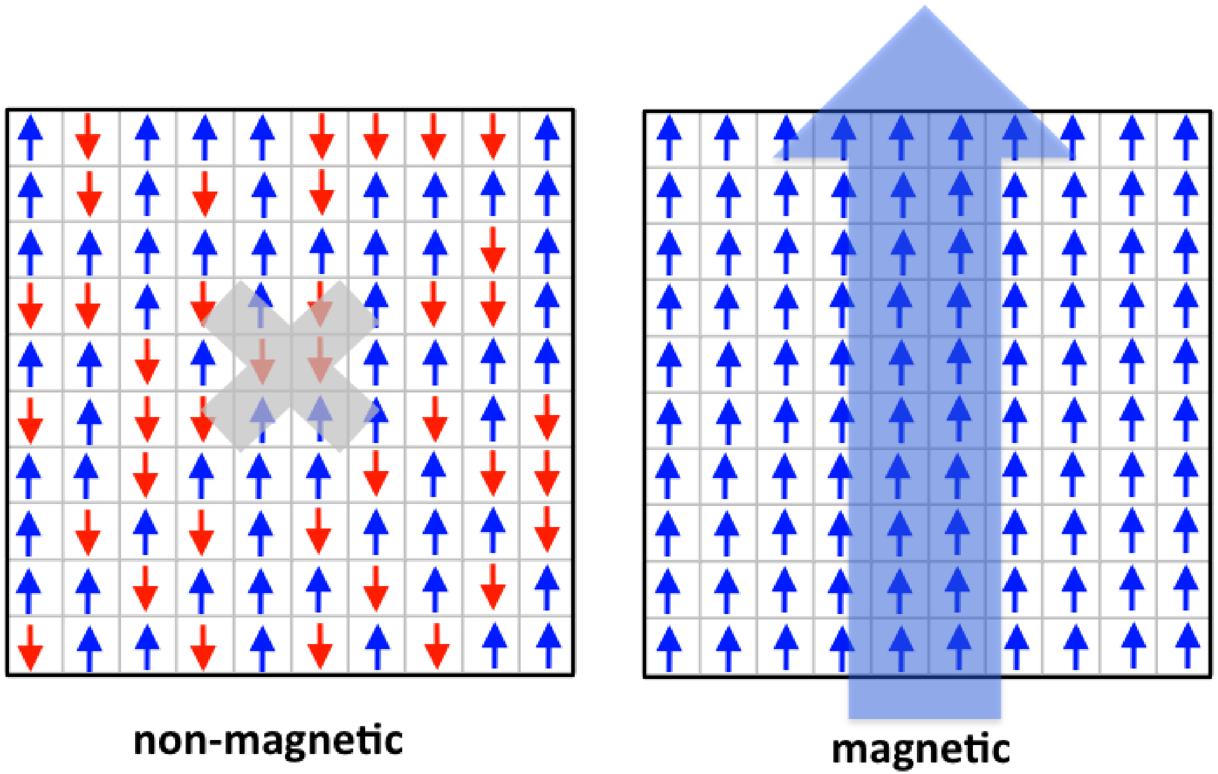
The Ising model: a laboratory for criticality

Originally a model for the magnetization of materials as a function of temperature.



- A lattice where each "*spin site*" can be in one of two orientations.
- We track the global **magnetization** of the lattice, which is the average of all site orientations:

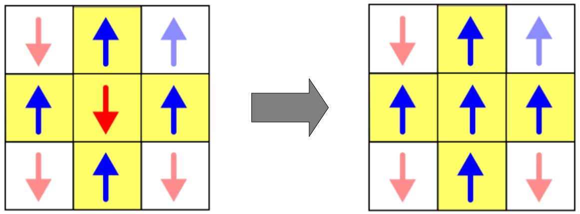
$$M \equiv \frac{1}{N} \sum_{ij} \sigma_{ij}$$

$$\sigma_{ij} \in \{-1, 1\}$$

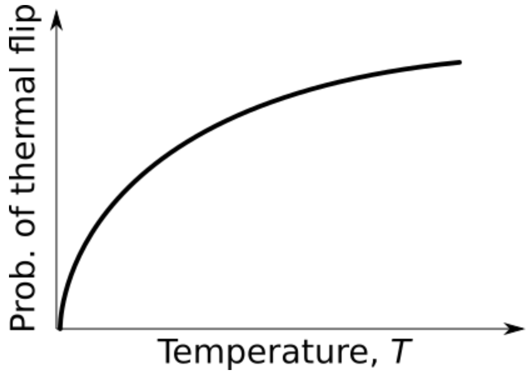
The Ising model: a laboratory for criticality

Two dynamics compete:

- On the one hand, the natural tendency of the spin sites is to **align with their neighbors**, thus increasing the global magnetization:

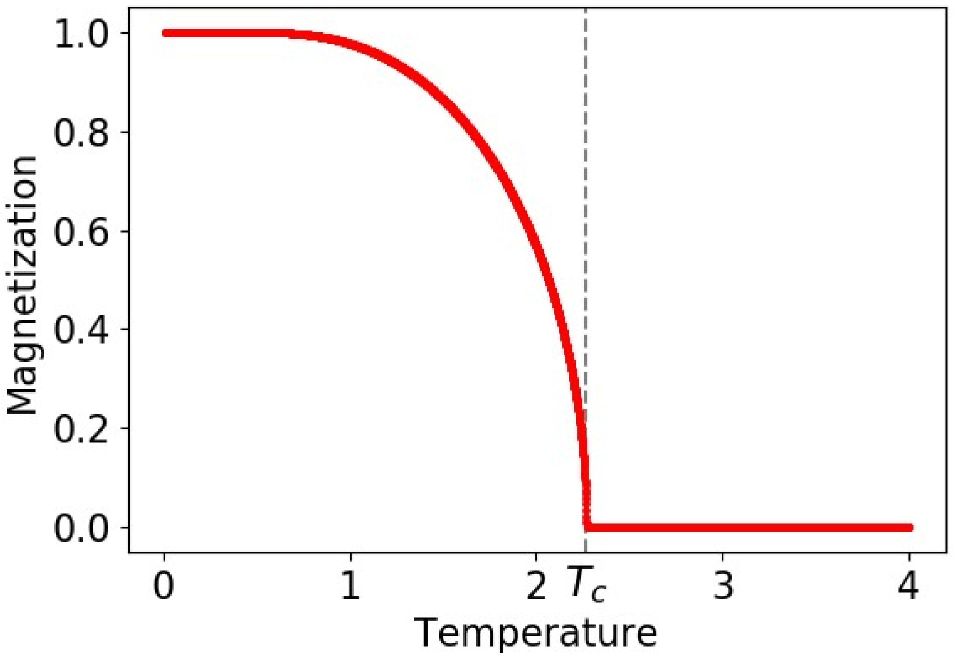


- On the other hand, the temperature introduces **random fluctuations** to the alignments, destroying the global magnetization; the probability of this *thermal flip* increases with the temperature.



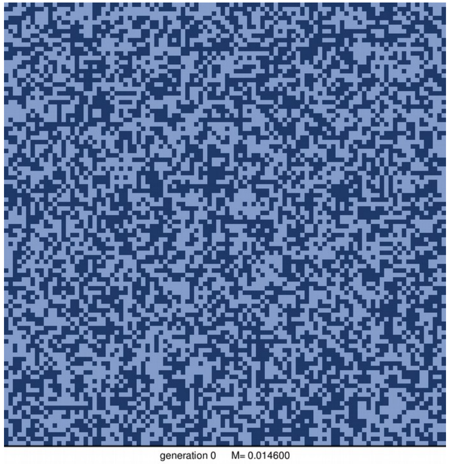
The Ising model: a laboratory for criticality

The result of these two competing dynamics is a **critical point**: there exists a **critical temperature** above which magnetization can't be sustained.

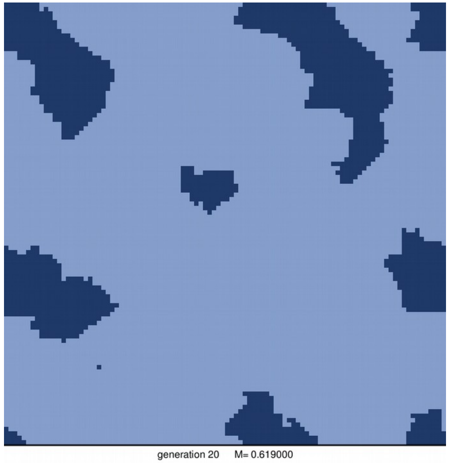


The Ising model: a laboratory for criticality

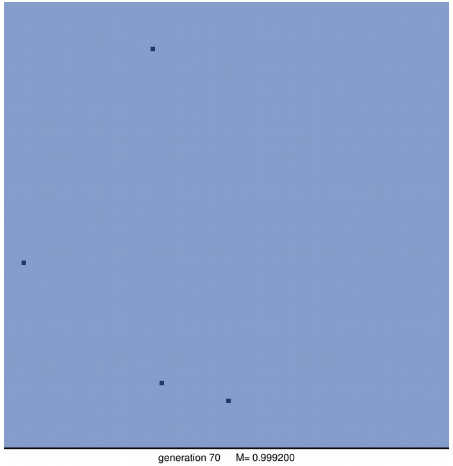
At temperatures **below** the critical point, one spin orientation eventually wins out and the lattice gains a global magnetization.



Initial condition (random)



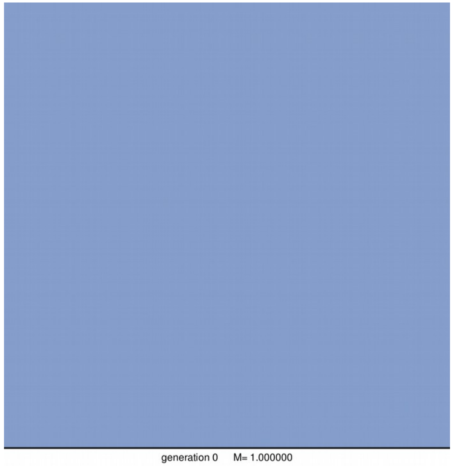
Transient state



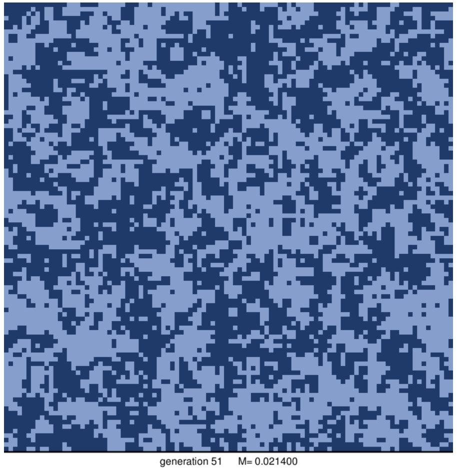
Stead state: (Nearly) all spins have the same alignment, M ~ 1

The Ising model: a laboratory for criticality

At temperatures **above** the critical point, the thermal fluctuations dominate and the spin sites can’t maintain a stable orientation, yielding a randomly fluctuating lattice and no global magnetization.



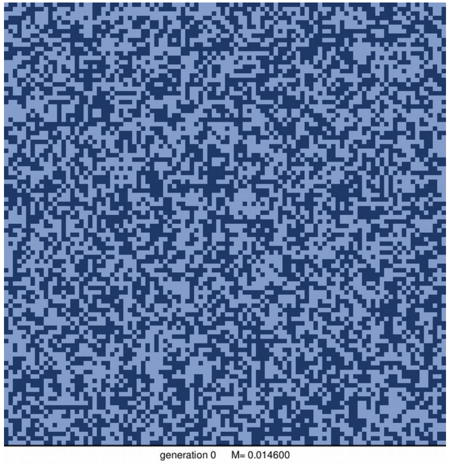
Initial condition
(uniform)



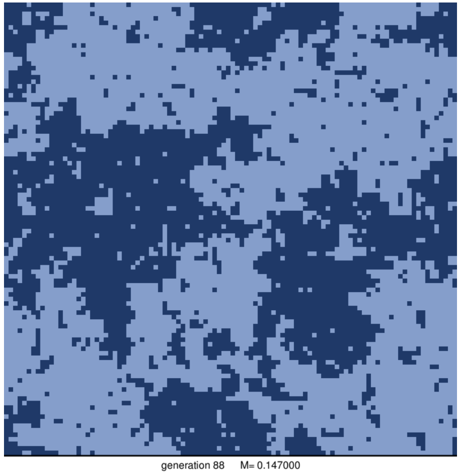
Steady state:
randomly fluctuating,
M ~ 0

The Ising model: a laboratory for criticality

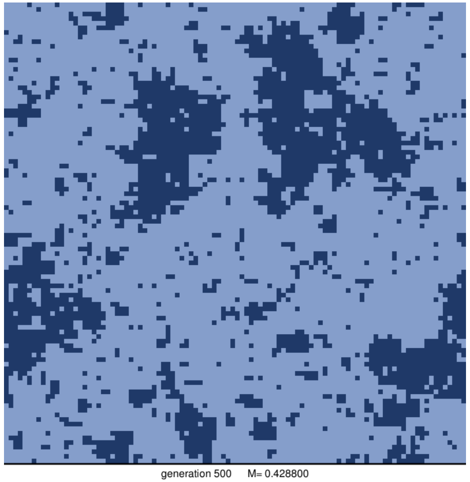
At the **critical temperature**, transient “islands of alignment” appear and disappear, their size distribution being fractal. The global magnetization never stabilizes, visiting all possible values.



Initial condition (random)



Fractal state

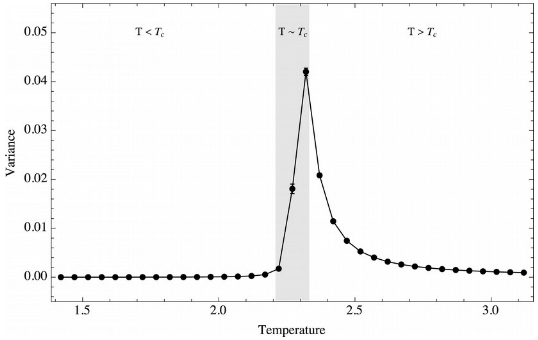


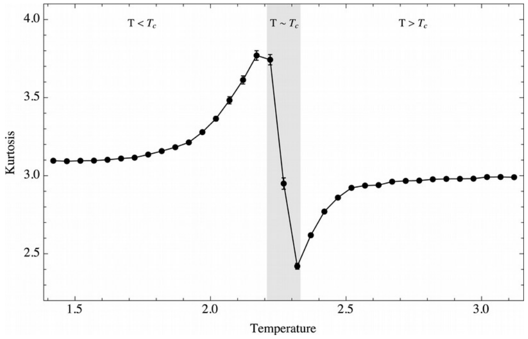
Different fractal state

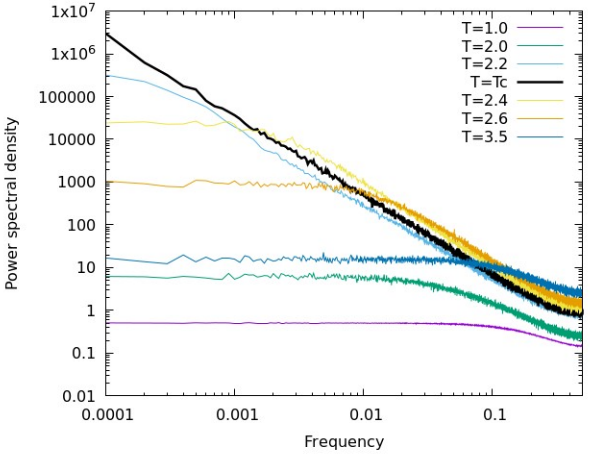
The Ising model: a laboratory for criticality

At this **critical point** many interesting properties emerge:

- The lattice structure becomes **fractal** (spatially scale-invariant);
- The fluctuations of the global magnetization become **temporally scale-invariant**, their power spectrum becoming a **power-law**;
- The statistics of the magnetization time series diverge or become non-Gaussian;
- Many others ...







Criticality in the brain

Recent results in neuroscience suggest that the human brain follows critical dynamics.

REVIEW ARTICLES | INSIGHT
PUBLISHED ONLINE: 1 OCTOBER 2010 | DOI: 10.1038/NPHYS1803
nature physics

Emergent complex neural dynamics

Dante R. Chialvo[1,2]*

PRL **110,** 178101 (2013) PHYSICAL REVIEW LETTERS 26 APRIL 2013

Brain Organization into Resting State Networks Emerges at Criticality on a Model of the Human Connectome

Ariel Haimovici,[1,2] Enzo Tagliazucchi,[3] Pablo Balenzuela,[1,2] and Dante R. Chialvo[2,4,5]

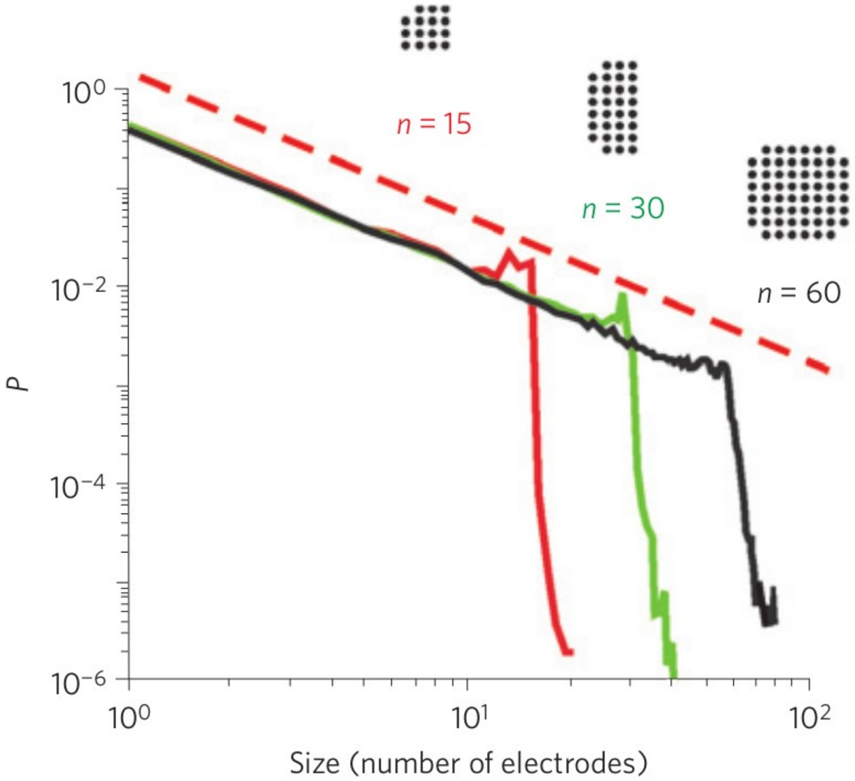
frontiers in SYSTEMS NEUROSCIENCE
EDITORIAL
published: 25 February 2015
doi: 10.3389/fnsys.2015.00022

Criticality as a signature of healthy neural systems

Paolo Massobrio[1], Lucilla de Arcangelis[2], Valentina Pasquale[3], Henrik J. Jensen[4] and Dietmar Plenz[5]*

Criticality in the brain

Size of ‘neuronal avalanches’

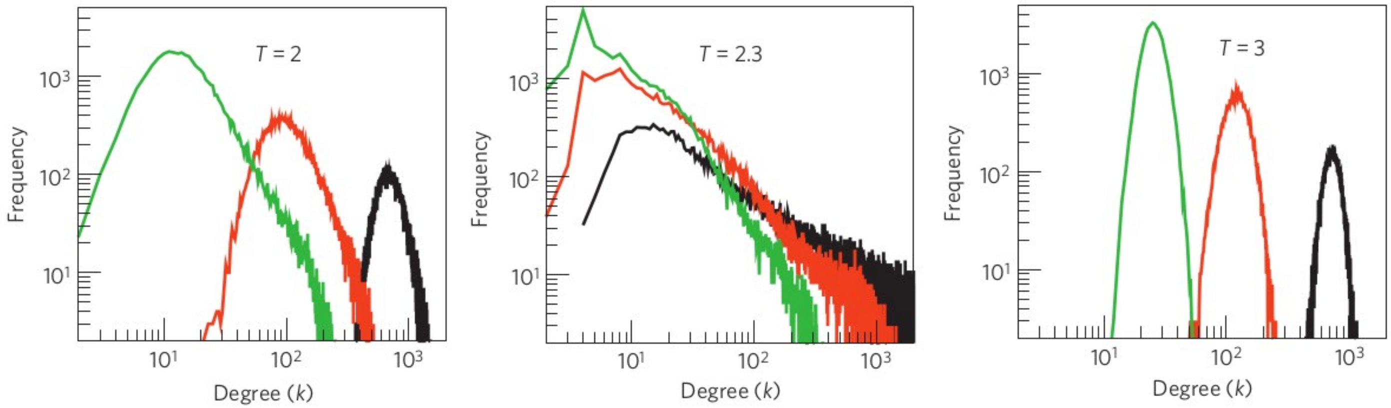


Beggs & Plenz (2003)

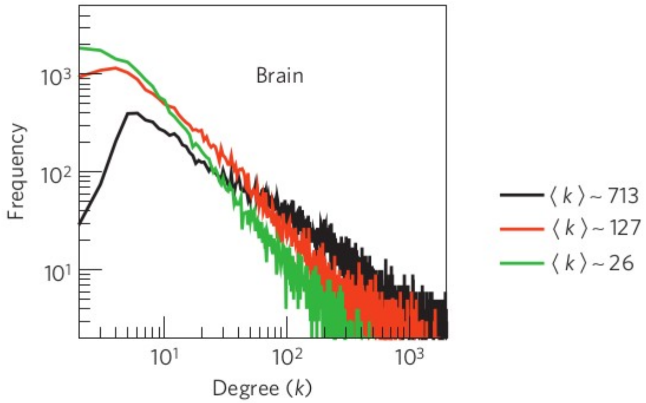
Criticality in the brain

Correlation networks analysis (degree distributions)

Ising



Brain fMRI

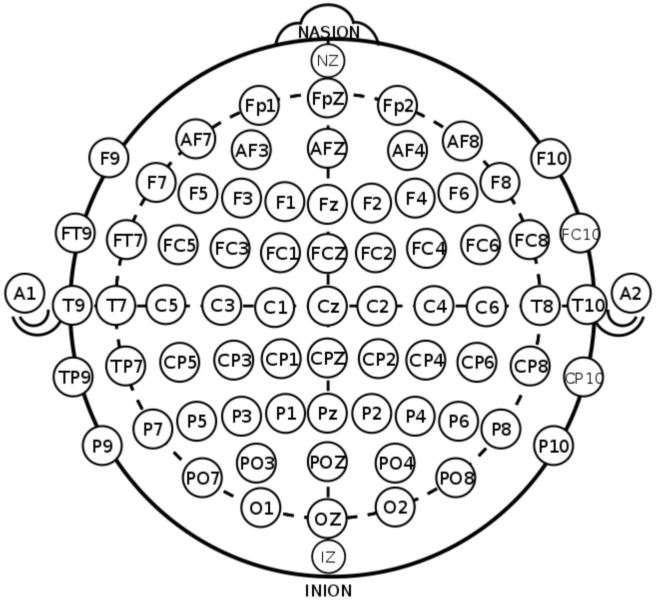


Fraiman et al. (2009)

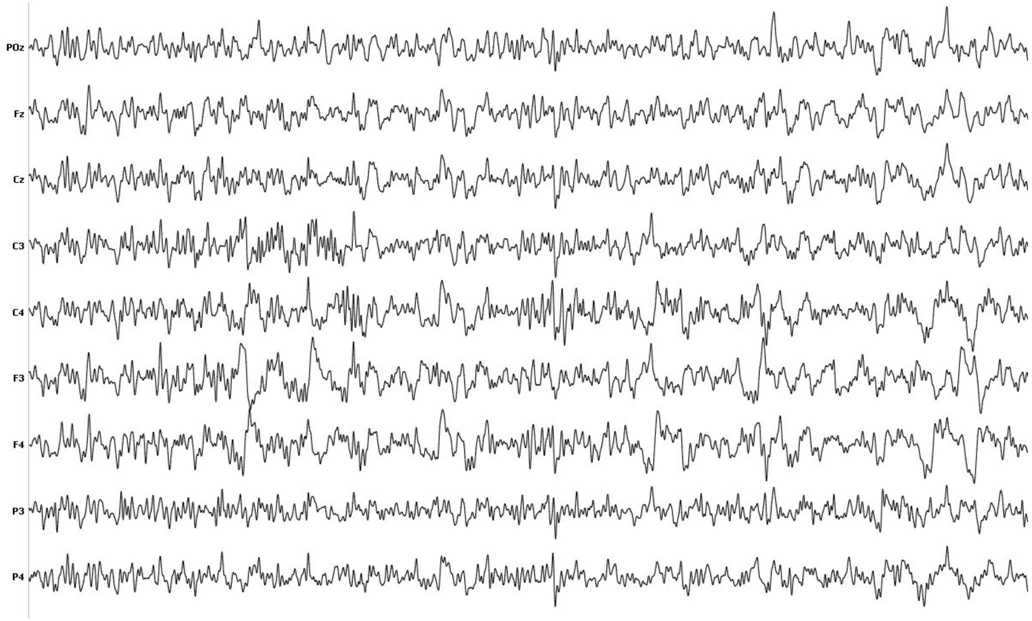
Multi-signal Criticality

The real-life systems that we study usually emit multiple simultaneous time signals.

How can we determine and measure criticality in such systems?



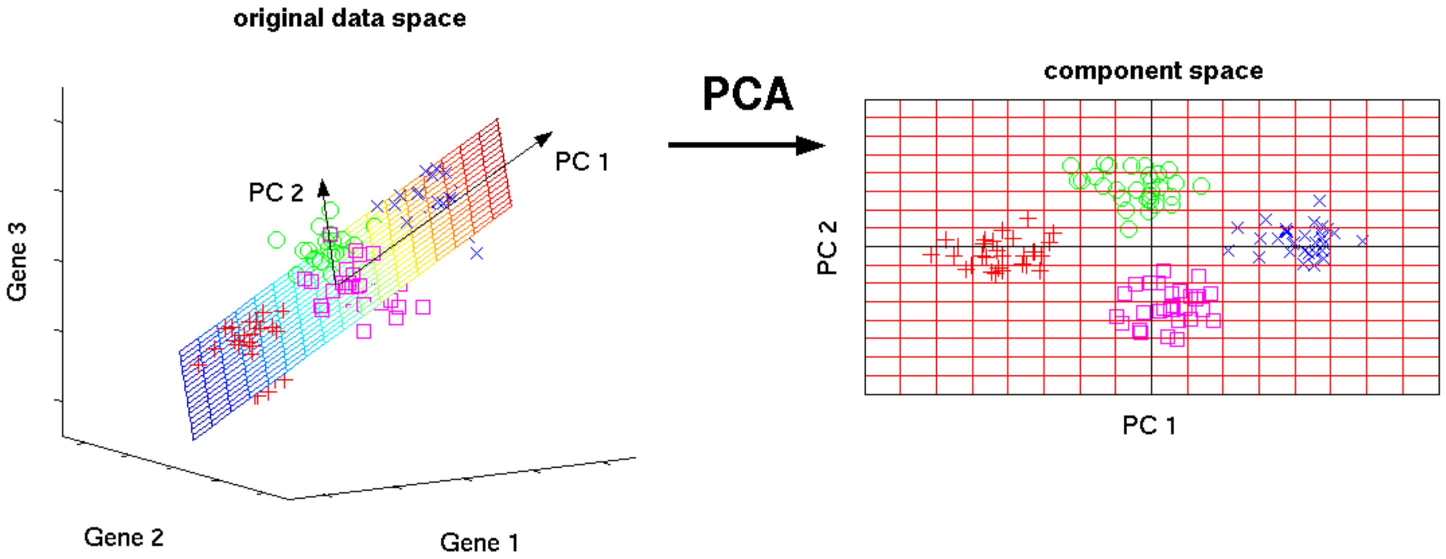
Electrodes (10-20 system)



Electroencephalographic (EEG) recording

Principal Component Analysis

Find a different basis for the data such that variance is maximized. Then we can more clearly see the “components” that make up the data.



Eigendecomposition of covariance matrix $\mathbf{X}^T\mathbf{X}$,
or equivalently Singular Value Decomposition of $\mathbf{X} = \mathbf{U\Sigma W}^T$

Principal Component Analysis

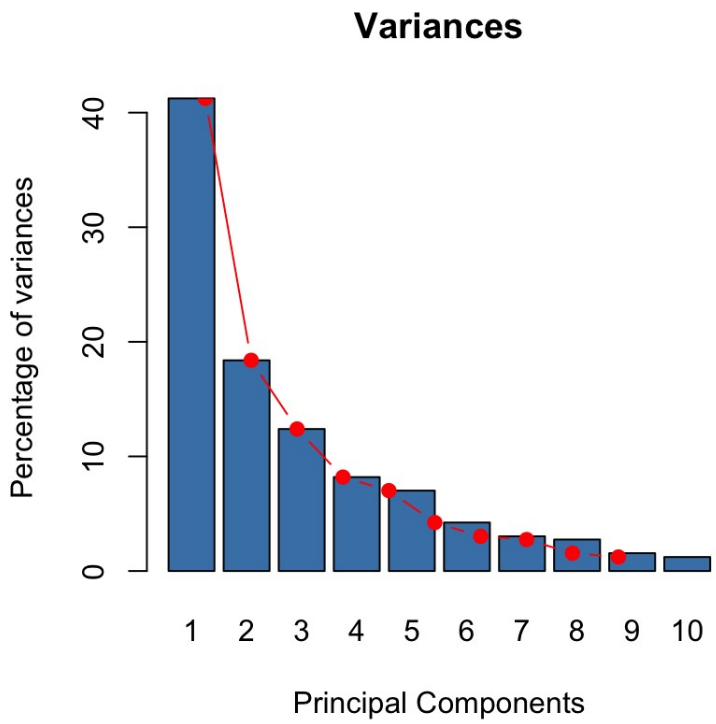
This has been applied, e.g., by the neurobiology team of R. Romo to clearly separate and identify the main areas in the monkey brain that participate in decision-making.

Stimulus-response tests

The principal components capture different parts of the stimulus-memory-decision-response process (Kobak et al. 2016)

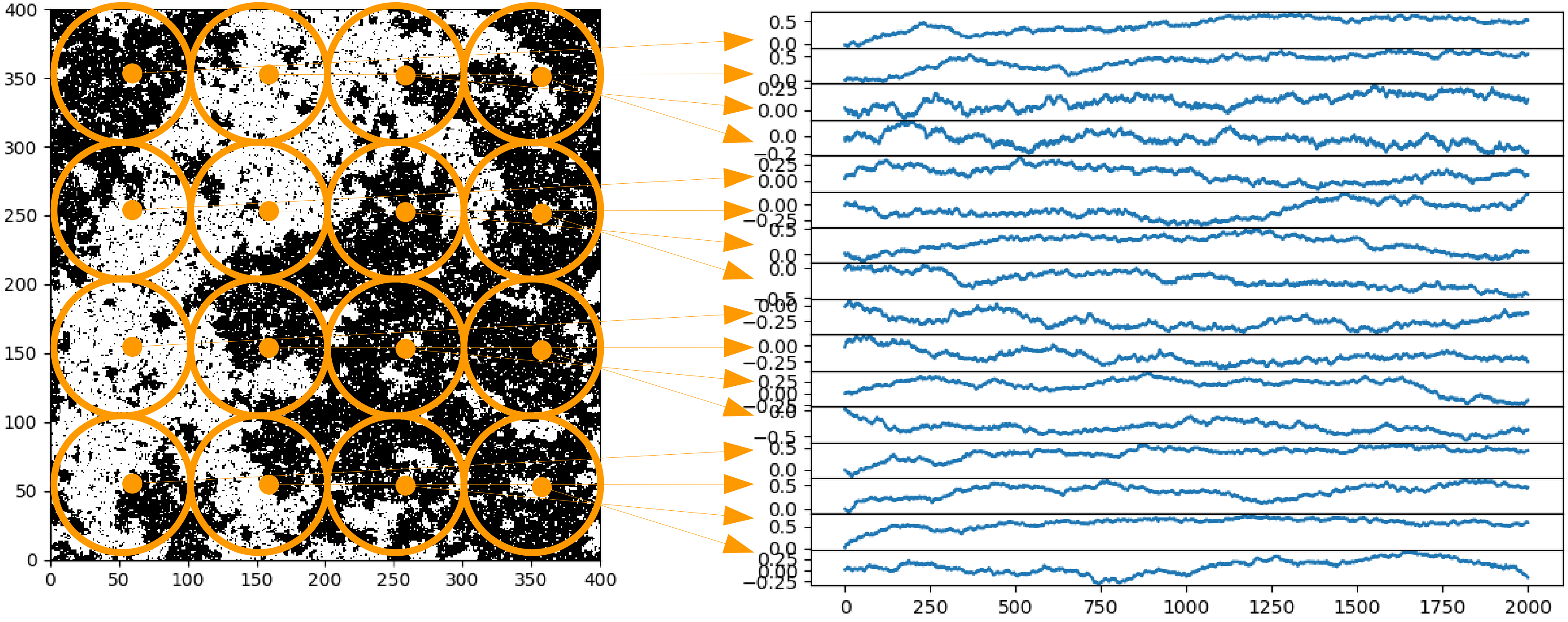
Principal Component Analysis

The eigenvalues (the “eigenspectrum”) tell us how much of the total variance is captured by each mode.



The “Ising Brain”

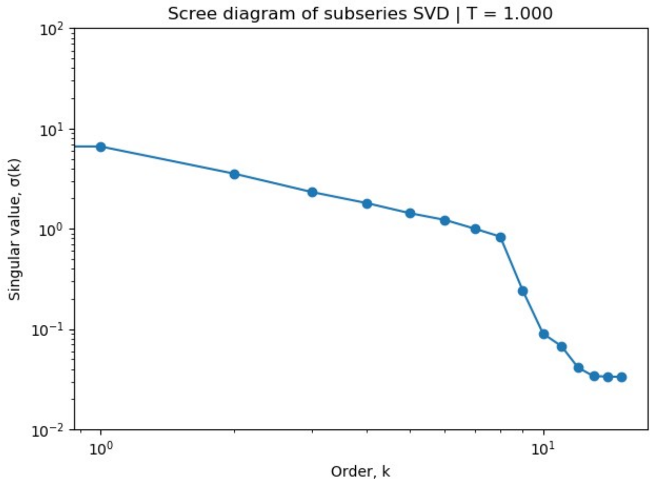
We can try to model this process by “doing an EEG to the Ising model”: we simultaneously sample a set of distinct regions in an Ising lattice, obtaining an ensemble of simultaneous signals.



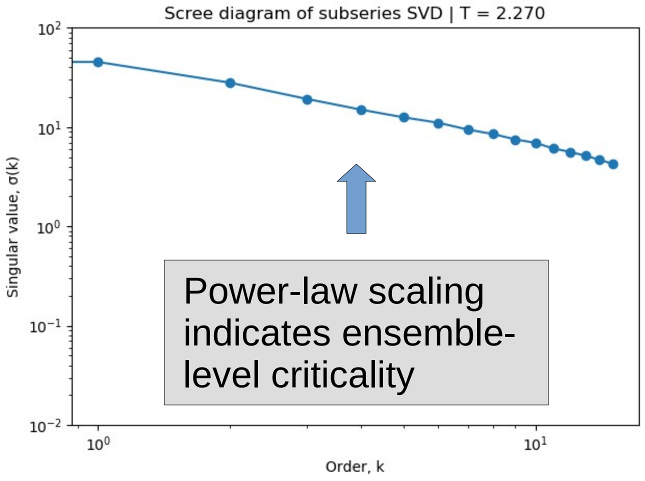
The “Ising Brain”

We then apply PCA, and study how the components scale in importance.

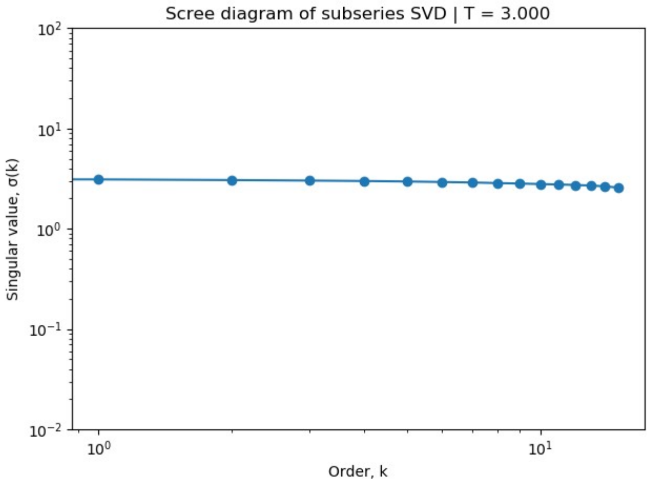
A near-perfect (nontrivial) power-law scaling is found only at the critical temperature!



T < Tc



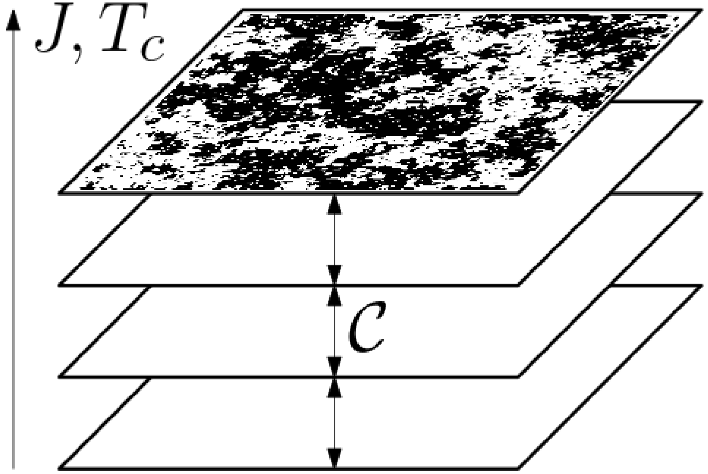
T = Tc

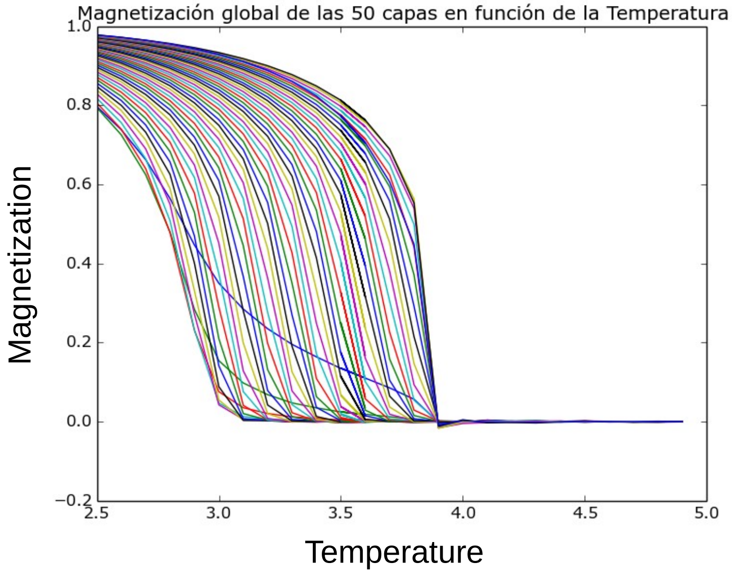


T > Tc

Multi-layer Ising

To better understand how criticality is manifested in multi-channel data, we are studying a version of the Ising model with multiple, coupled layers. Work of Miguel Sánchez-Islas.

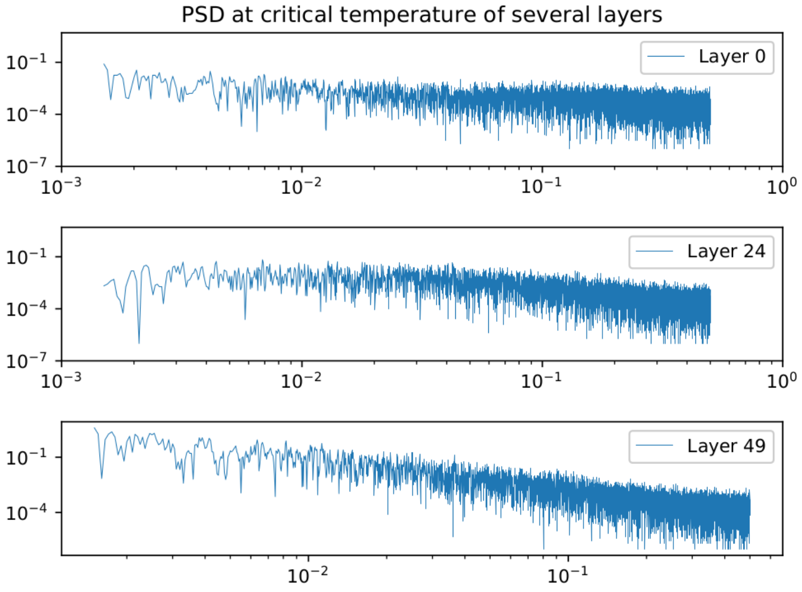


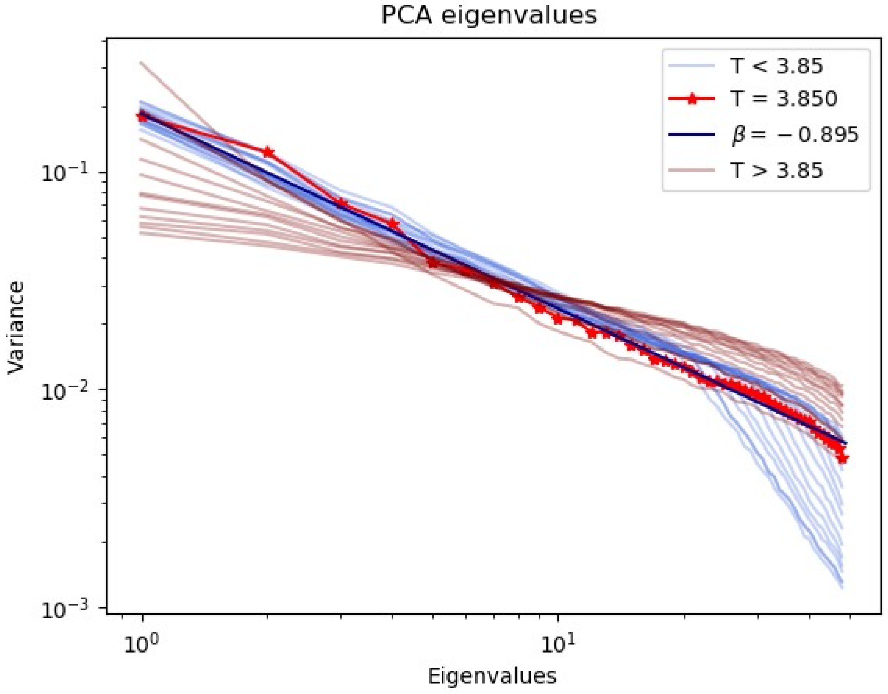


Each layer has its own transition temperature.

Multi-layer Ising

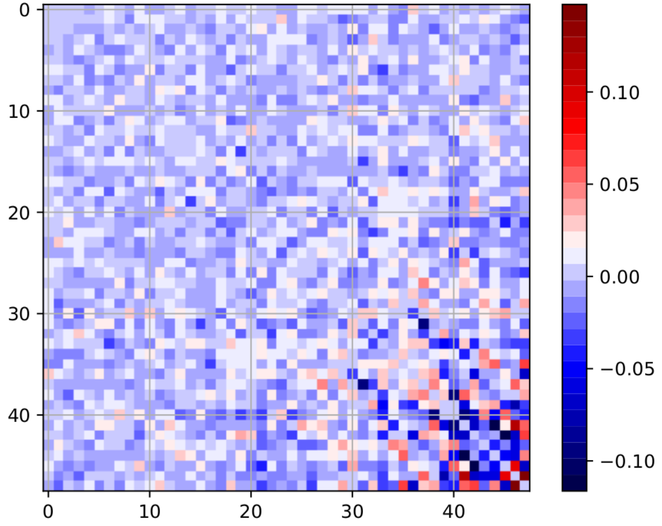
The PCA eigenspectrum reveals a power-law scaling, being strongest when all layers have begun transitioning, even if not all layers exhibit a critical power spectrum.



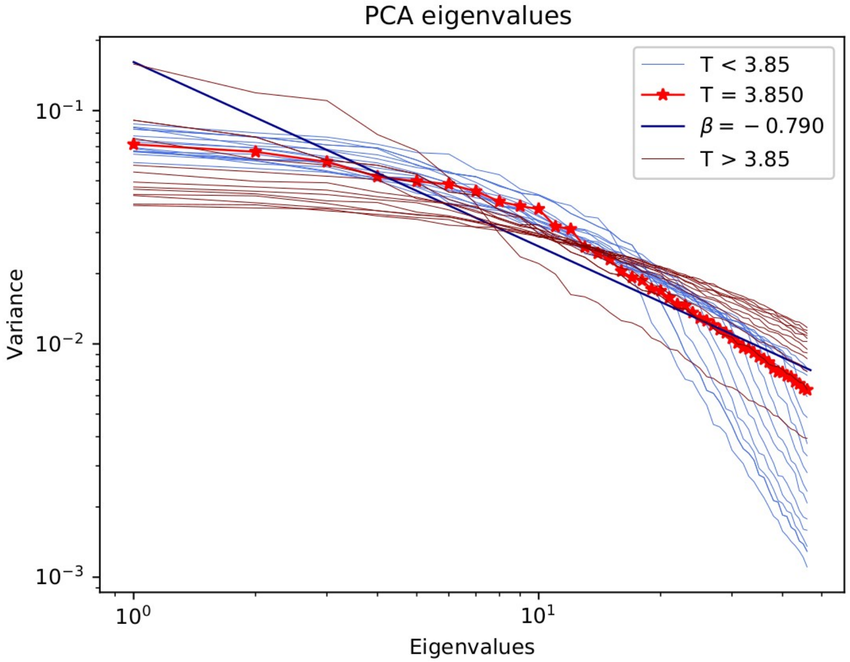


Multi-layer Ising

This clean power-law scaling is destroyed when the correlations between the time series of the layers are artificially removed (by randomizing the phases in Fourier space).



Correlation matrix of the time series

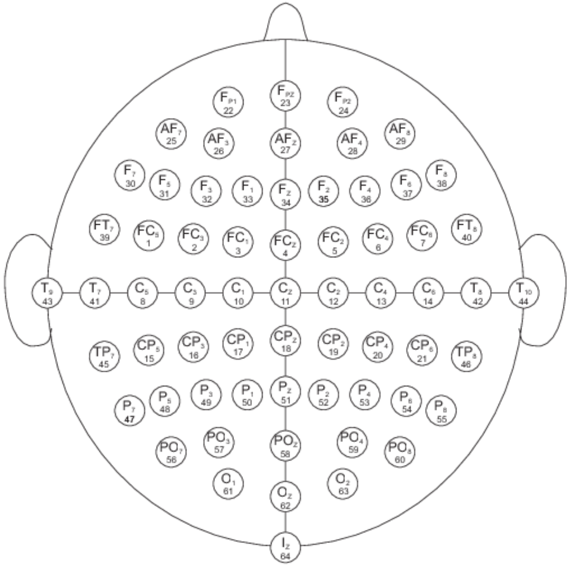


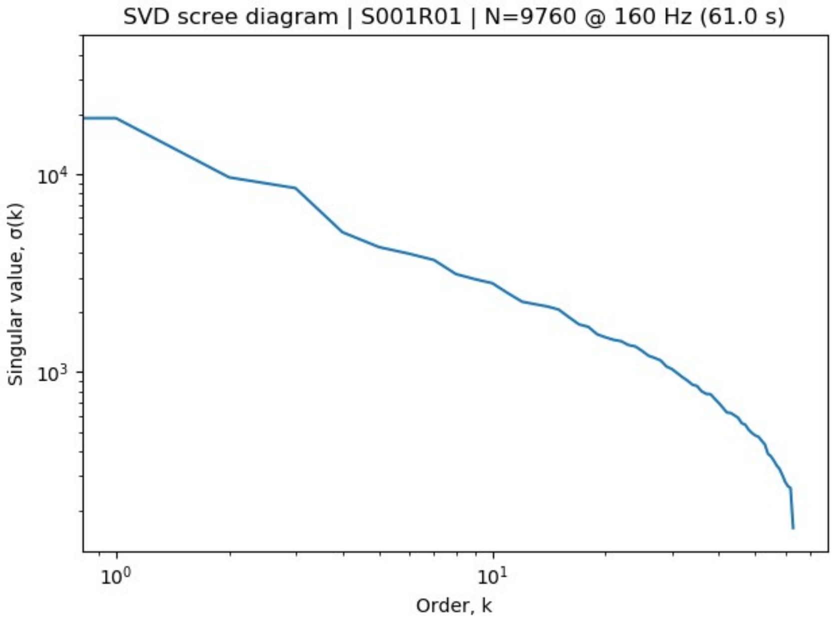
Hence, the PCA seems to be capturing a "**collective criticality**".

The real brain?

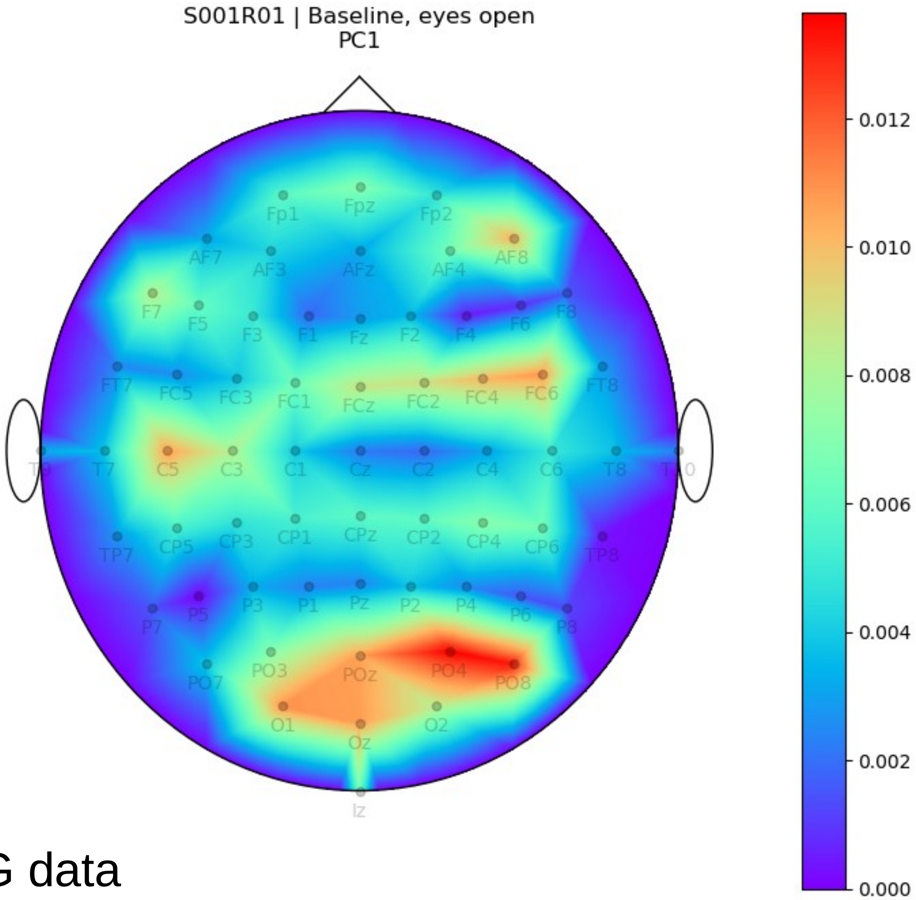
Similar results can be found, approximately, in real EEGs obtained from Physionet.

We apply PCA and study the scaling of the components to 64-channel EEGs in resting conditions.





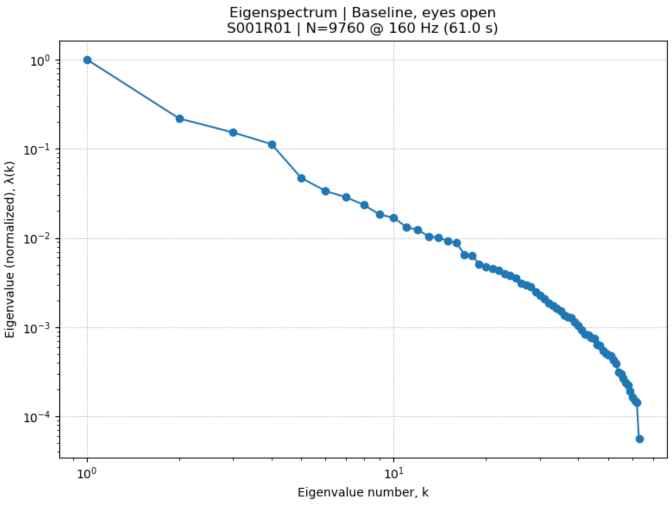
We can spatially plot the eigenmodes

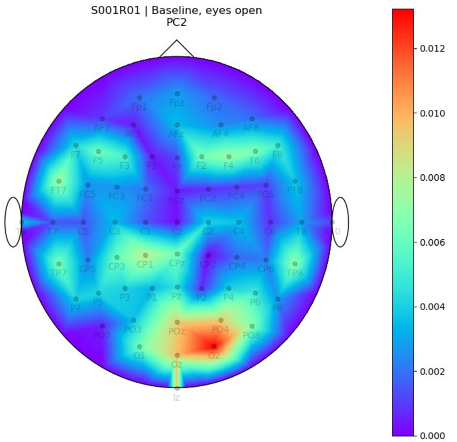


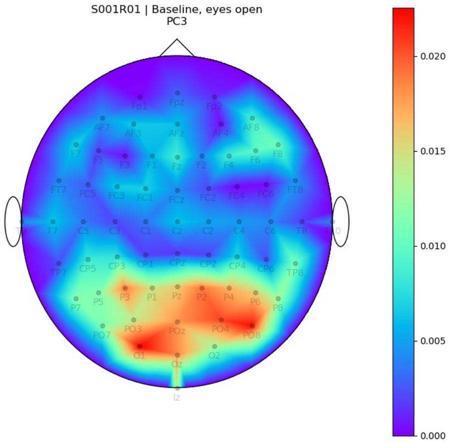
64-channel Physionet EEG data

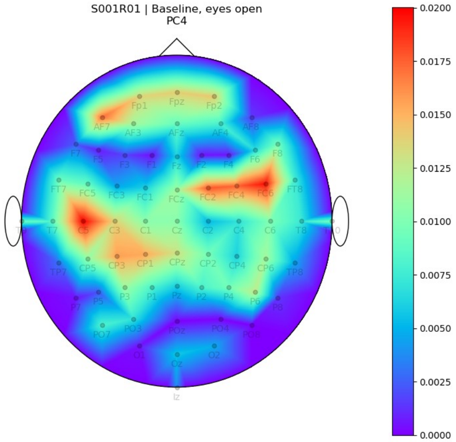
We can spatially plot the eigenmodes

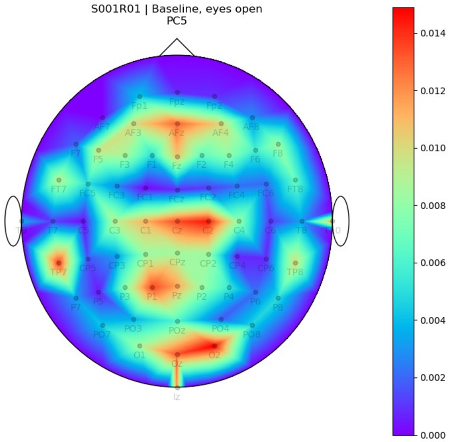
Next 4 eigenmodes











But a lot of variation between individuals

PC1 for 5 different individuals