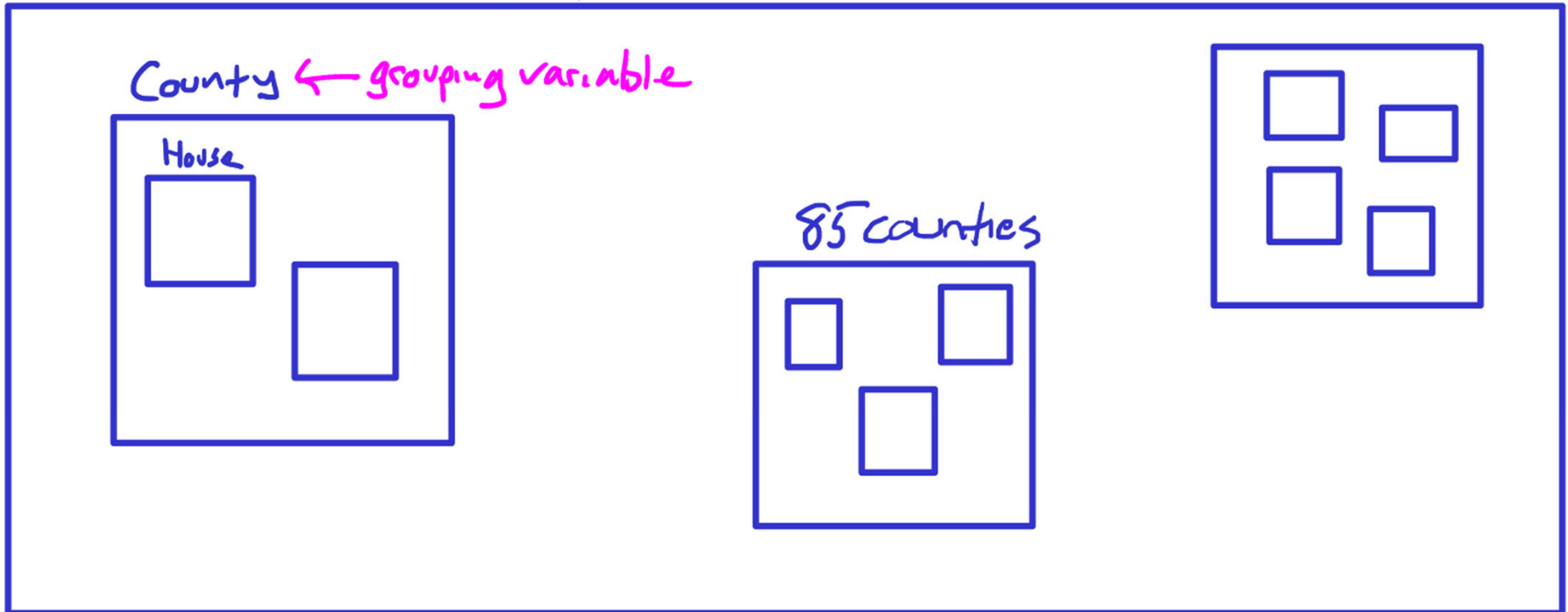


For each analysis problem

- Sketch data design
- Math equations
- Linear model syntax

Model 1: random effects

State



$$y_i \sim \text{Normal}(\mu_{j[i]}, \sigma_h^2) \quad \text{House scale}$$

$$\mu_j \sim \text{Normal}(\bar{\mu}, \sigma_c^2) \quad \text{County scale}$$

Alt parameterization

Additive decomposition

Observed value for house i $\rightarrow y_i = \bar{\mu} + c_{j[i]} + h_i$ \leftarrow Deviation of house i from county j mean (aka residual deviation, e_i)

Mean among counties (aka β_0) \rightarrow $\bar{\mu}$ \leftarrow Deviation of county j mean from mean among counties $\rightarrow c_{j[i]}$

$c_j \sim \text{Normal}(0, \sigma_c^2)$ \leftarrow Variance among counties within state County-scale stochastic model

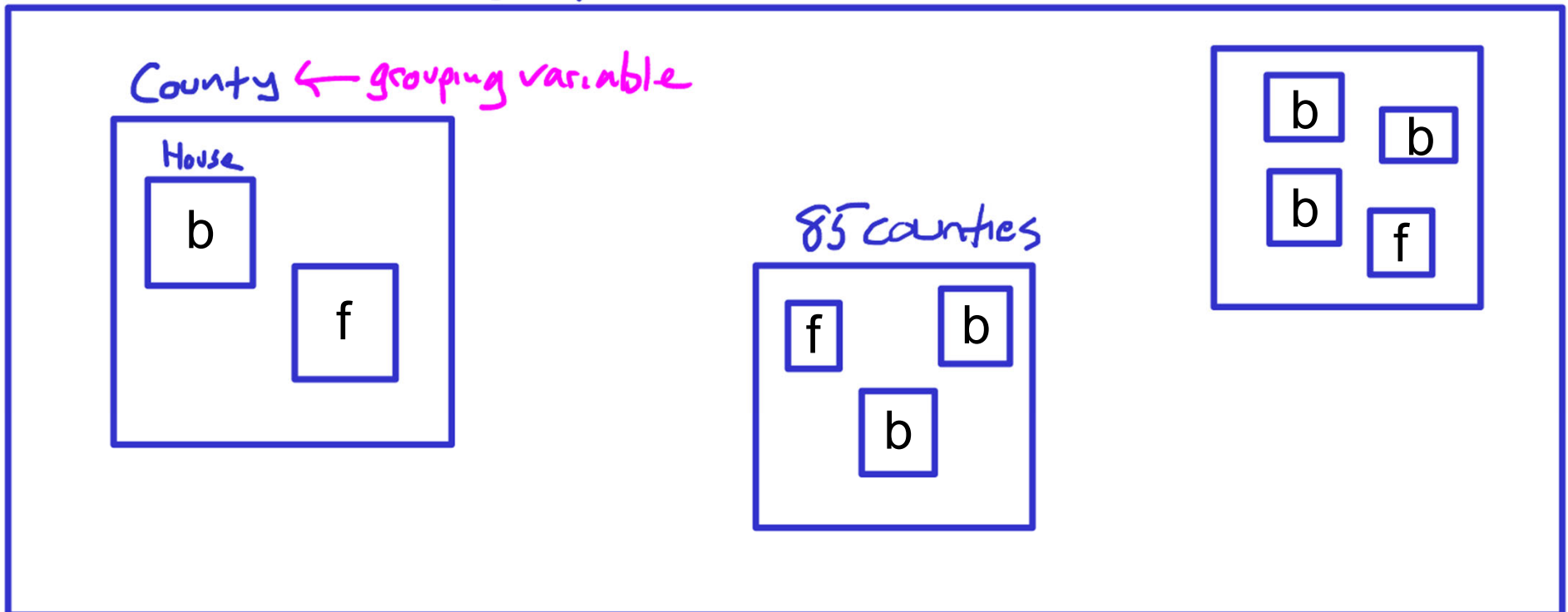
$h_i \sim \text{Normal}(0, \sigma_h^2)$ \leftarrow Variance among houses within counties House-scale stochastic model

Linear model syntax

```
log_radon ~ 1 + (1|county)
```

Model 2: 1 predictor

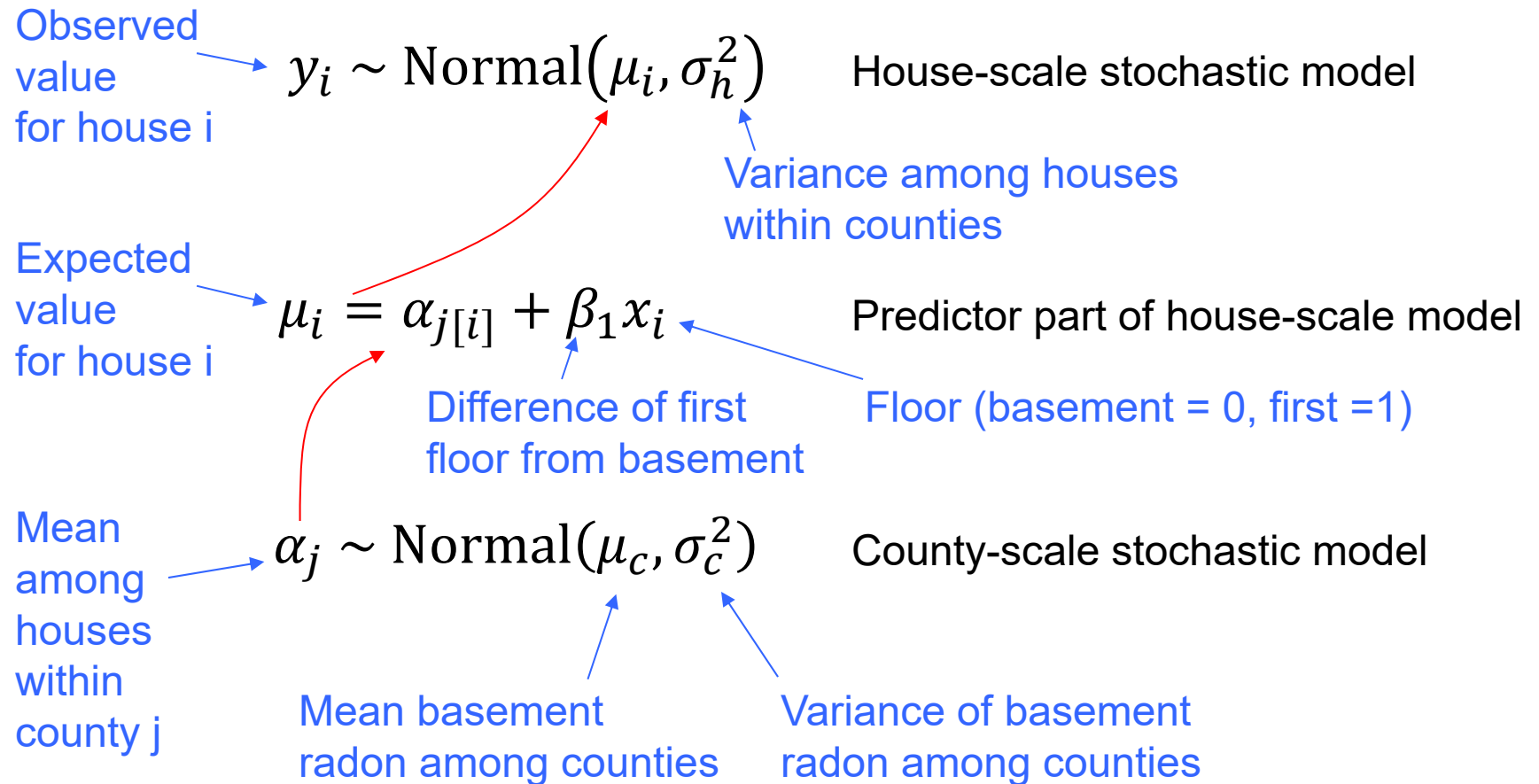
State



Predictor (fixed effect) is at house scale

Writing model 2

Multilevel model, with 1 predictor at house scale



Model 2: Alt parameterization

Multilevel model, with 1 predictor at house scale

Difference of first floor from basement

Floor (basement = 0, first = 1)

$$y_i = \beta_0 + \beta_1 x_1 + c_{j[i]} + e_i$$

Mean basement radon among counties

Deviation of house i from county j mean

Deviation of county j mean from mean among counties

$c_j \sim \text{Normal}(0, \sigma_c^2)$

Variance among counties within state

County-scale stochastic model

$e_i \sim \text{Normal}(0, \sigma_e^2)$

Variance among houses within counties

House-scale stochastic model

Linear model syntax

```
log_radon ~ floor + (1|county)
```

Equivalent:

```
log_radon ~ 1 + floor + (1|county)
```


Model with one house-scale predictor

**See 11_2_radon_multilevel_2.R
11_2_radon_multilevel_2.md**