## SUPPLEMENTARY MATERIALS

1. Endemic Equilibrium ( $E_{Vb}^*$ ) in the Stability Analysis of the Avian-Only Vaccination Model. Using the Wolfram Mathematica, we let Equations (49) ~ (51) = 0 to solve for  $S_b, V_b$ , and  $I_b$  at EE.

## Input

Solve  $[(1-p)*\Lambda b-\mu b*Sb-(\beta b*Sb*Ib)/(Hb+Ib)==0 \&\& p*\Lambda b-(1-\phi)*(\beta v*Vb*Ib)/(Hv+Ib)-\mu b*Vb == 0 \&\& (\beta b*Sb*Ib)/(Hb+Ib)+(1-\phi)*(\beta v*Vb*Ib)/(Hv+Ib) - (\mu b+\delta b)*Ib == 0, {Sb,Vb,Ib}]$ 

## **Output**

{ $Sb > (\Lambda b - p \Lambda b)/\mu b, Vb > (p \Lambda b)/\mu b, Ib > 0$ },

 ${\bf Sb}$ ->(Hb βv Λb-Hb Hv δb μb+Hb Λb μb-Hv Λb μb-Hb p Λb μb+Hv p Λb μb-Hb Hv μb<sup>2</sup>-Hb βν Λb φ-(Hb βν δb (βb βν Λb-Ην βb δb μb-Hb βν δb μb+βb Λb μb-p βb Λb μb+p βν Λb μb-Hν  $βb μb^2$ -Hb  $βv μb^2$ -Hb  $δb μb^2$ -Hv  $δb μb^2$ -Hb  $μb^3$ -Hv  $μb^3$ -βb βv Λb φ+Hb <math>βv δb μb φ-p <math>βv Λb μbΦ+Hb βv μb<sup>2</sup> Φ-\[Sqrt](-4 (-Hv βb Λb μb+Hv p βb Λb μb-Hb p βv Λb μb+Hb Hv δb μb<sup>2</sup>+Hb Hv  $\mu b^3 + Hb p \beta v \Lambda b \mu b \phi$ ) ( $\beta b \beta v \delta b + \beta b \beta v \mu b + \beta b \delta b \mu b + \beta v \delta b \mu b^2 + \beta v \mu b^2 + \delta b \mu b^2 + \mu b^3 - \beta b$ βv δb φ-βb βv μb φ-βv δb μb φ-βv  $μb^2$  φ)+(-βb βv Λb+Hv βb δb μb+Hb βv δb μb-βb Λb μb+p βbΛb μb-p βv Λb μb+Hv βb μb^2+Hb βv μb^2+Hb δb μb^2+Hv δb μb^2+Hb μb^3+βb  $\beta$ v Λb φ-Hb  $\beta$ v δb  $\mu$ b φ+p  $\beta$ v Λb  $\mu$ b φ-Hb  $\beta$ v  $\mu$ b^2 φ)<sup>2</sup>))/(2 ( $\beta$ b  $\beta$ v δb+ $\beta$ b  $\beta$ v  $\mu$ b+ $\beta$ b δb  $\mu$ b+ $\beta$ v δb  $\mu$ b+βb  $\mu$ b<sup>2</sup>+βv  $\mu$ b<sup>2</sup>+δb  $\mu$ b<sup>2</sup>+ $\mu$ b<sup>3</sup>-βb βv δb φ-βb βv  $\mu$ b φ-βv δb  $\mu$ b φ-βv  $\mu$ b<sup>2</sup> φ))-(Hb βv  $\mu$ b (βb βv  $\Lambda$ b-Hv βb δb μb-Hb βv δb μb+βb  $\Lambda$ b μb-p βb  $\Lambda$ b μb+p βv  $\Lambda$ b μb-Hv βb μb²-Hb βv μb²-Hb δb  $\mu$ b<sup>2</sup>-Hv δb  $\mu$ b<sup>2</sup>-Hb  $\mu$ b<sup>3</sup>-Hv  $\mu$ b<sup>3</sup>-βb  $\beta$ v Λb  $\phi$ +Hb  $\beta$ v δb  $\mu$ b  $\phi$ -p  $\beta$ v Λb  $\mu$ b  $\phi$ +Hb  $\beta$ v  $\mu$ b<sup>2</sup>  $\phi$ -\[Sqrt](-4 (-Hv βb Λb μb+Hv p βb Λb μb-Hb p βv Λb μb+Hb Hv δb μb<sup>2</sup>+Hb Hv μb<sup>3</sup>+Hb p βv Λb μb  $\phi$ ) (βb βv δb+βb βv μb+βb δb μb+βv δb μb+βb μb<sup>2</sup>+βv μb<sup>2</sup>+δb μb<sup>2</sup>+μb<sup>3</sup>-βb βv δb φ-βb βv μb φ-βv $\mu b + H v \beta b \mu b^2 + H b \beta v \mu b^2 + H b \delta b \mu b^2 + H v \delta b \mu b^2 + H b \mu b^3 + H v \mu b^3 + \beta b \beta v \Lambda b \phi - H b \beta v$  $\delta b$  μb  $\phi$ +p  $\beta v$   $\Delta b$  μb  $\phi$ -Hb  $\beta v$  μb $^2$   $\phi)^2)))/(2$  ( $\beta b$   $\beta v$   $\delta b$ + $\beta b$   $\beta v$  μb+ $\beta b$   $\delta b$  μb+ $\beta v$   $\delta b$  μb+ $\beta b$  μb $^2$ + $\beta v$  $\mu b^2 + \delta b \ \mu b^2 + \mu b^3 - \beta b \ \beta v \ \delta b \ \phi - \beta b \ \beta v \ \mu b \ \phi - \beta v \ \delta b \ \mu b \ \phi - \beta v \ \mu b^2 \ \phi))$ -(Hb  $\delta b \ \mu b \ (\beta b \ \beta v \ \Lambda b - Hv \ \beta b \ \delta b \ A b - Hv \ \beta b \ \delta b \ A b - Hv \ \beta b \ \delta b$  $\mu$ b-Hb βv δb  $\mu$ b+βb Λb  $\mu$ b-p βb Λb  $\mu$ b+p βv Λb  $\mu$ b-Hv βb  $\mu$ b<sup>2</sup>-Hb βv  $\mu$ b<sup>2</sup>-Hb δb  $\mu$ b<sup>2</sup>-Hv δb  $\mu$ b<sup>2</sup>-Hb  $\mu$ b<sup>3</sup>-Hv  $\mu$ b<sup>3</sup>-βb βv Λb  $\phi$ +Hb βv δb  $\mu$ b  $\phi$ -p βv Λb  $\mu$ b  $\phi$ +Hb βv  $\mu$ b<sup>2</sup>  $\phi$ -\[Sqrt](-4 (-Hv βb Λb  $\mu$ b+Hv p βb Λb  $\mu$ b-Hb p βv Λb  $\mu$ b+Hb Hv δb  $\mu$ b<sup>2</sup>+Hb Hv  $\mu$ b<sup>3</sup>+Hb p βv Λb  $\mu$ b φ) (βb βv δb+βb βν μb+βb δb μb+βν δb μb+βν μb<sup>2</sup>+βν μb<sup>2</sup>+δb μb<sup>2</sup>+μb<sup>3</sup>-βb βν δb φ-βb βν μb φ-βν δb μb φ-βν μb<sup>2</sup>  $\phi$ )+(-βb βv Λb+Hv βb δb μb+Hb βv δb μb-βb Λb μb+p βb Λb μb-p βv Λb μb+Hv βb μb^2+Hb  $\beta v \mu b^2 + Hb \delta b \mu b^2 + Hv \delta b \mu b^2 + Hb \mu b^3 + Hv \mu b^3 + \beta b \beta v \Lambda b \phi - Hb \beta v \delta b \mu b \phi + p \beta v \Lambda b \mu b$  $\phi$ -Hb βv μb<sup>2</sup>  $\phi$ )))/(2 (βb βv δb+βb βv μb+βb δb μb+βv δb μb+βb μb<sup>2</sup>+βv μb<sup>2</sup>+δb μb<sup>2</sup>+μb<sup>3</sup>-βb βv δb φ-βb βv μb φ-βv δb μb φ-βv  $μb^2$   $φ))-(Hb <math>μb^2$  (βb βv Λb-Hv βb δb μb-Hb βv δb μb+βb Λb $\mu$ b-p βb Λb  $\mu$ b+p βv Λb  $\mu$ b-Hv βb  $\mu$ b<sup>2</sup>-Hb βv  $\mu$ b<sup>2</sup>-Hb δb  $\mu$ b<sup>2</sup>-Hv δb  $\mu$ b<sup>3</sup>-Hv  $\mu$ b<sup>3</sup>-βb βv  $\Lambda b \phi + Hb \beta v \delta b \mu b \phi - p \beta v \Lambda b \mu b \phi + Hb \beta v \mu b^2 \phi - [Sqrt](-4 (-Hv βb Λb μb + Hv p βb Λb μb - Hb p)$ βv Λb μb+Hb Hv δb μb<sup>2</sup>+Hb Hv μb<sup>3</sup>+Hb p βv Λb μb φ) (βb βv δb+βb βv μb+βb δb μb+βv δb  $\mu b + \beta b \mu b^2 + \beta v \mu b^2 + \delta b \mu b^2 + \mu b^3 - \beta b \beta v \delta b \phi - \beta b \beta v \mu b \phi - \beta v \delta b \mu b \phi - \beta v \mu b^2 \phi) + (-\beta b \beta v \Lambda b + Hv \beta b \mu b^2 + \delta b$  $\delta b \mu b + H b \beta v \delta b \mu b - \beta b \Lambda b \mu b + p \beta b \Lambda b \mu b - p \beta v \Lambda b \mu b + H v \beta b \mu b^2 + H b \beta v \mu b^2 + H b \delta b$  $\mu b^2 + Hv \delta b \mu b^2 + Hb \mu b^3 + Hv \mu b^3 + \beta b \beta v \delta b \mu b \delta + p \beta v \delta b \mu b \delta + Hb \beta v \mu b^2$  $(4)^2$ ))/(2 ( $(3b \beta v \delta b + \beta b \beta v \mu b + \beta b \delta b \mu b + \beta v \delta b \mu b + \beta b \mu b^2 + \beta v \mu b^2 + \delta b \mu b^2 + \mu b^3 - \beta b \beta v \delta b \phi - \beta b \beta v$  $\mu$ b  $\phi$ -βν δb  $\mu$ b  $\phi$ -βν  $\mu$ b<sup>2</sup>  $\phi$ ))+(Hb βν δb  $\phi$  (βb βν Λb-Hν βb δb  $\mu$ b-Hb βν δb  $\mu$ b+βb Λb  $\mu$ b-p βb  $\Lambda$ b μb+p βv  $\Lambda$ b μb-Hv βb μb<sup>2</sup>-Hb βv μb<sup>2</sup>-Hb δb μb<sup>2</sup>-Hv δb μb<sup>2</sup>-Hv μb<sup>3</sup>-βb βv  $\Lambda$ b φ+Hb  $\beta$ ν δb μb  $\phi$ -p  $\beta$ ν Λb μb  $\phi$ +Hb  $\beta$ ν μb<sup>2</sup>  $\phi$ -\[Sqrt](-4 (-Hν  $\beta$ b Λb μb+Hν p  $\beta$ b Λb μb-Hb p  $\beta$ ν Λb  $\mu$ b+Hb Hv δb  $\mu$ b<sup>2</sup>+Hb Hv  $\mu$ b<sup>3</sup>+Hb p βv Λb  $\mu$ b φ) (βb βv δb+βb βv  $\mu$ b+βb δb  $\mu$ b+βv δb  $\mu$ b+βb  $\mu b^2 + \beta v \mu b^2 + \delta b \mu b^2 + \mu b^3 - \beta b \beta v \delta b \phi - \beta b \beta v \mu b \phi - \beta v \delta b \mu b \phi - \beta v \mu b^2 \phi) + (-\beta b \beta v \Lambda b + Hv \beta b \delta b)$  $\mu b+Hb$  βν δb  $\mu b-\beta b$  Λb  $\mu b+p$  βb Λb  $\mu b-p$  βν Λb  $\mu b+Hv$  βb  $\mu b^2+Hb$  βν  $\mu b^2+Hb$  δb  $\mu b^2+Hv$  $\delta b \mu b^2 + Hb \mu b^3 + Hv \mu b^3 + \beta b \beta v \Lambda b \phi - Hb \beta v \delta b \mu b \phi + p \beta v \Lambda b \mu b \phi - Hb \beta v \mu b^2 \phi^2))/(2 (βb)$ βv δb+βb βv μb+βb δb μb+βv δb μb+βb  $μb^2+βv$   $μb^2+δb$   $μb^2+μb^3-βb$  βv δb φ-βb βv μb φ-βv δb $\mu$ b  $\phi$ -βν  $\mu$ b<sup>2</sup>  $\phi$ ))+(Hb βν  $\mu$ b  $\phi$  (βb βν Λb-Hν βb δb  $\mu$ b-Hb βν δb  $\mu$ b+βb Λb  $\mu$ b-p βb Λb  $\mu$ b+p βν  $\Lambda$ b μb-Hv βb μb<sup>2</sup>-Hb βv μb<sup>2</sup>-Hb δb μb<sup>2</sup>-Hv δb μb<sup>3</sup>-Hv μb<sup>3</sup>-βb βv  $\Lambda$ b φ+Hb βv δb μb φ-p βv Λb μb φ+Hb βv μb φ-[Sqrt](-4 (-Hv <math>βb Λb μb+Hv p βb Λb μb-Hb p βv Λb μb+Hb Hv δb $\mu b^2 + Hb \ Hv \ \mu b^3 + Hb \ p \ \beta v \ \Delta b \ \mu b \ \phi) \ (\beta b \ \beta v \ \delta b + \beta b \ \beta v \ \mu b + \beta b \ \delta b \ \mu b + \beta v \ \delta b \ \mu b + \beta b \ \mu b^2 + \beta v \ \mu b^2 + \delta b$  $\mu b^2 + \mu b^3 - \beta b$  by  $\delta b$   $\phi - \beta b$  by  $\mu b$   $\phi - \beta v$   $\delta b$   $\mu b$   $\phi - \beta v$   $\mu b^2$   $\phi$ ) + (-\beta b \beta v \Lambda b + Hv \beta b \delta b \mu b + Hb \beta v \delta b \mu b - \beta b Λb μb+p βb Λb μb-p βv Λb μb+Hv βb μb $^2$ +Hb βv μb $^2$ +Hb δb μb $^2$ +Hv δb μb $^2$ +Hb  $\mu$ b^3+Hv  $\mu$ b^3+βb  $\beta$ v  $\Lambda$ b  $\phi$ -Hb  $\beta$ v  $\delta$ b  $\mu$ b  $\phi$ +p  $\beta$ v  $\Lambda$ b  $\mu$ b  $\phi$ -Hb  $\beta$ v  $\mu$ b^2  $\phi$ )))/(2 ( $\beta$ b  $\beta$ v  $\delta$ b+ $\beta$ b  $\beta$ v  $\mu b + \beta b \ \delta b \ \mu b + \beta v \ \delta b \ \mu b^2 + \beta v \ \mu b^2 + \delta b \ \mu b^2 + \mu b^3 - \beta b \ \beta v \ \delta b \ \phi - \beta b \ \beta v \ \phi b \ \phi - \beta v \ \delta b \ \mu b \ \phi - \beta v \ \mu b^2$  $\phi$ )))/(-Hv  $\beta$ b  $\mu$ b+Hb  $\beta$ v  $\mu$ b+Hb  $\mu$ b<sup>2</sup>-Hv  $\mu$ b<sup>2</sup>-Hb  $\beta$ v  $\mu$ b  $\phi$ ),

Vb->(-Hv βb  $\Lambda b$ +Hb Hv  $\delta b$  µb+Hb p  $\Lambda b$  µb-Hv p  $\Lambda b$  µb+Hb Hv µb<sup>2</sup>+(Hv βb  $\delta b$  (βb βv  $\Lambda b$ -Hv βb δb μb-Hb βv δb μb+βb Λb μb-p βb Λb μb+p βv Λb μb-Hv βb  $μb^2-Hb$  δv  $μb^2-Hv$  $\delta b \mu b^2$ -Hb  $\mu b^3$ -Hv  $\mu b^3$ -βb βv  $\Lambda b \phi$ +Hb βv  $\delta b \mu b \phi$ -p βv  $\Lambda b \mu b \phi$ +Hb βv  $\mu b^2 \phi$ -\[Sqrt](-4 (-Hv βb  $\Lambda$ b μb+Hv p βb  $\Lambda$ b μb-Hb p βv  $\Lambda$ b μb+Hb Hv δb μb<sup>2</sup>+Hb Hv μb<sup>3</sup>+Hb p βv  $\Lambda$ b μb  $\phi$ ) (βb βv  $\delta b + \beta b \beta v \mu b + \beta b \delta b \mu b + \beta v \delta b \mu b + \beta b \mu b^2 + \beta v \mu b^2 + \delta b \mu b^2 + \mu b^3 - \beta b \beta v \delta b \phi - \beta b \beta v \mu b \phi - \beta v \delta b \mu b \phi - \delta v \delta b \mu b \phi -$  $\beta$ ν μ $b^2$  φ)+(-βb βν Λb+Hν βb δb μb+Hb βν δb μb-βb Λb μb+p βb Λb μb-p βν Λb μb+Hν βb $\mu b^2 + Hb \beta v \mu b^2 + Hb \delta b \mu b^2 + Hv \delta b \mu b^2 + Hb \mu b^3 + Hv \mu b^3 + \beta b \beta v \Lambda b \phi - Hb \beta v \delta b \mu b \phi + p$  $\beta v \ \Lambda b \ \mu b \ \varphi - Hb \ \beta v \ \mu b^2 + \beta v \ \mu b^2 + \beta b \ \beta v \ \mu b + \beta b \ \delta b \ \mu b + \beta v \ \delta b \ \mu b^2 + \beta v \ \mu b^2 + \delta b$  $\mu$ b<sup>2</sup>+ $\mu$ b<sup>3</sup>-βb βv δb φ-βb βv  $\mu$ b φ-βv δb  $\mu$ b φ-βv  $\mu$ b<sup>2</sup> φ))+(Hv βb  $\mu$ b (βb βv Λb-Hv βb δb  $\mu$ b-Hb βv  $\delta b$  μb+βb  $\Lambda b$  μb-p βb  $\Lambda b$  μb+p βv  $\Lambda b$  μb-Hv βb μb<sup>2</sup>-Hb βv μb<sup>2</sup>-Hb  $\delta b$  μb<sup>2</sup>-Hv  $\delta b$  μb<sup>2</sup>-Hb μb<sup>3</sup>-Hv  $\mu$ b<sup>3</sup>-βb βv Λb φ+Hb βv δb  $\mu$ b φ-p βv Λb  $\mu$ b φ+Hb βv  $\mu$ b<sup>2</sup> φ-\[Sqrt](-4 (-Hv βb Λb  $\mu$ b+Hv p βb  $\Lambda$ b μb-Hb p βv  $\Lambda$ b μb+Hb Hv δb μb<sup>2</sup>+Hb Hv μb<sup>3</sup>+Hb p βv  $\Lambda$ b μb φ) (βb βv δb+βb βv μb+βb δb  $\Lambda b$ +Hv βb δb  $\mu b$ +Hb βv δb  $\mu b$ -βb  $\Lambda b$   $\mu b$ +p βb  $\Lambda b$   $\mu b$ -p βv  $\Lambda b$   $\mu b$ +Hv βb  $\mu b$ ^2+Hb βv  $\mu b$ ^2+Hb  $\delta b \ \mu b^2 + Hv \ \delta b \ \mu b^2 + Hb \ \mu b^3 + Hv \ \mu b^3 + \beta b \ \beta v \ \Lambda b \ \phi - Hb \ \beta v \ \delta b \ \mu b \ \phi + p \ \beta v \ \Lambda b \ \mu b \ \phi - Hb \ \beta v \ \mu b^2$  $(4)^2$ ))/(2 ( $(3b \ \beta v \ \delta b + \beta b \ \beta v \ \mu b + \beta b \ \delta b \ \mu b + \beta v \ \delta b \ \mu b + \beta b \ \mu b^2 + \beta v \ \mu b^2 + \delta b \ \mu b^2 + \mu b^3 - \beta b \ \beta v \ \delta b \ \phi - \beta b \ \beta v$  $\mu$ b φ-βν δb  $\mu$ b φ-βν  $\mu$ b<sup>2</sup> φ))+(Hν δb  $\mu$ b (βb βν Λb-Hν βb δb  $\mu$ b-Hb βν δb  $\mu$ b+βb Λb  $\mu$ b- $\mu$ b βb Λb  $\mu$ b+p βv Λb  $\mu$ b-Hv βb  $\mu$ b<sup>2</sup>-Hb βv  $\mu$ b<sup>2</sup>-Hb δb  $\mu$ b<sup>2</sup>-Hv δb  $\mu$ b<sup>2</sup>-Hv  $\mu$ b<sup>3</sup>-βb βv Λb  $\phi$ +Hb βv δb μb φ-p βν Λb μb φ+Hb βν μb² φ-\[Sqrt](-4 (-Hv βb Λb μb+Hv p βb Λb μb-Hb p βν Λb μb+Hb Hv δb μb²+Hb Hv μb³+Hb p βν Λb μb φ) (βb βν δb+βb βν μb+βb δb μb+βν δb μb+βb αb μb+βb αb μb²+βν δb μb+βb βν μb φ-βν βb μb φ-βν μb φ-βν μb² φ)+(-βb βν Λb+Hv βb δb μb+Hb βν δb μb-βb Λb μb+p βb Λb μb-p βν Λb μb+Hv βb μb^2+Hb βν μb^2+Hb δb μb^2+Hv δb μb^2+Hb μb^3+Hv μb^3+βb βν Λb φ-Hb βν δb μb φ+p βν Λb μb φ-Hb βν μb^2 φ)²)))/(2 (βb βν δb+βb βν μb+βb δb μb+βν δb μb+βb μb²+βν μb²+βν μb²+βb βν μb-βb βν μb φ-βν δb μb φ-βν δb μb-βν δb μb-βν δb μb+βb Λb μb-p βb Λb μb+p βν Λb μb-Hv βb μb²-Hb βν μb²-Hb δb μb²-Hv δb μb²-Hb μb³-Hv μb³-βb Λb μb-βν δb μb+βb Λb μb+βb Λb

**Ib**->(βb βν Λb-Hν βb δb μb-Hb βν δb μb+βb Λb μb-p βb Λb μb+p βν Λb μb-Hν βb μb²-Hb βν μb²-Hb δb μb²-Hν δb μb²-Hν μb³-βν Λb μb+Hb βν δb μb φ-p βν Λb μb φ+Hb βν μb² φ-\[Sqrt](-4 (-Hν βb Λb μb+Hν p βb Λb μb-Hb p βν Λb μb+Hb Hν δb μb²+Hb Hν μb³+Hb p βν Λb μb φ) (βb βν δb+βb βν μb+βb δb μb+βν δb μb+βν μb²+βν μb²+δb μb²+μb³-βb βν δb φ-βb βν μb φ-βν δb μb φ-βν μb² φ)+(-βb βν Λb+Hν βb δb μb+Hb βν δb μb-βb Λb μb+p βb Λb μb-p βν Λb μb+Hν βb μb^2+Hb βν μb^2+Hb βν μb^3+βb βν Λb φ-Hb βν δb μb φ-βν μb φ-Hb βν μb^2+Hb βν μb^2+βb βν μb+βb δb μb+βb δb μb+βν δb