Homework 12

● Graded

Student

苏慧哲

Total Points

30 / 40 pts

Question 1

Multiple Choice

10 / 10 pts

✔ **– 0 pts** Correct

– 3 pts Question 1 Wrong

– 3 pts Question 2 Wrong

– 2 pts Question 3 half wrong

– 4 pts Question 3 Wrong

Question 2

4-COLOR

10 / 10 pts

✔ **+ 3 pts** Prove it is in NP correctly.

✔ **+ 3 pts** Construct the reduction correctly.

✔ **+ 2 pts** Prove "$\Rightarrow$" correctly.

✔ **+ 2 pts** Prove "$\Leftarrow$" correctly.

+ 0 pts All wrong.

Question 3

Reduction from Independent Set

10 / 10 pts

✔ **– 0 pts** Correct

– 2 pts Not prove this problem is in NP.

– 6 pts Wrong or No Reduction: Correct Disjoint Subsets instance construction for Independent Set in $G = (V, E)$ should be $E = E, m = |V|$ and $S_i = \{e \in E | v_i \text{ is incident with } e\}$ i.e. $S_i = \{(v_i, v_j) | v_j \text{ is adjacent to } v_i\}$.

– 3 pts Incomplete Reduction: What is your exact E and S_i construction corresponding to $G = (V, E)$? You cannot simply say two subsets are disjoint if their corresponding vertices are not adjacent.

– 5 pts Wrong Reduction: You should show **Independent Set** $\leq_P$ **Disjoint Subsets** instead of Disjoint Subsets $\leq_P$ Independent Set i.e. You should **construct subsets from a graph** instead of constructing a graph from subsets.

– 2 pts Wrong or Incomplete proof for Disjoint Subsets => Independent Set

– 2 pts Wrong or Incomplete proof for Disjoint Subsets <= Independent Set

– 10 pts Wrong or Blank

Question 4

Exact 4-SAT problem

0 / 10 pts

– 0 pts Correct

– 2 pts Not prove the problem is in NP.

– 5 pts We should not define the value of the variable in 4 SAT since it is the problem that should be solved by us.

– 6 pts Not solve an **EXACT** 4-SAT problem. Please read the question carefully.

– 4 pts Not a proper reduction from 3-SAT.

– 2 pts the proof of 3SAT => 4SAT is wrong or not completed.

– 2 pts the proof of 3SAT <= 4SAT is wrong or not completed.

– 3 pts Proof not completed.

✔ **– 10 pts** blank or mis-matched.

– 8 pts You prove 3 SAT by reduction from 4 SAT.

Questions assigned to the following page: 1 and 2

1.

Q1: B.

Q2: D.

Q3: B C.

2.

(1) 4-color $\in NP$

~~For any~~ Given a graph $G(V, E)$. We can go through all the edges $e(u,v)$ to check whether u, v has the same color. Therefore, 4-color is NP problem.

(2). 3-color $\le_p$ 4-color.

Given any instance $I = G = (V, E)$ of 3-color. Let $I' = G' = (V', E')$ where $V' = V \cup \{n\}$, $E' = E \cup \{\{v, n\} \mid v \in V\}$ and I' is an instance of 4-color instance.

(a) $\Rightarrow$.

Suppose I is a yes-problem of 3-color. Let $f: V \to \{R, G, B\}$.
Let $f': V' \to \{R, G, B, Y\}$. $f'(v) = \begin{cases} f(v), & v \in V \\ Y, & v = n \end{cases}$. $G' = (V', E')$ is a yes-instance of 4-color.
f' is a valid transform from 3-color to 4-color problem.

(b) $\Leftarrow$.

Suppose G' is a yes-instance of 4-color and let $g: V' \to \{R, G, B, Y\}$.
Without loss of generality, let $g(n) = Y$, Since every ~~node~~ vertex is adjacent to n. Only n can be colored to Y. Let $V = V' \setminus \{n\}$. $E = \{(u, v) \mid u, v \in V'$ and $u, v \neq n\}$
$f: V \to \{R, G, B\}$: $f(v) = g(v)$. $G(V, E)$ is a yes-instance of 3-color problem. ~~Therefore f is a valid 3-color~~

Therefor $G \in$ 3-color $\Rightarrow G' \in$ 4-color.

Since 3-color $\in$ NPC, 4-color $\in$ NPC.

Question assigned to the following page: 3

3. Let Q = K-PairwiseDisjoint Subset.

(1) $Q \in NP$.

Given a insme of Q, $I = (E, \{S_1, S_2 \dots S_m\})$

where chose $I' = \{S_{a_1} \dots S_{a_k}\}$ from $\{S_1, \dots S_m\}$, then we can check any two set $S_1, S_2 \in I'$ are disjoint is $O(|E|)$ where E has polynomial length.

we only have to chec $C_2^k = O(m^2)$ pairs.

Therefor $Q \in NP$.

(2) K-IndependentSet $\le_p$ K-PairwiseDisjointSubset

For ever insance of Q $I = (E, \{S_1 \dots S_m\})$ let $G' = (V', E')$ be an instance of K-IndependenSet problem. Let $V' = \{1, 2 \dots, m\}$ be the m subsets of E.

$E = \{\{i, j\} \mid S_i \cap S_j \ne \phi\}$

(a) $\Rightarrow$ Suppose $I = (E, \{S_1 \dots S_m\}) \in Q$ is an yes-insance of Q.
there fore we have $A = \{a_{k_1} \dots a_{k_m}\}$ for any $a_i, a_j \in A$ a_i and a_j are disjoint. Therefor, in G', where have $\{k_1 \dots k_m\}$ $\forall i, j \in A$, i and j are not connected. There fore G' is also an yes-insance of K-IndependantSet.

(b) $\Leftarrow$ Suppose G' is a yes-insance of K-Independent set.
let $A = \{a_1, \dots, a_k\} \in \{1, 2 \dots m\} = V'$ be an independent set. for every pair a_i, a_j in the independant set., a_i, a_j are not connected. $S_{a_i} \cap S_{a_j} = \phi$.
there for $A = \{S_{a_1} \dots S_{a_k}\}$ are pairwise disjoint.
There fore $I = (E, \{S_1, \dots S_m\})$ is a yes-instance of Q.

There fore, ~~Q polynomial transforms to K-IndependentSet~~
K-IndepentSet can be polynomial transform to Q

Since k-Independent set $\in$ NPC, $Q \in$ NPC.