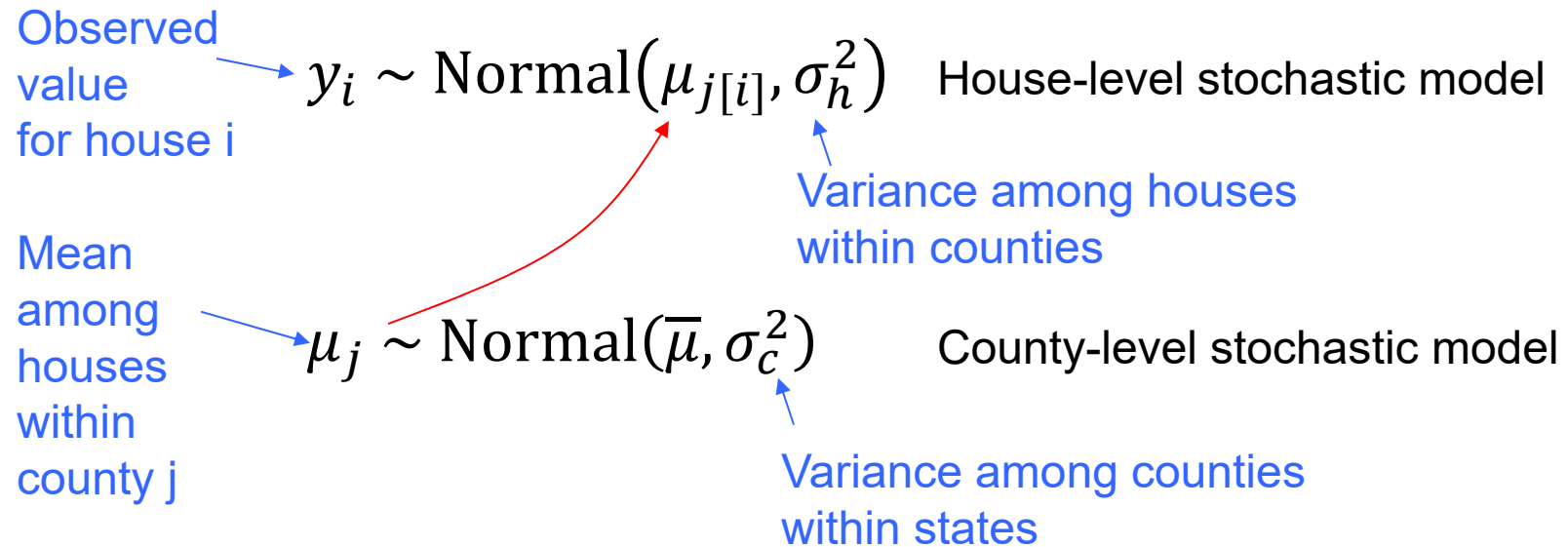


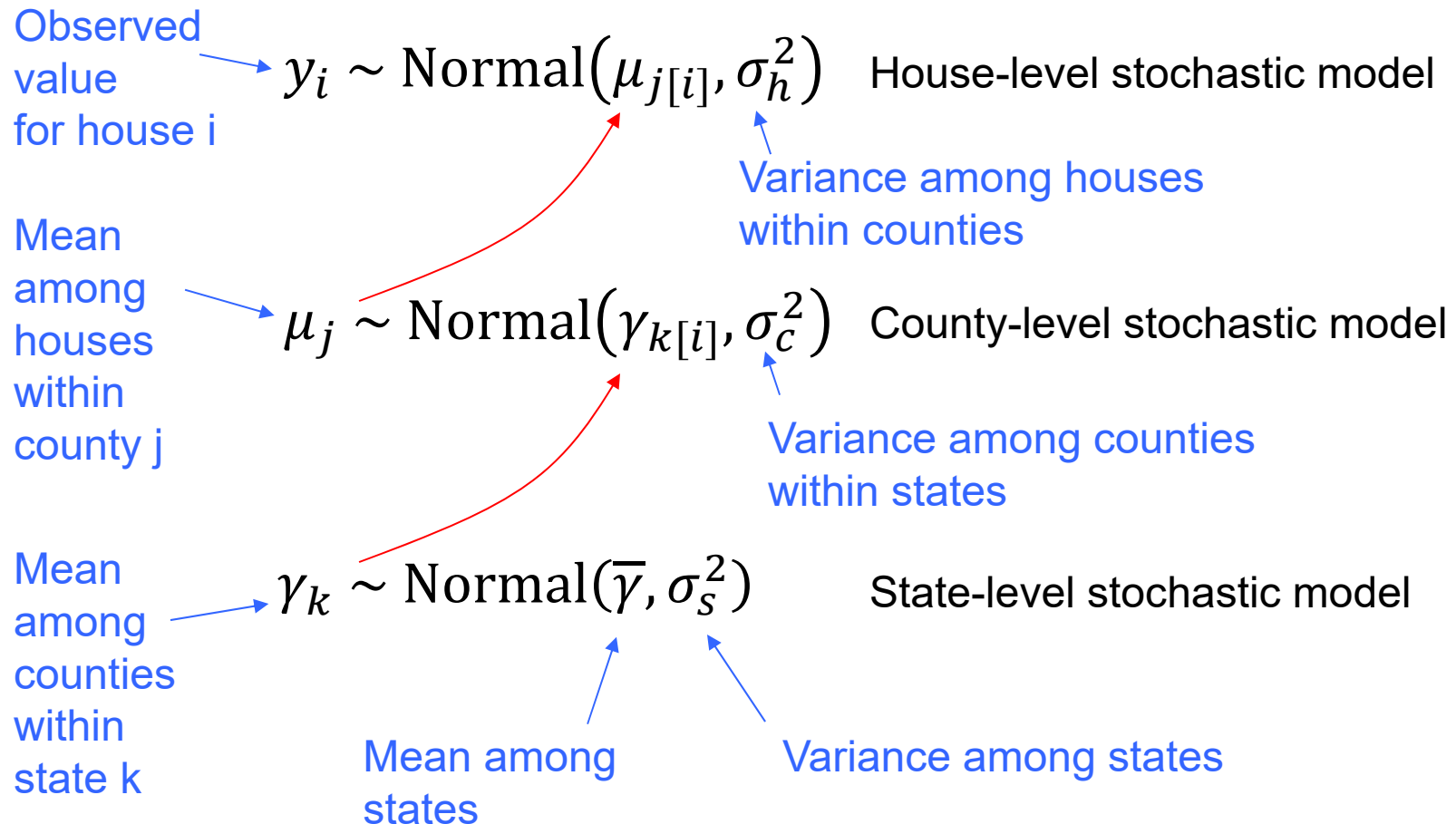
Multiple nested scales

Multilevel model, with 3 nested levels



Multiple nested scales

Multilevel model, with 3 nested levels



Multiple nested scales

Alternative parameterization

$$y_i = \beta_0 + s_{k[i]} + c_{j[i]} + e_i$$

Residual: deviation of house i from county j mean

Overall mean among states

Deviation of state k mean from overall mean

Deviation of county j mean from state k mean

$$s_k \sim \text{Normal}(0, \sigma_s^2)$$

Variance among states

$$c_j \sim \text{Normal}(0, \sigma_c^2)$$

Variance among counties within states

$$e_i \sim \text{Normal}(0, \sigma_e^2)$$

Variance among houses within counties

Multiple nested scales

Linear model syntax for lmer

If states and counties uniquely identified in data:

```
y ~ 1 + (1|state) + (1|county)
```

Otherwise you can form unique identifiers with the nested operator:

```
y ~ 1 + (1|state/county)
```

which expands to (you can also use this):

```
y ~ 1 + (1|state) + (1|state:county)
```

thus, the interaction `state:county` uniquely identifies all counties in the dataset

Autocorrelation

- When present: **overoptimistic** about the **amount of information** in the data
- The grouping terms **account for autocorrelation** (units within groups are more correlated than between groups)
- Specifically, their **deviations are correlated**. The estimated group deviations (i.e. "random effects") properly partition the correlation into scales of variation

Autocorrelation

How the multilevel model accounts for autocorrelation

All the counties from state k have the same state deviation

Partitions the deviations/residuals so all the houses from county j have the same county deviation (i.e. the deviations are correlated)

$$y_i = \beta_0 + s_{k[i]} + c_{j[i]} + e_i$$

Residual: deviation of house i from county j mean

Overall mean among states

Deviation of state k mean from overall mean

Deviation of county j mean from state k mean

LMMs

- Linear **Mixed** Models
 - aka linear mixed effects models
- Class of multilevel models
- Mixed = **fixed** + **random** effects
 - aka deterministic + stochastic effects
 - aka no pooling + partial pooling
- Fixed effects are **linear**
- Random effects are all **Normal**

GLMMs

- Generalized linear mixed models
- Fixed effects have linear predictor but link function can be nonlinear (e.g. log, logistic)
- Data level: exponential family distribution
 - e.g. Poisson, Binomial, Exponential, Normal
- Upper levels: Normal distribution