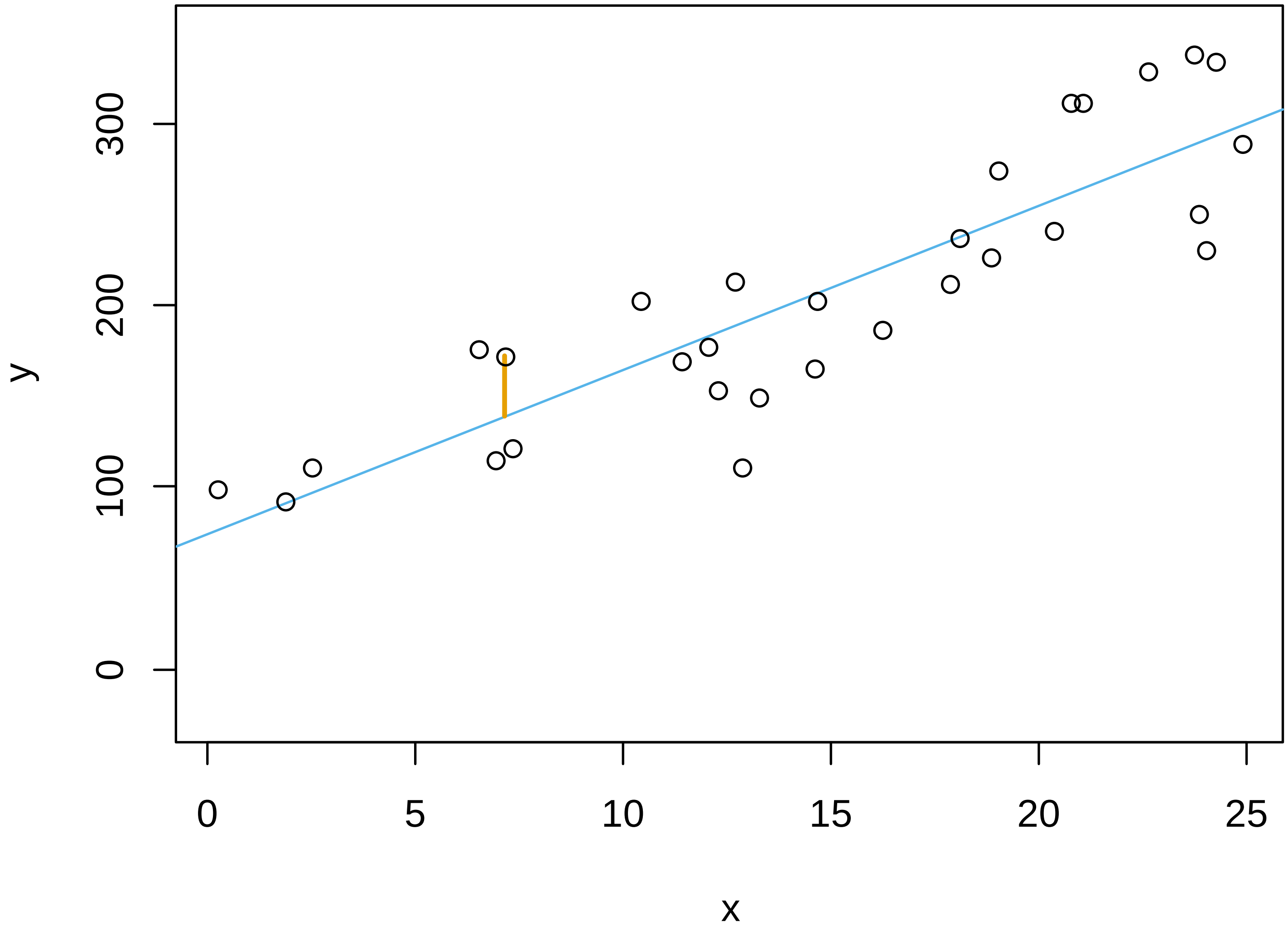
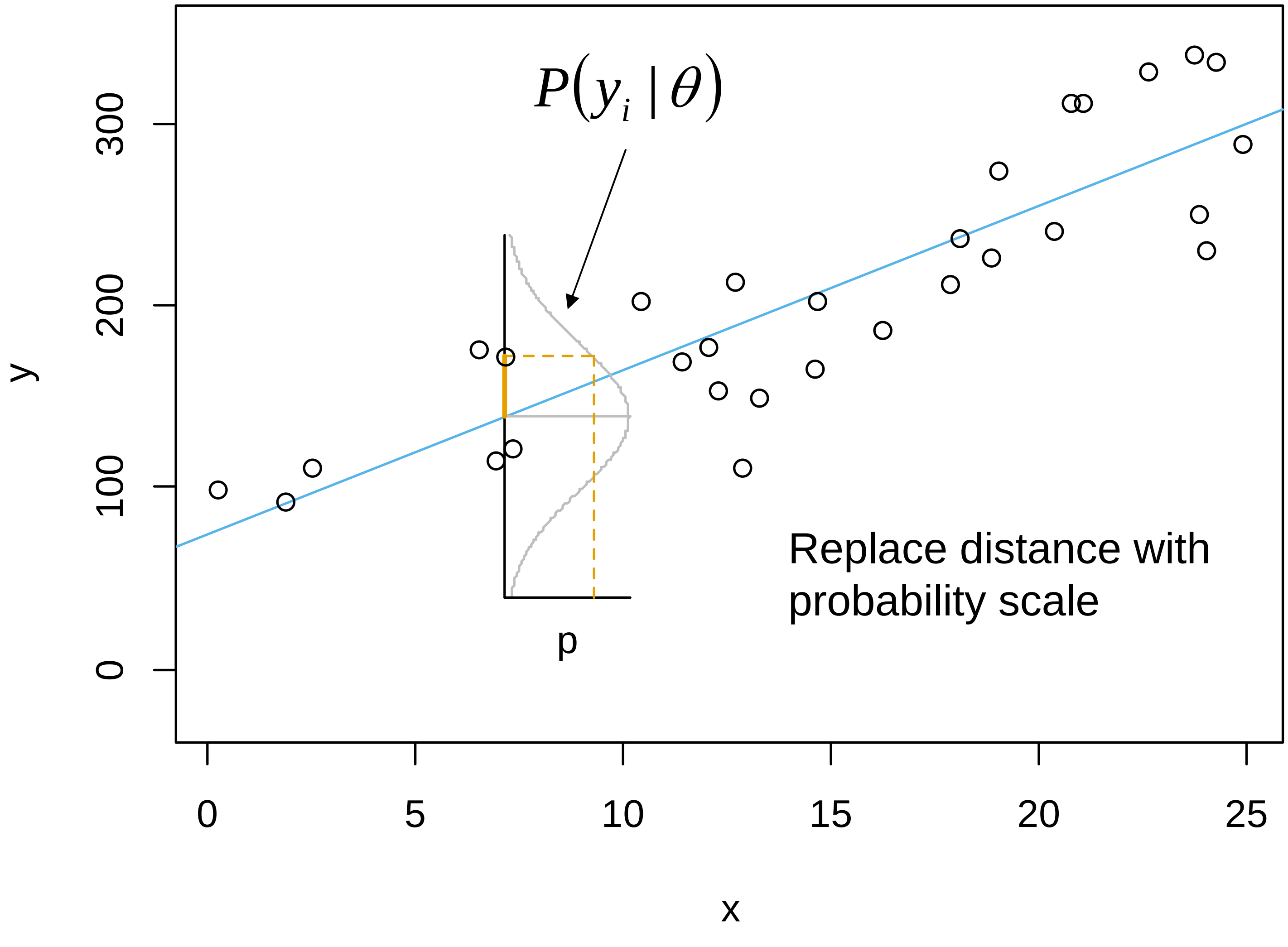
Likelihood inference for the linear model

Likelihood (linear, Normal)



Likelihood (linear, Normal)




```
dnorm( y[i], mean = beta_0 + beta_1*x[i], sd = sd_pred )
```

Likelihood (linear, Normal)

Writing down the model:

$$y \sim \mathrm{Normal}(\mu, \sigma)$$

$$\mu = \beta_0 + \beta_1 x$$

Likelihood for the model:

$$L(\theta) = P(y \mid \theta) = P(y \mid \beta_0, \beta_1, \sigma)$$

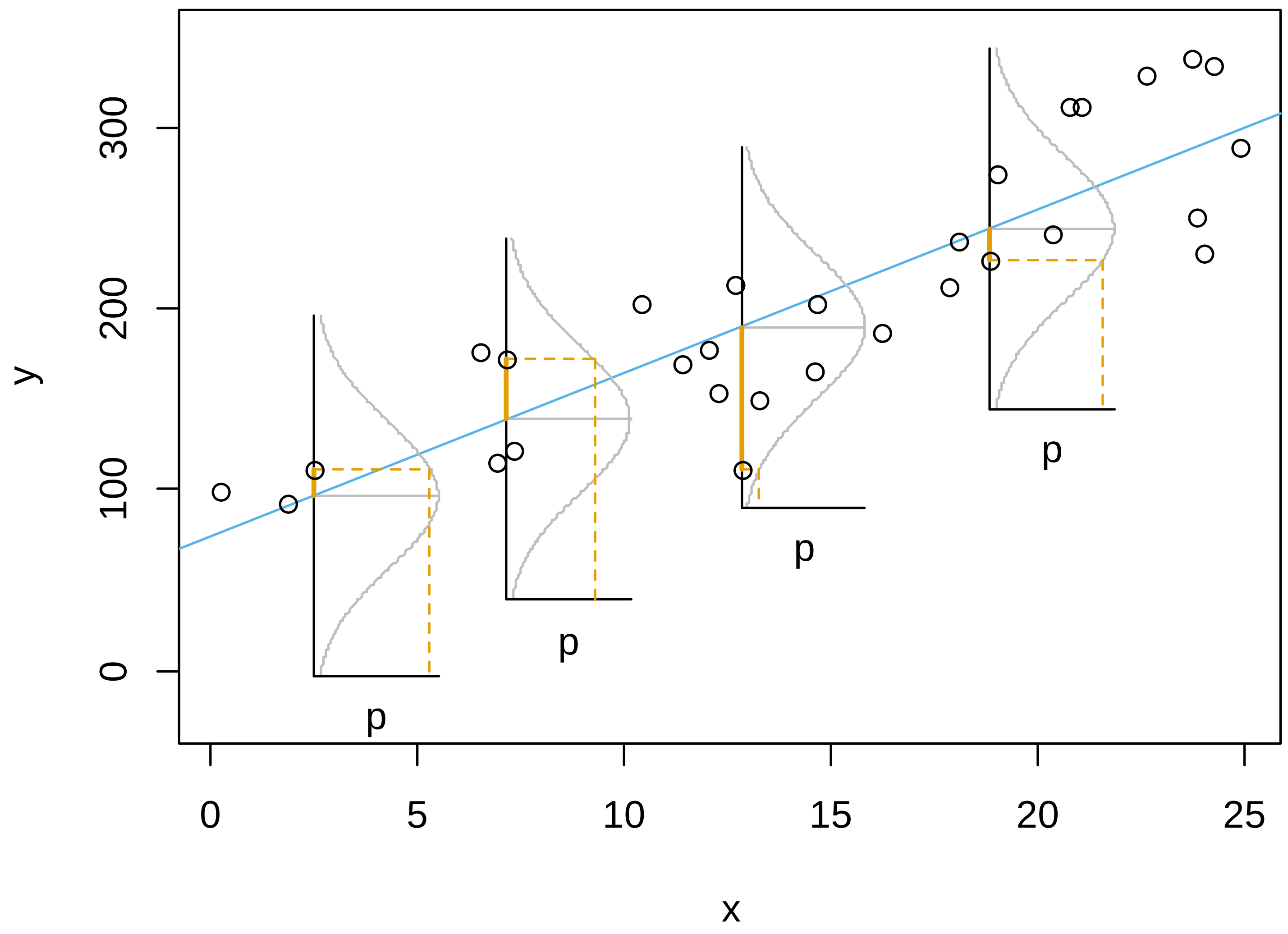
Total likelihood for a data set

One data point: $P(y_1 \mid \theta)$

All data points: $\prod_{i}^{n} P(y_i \mid \theta)$

because probabilities multiply together to give total probability (*n* is the number of datapoints). Independence is assumed.

Likelihood (linear, Normal)



Support function

The log likelihood:

$$\sum_{i}^{n} \ln P\left(y_i \mid \theta\right)$$

Instead of multiplying small probabilities,
it is more accurate and convenient to sum
their logs.

```
sum( dnorm(y, mean=beta_0+beta_1*x, sd=sd_pred, log=TRUE) )
```

Training algorithm: Maximum likelihood

The values of the parameters that maximize the likelihood. In other words, the model that maximizes the probability of the data.

An optimization problem.

In practice: minimize the negative log likelihood. The model with the most support, has the smallest negative log likelihood.

Maximum likelihood

- Linear, Normal model, 3 parameters
  - intercept
  - slope
  - standard deviation of Normal
- We find maximum likelihood estimates (MLE) for all 3

Inference algorithm

$$\frac{P(y \mid \theta_2)}{P(y \mid \theta_1)}$$

Likelihood ratio

Bayes rule to the rescue:

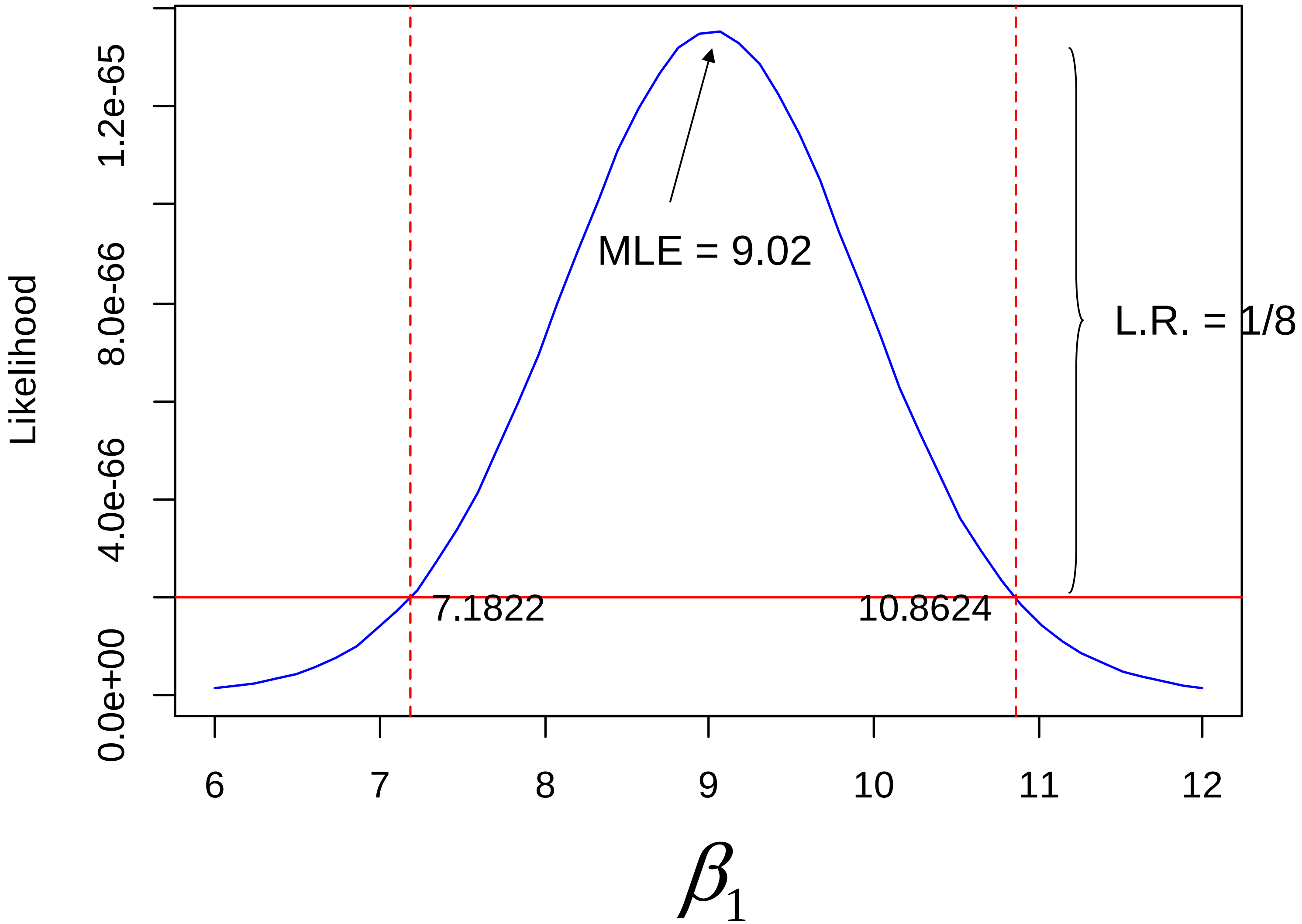
$$\frac{P(\theta_2 | y)}{P(\theta_1 | y)} = \frac{i \cdot P(y | \theta_2)}{i \cdot P(y | \theta_1)} = \frac{P(y | \theta_2)}{P(y | \theta_1)} = LR.$$

for each pair of models in a set
    calculate likelihood ratio
judge the relative evidence for the models

$$\frac{P(y \mid \beta_i)}{P(y \mid \beta_{MLE})}$$

Compare β values against MLE

Likelihood profile & interval



Calibrating likelihood ratio

- Measure strength of evidence
- How strong do you think it is?
- Two bags with many marbles
  - Bag 1: all white
  - Bag 2: half white, half blue
- 3 whites LR = $1 / 0.5^3 = 2^3 = 8$
- 5 whites LR = $1 / 0.5^5 = 2^5 = 32$
- 10 whites LR = $1 / 0.5^{10} = 2^{10} = 1024$

Compared to SSQ

- Likelihood with a Normal distribution

Likelihood for a dataset

pdf of the Normal distribution

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\cdot\frac{(x_i-\mu_i)^2}{\sigma^2}}$$

x_i are the data points
μ_i is the mean relationship
σ^2 is the variance

Negative log likelihood

$$-\ln(L(\theta)) = \underbrace{n\left[\ln(\sigma) + \frac{1}{2}\ln(2\pi)\right]}_{\text{Constant (no } \mu\text{)}} + \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu_i)^2$$

This is the SSQ!

So, minimizing the nll is the same as minimizing the SSQ