

# Optimization algorithms

- Systematically try all combinations for  $\beta_0$  and  $\beta_1$  - Grid search algorithms
- Narrowing in - keep changing parameters in the direction that leads to lower SSQ - Descent algorithms
- Try random values for  $\beta_0$  and  $\beta_1$  - Monte Carlo algorithms
- Solve for parameters using math - Analytical or numerical algorithms

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**Descent algorithms** are ubiquitous and dependable

optim() ?optim for more info and references

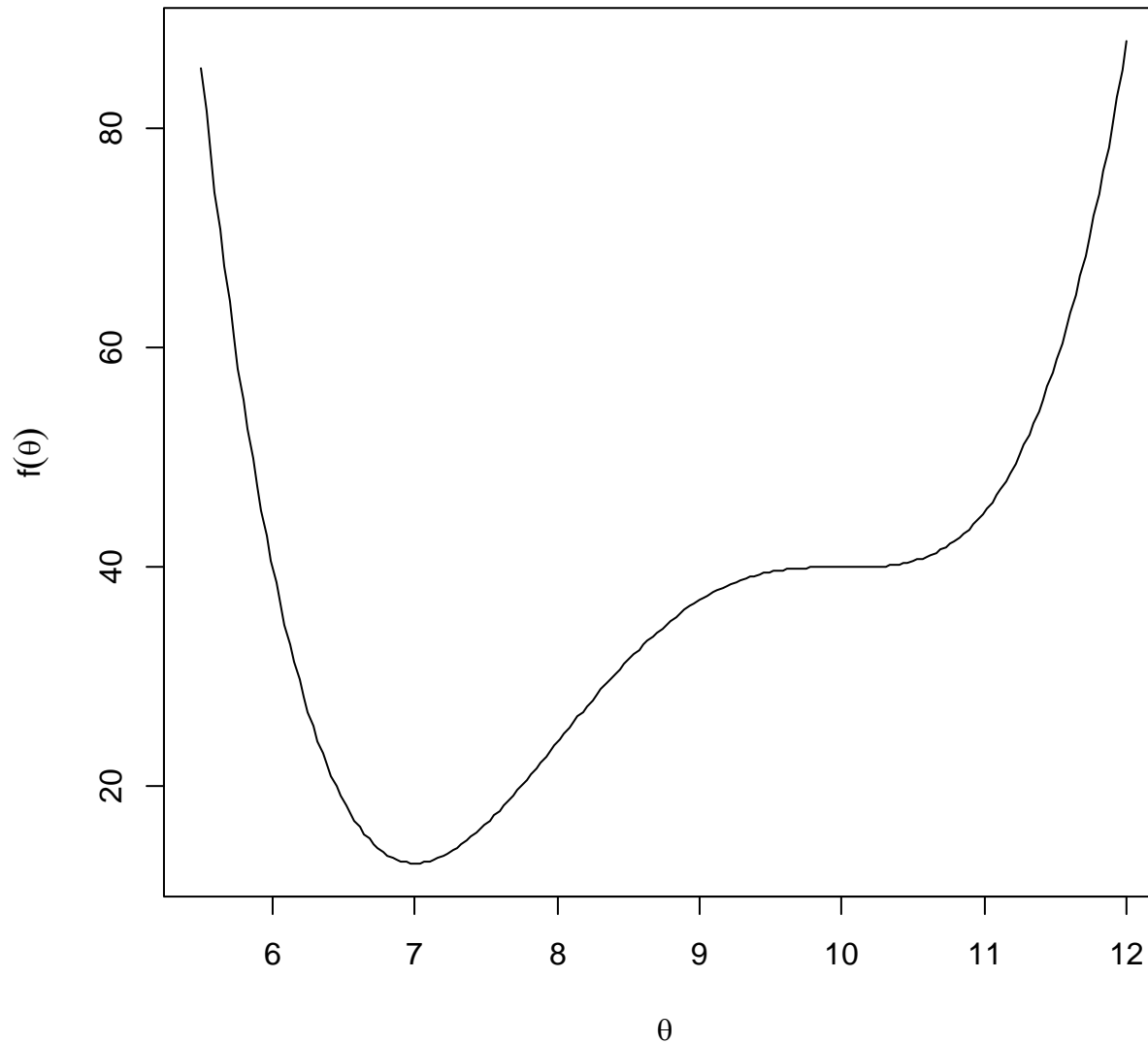
Nelder-Mead simplex algorithm

BFGS algorithm

} e.g. lme4 (lmer, glmer)

# A feel for descent algorithms

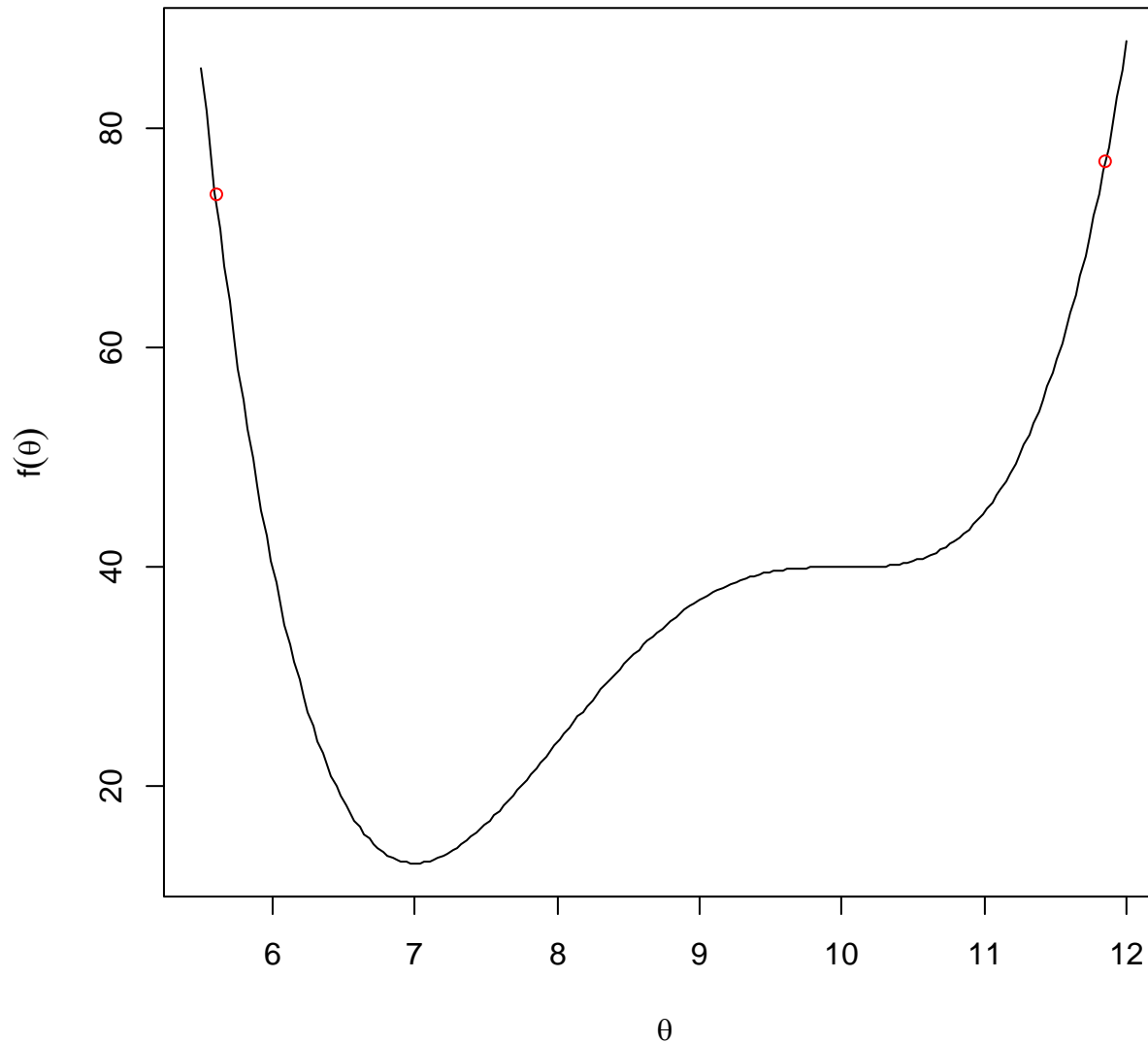
Optimize  $\theta$ : find  $\theta$  such that  $f(\theta)$  is minimum



Bisection  
algorithm

# A feel for descent algorithms

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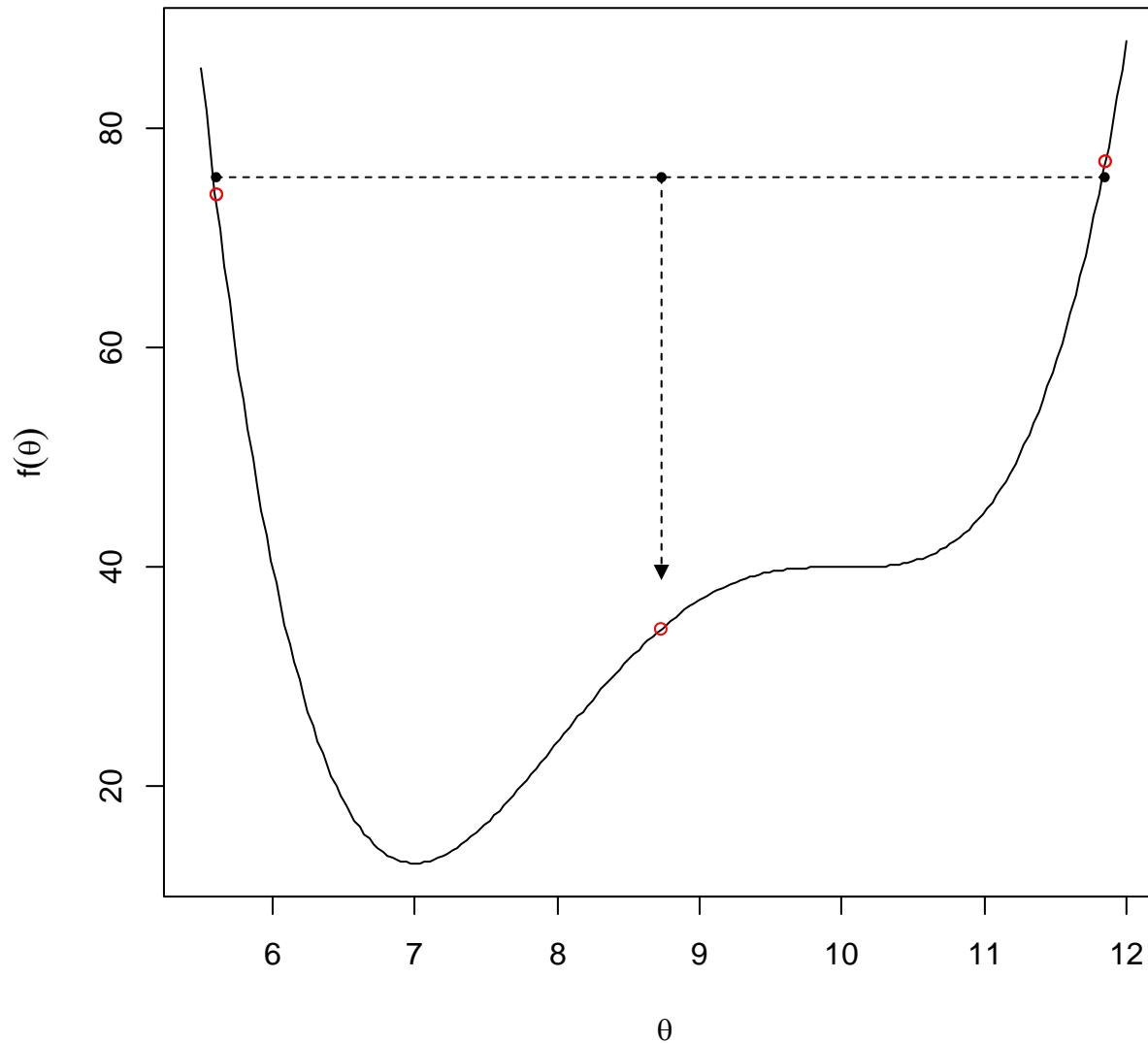


Bisection  
algorithm

Start with 2 points

# A feel for descent algorithms

Optimize  $\theta$ : find  $\theta$  such that  $f(\theta)$  is minimum



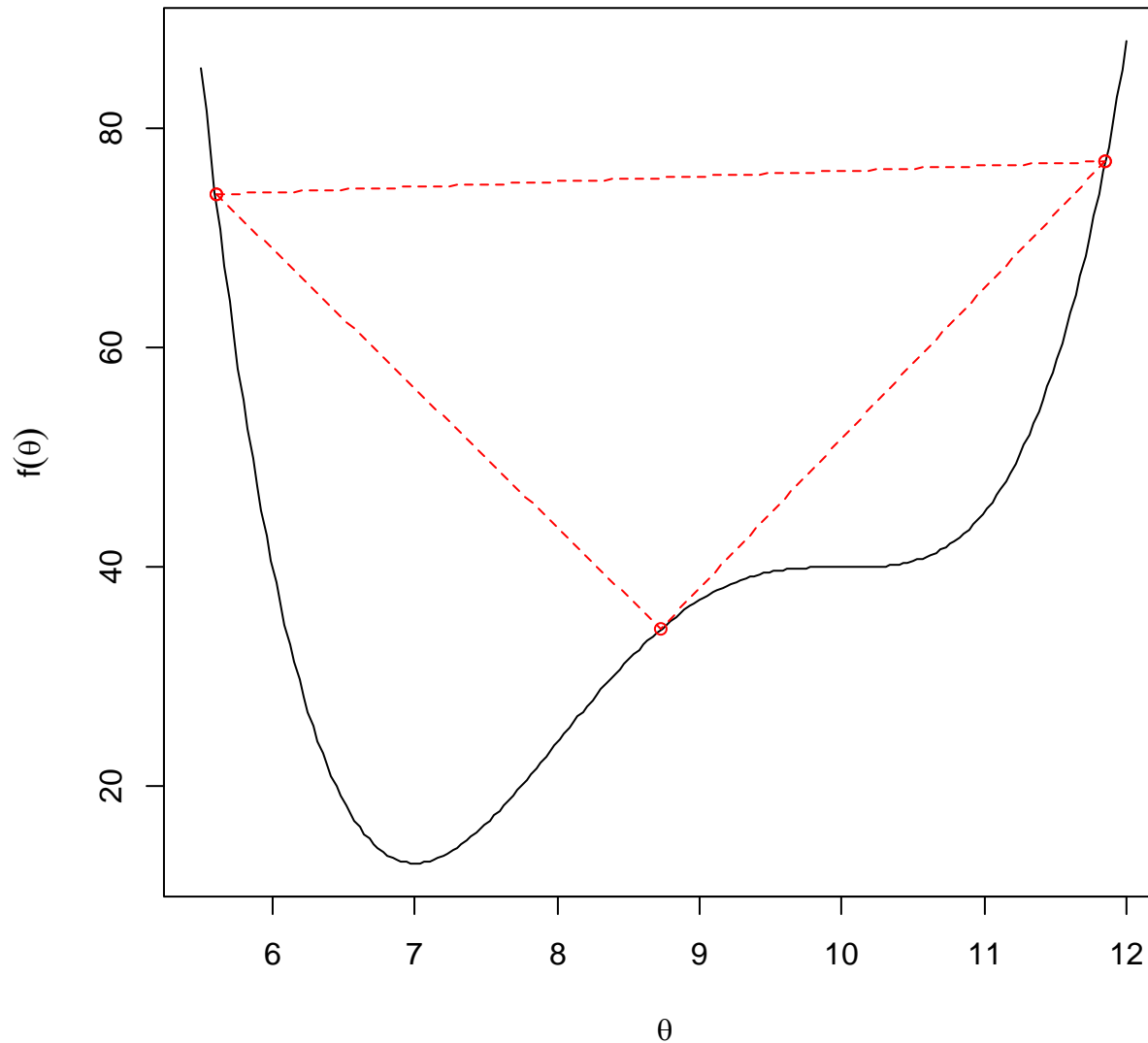
Bisection  
algorithm

Start with 2 points

Bisect

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Bisection  
algorithm

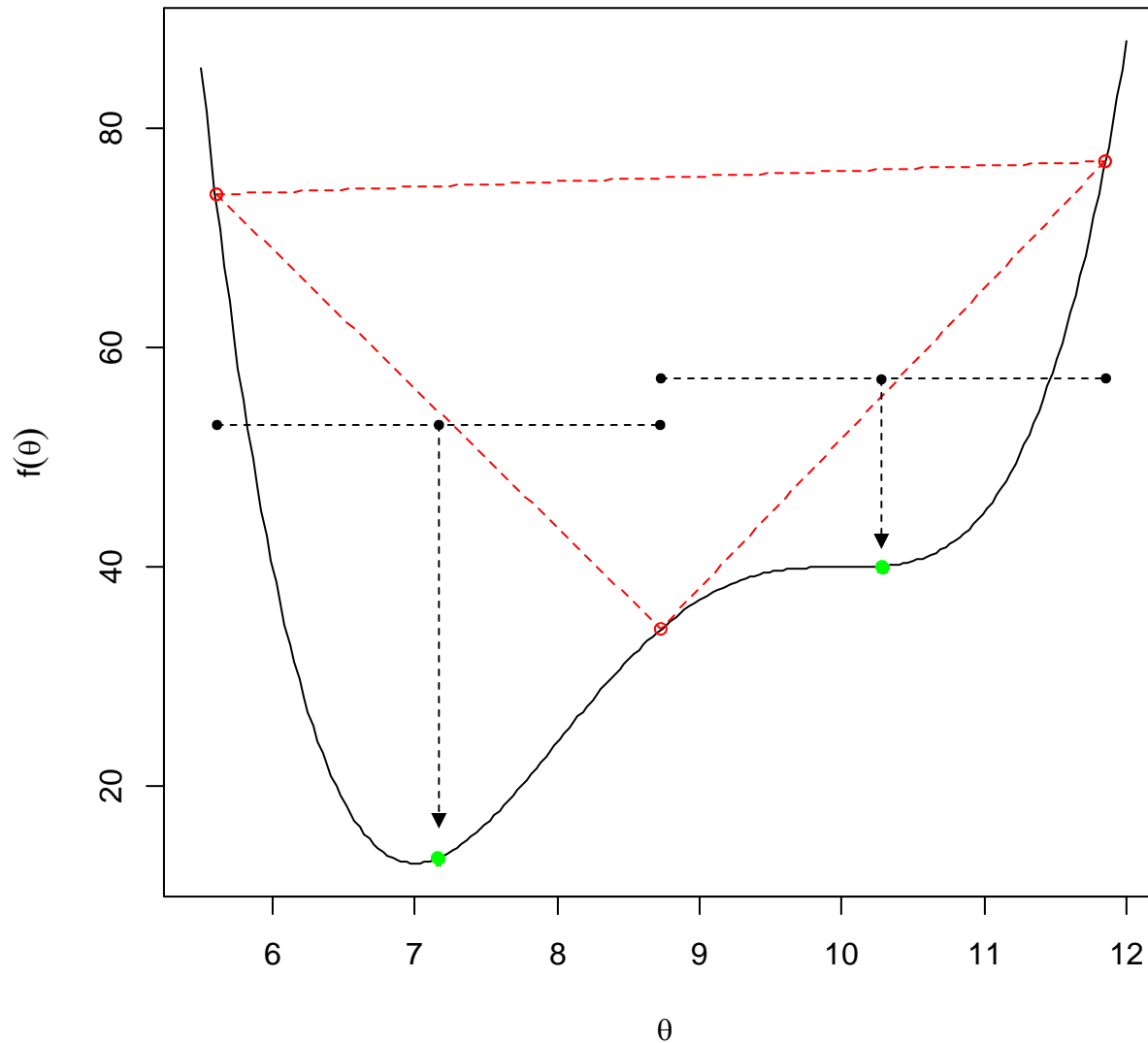
Start with 2 points

Bisect

Make triangle

# A feel for descent algorithms

Optimize  $\theta$ : find  $\theta$  such that  $f(\theta)$  is minimum



Bisection  
algorithm

Start with 2 points

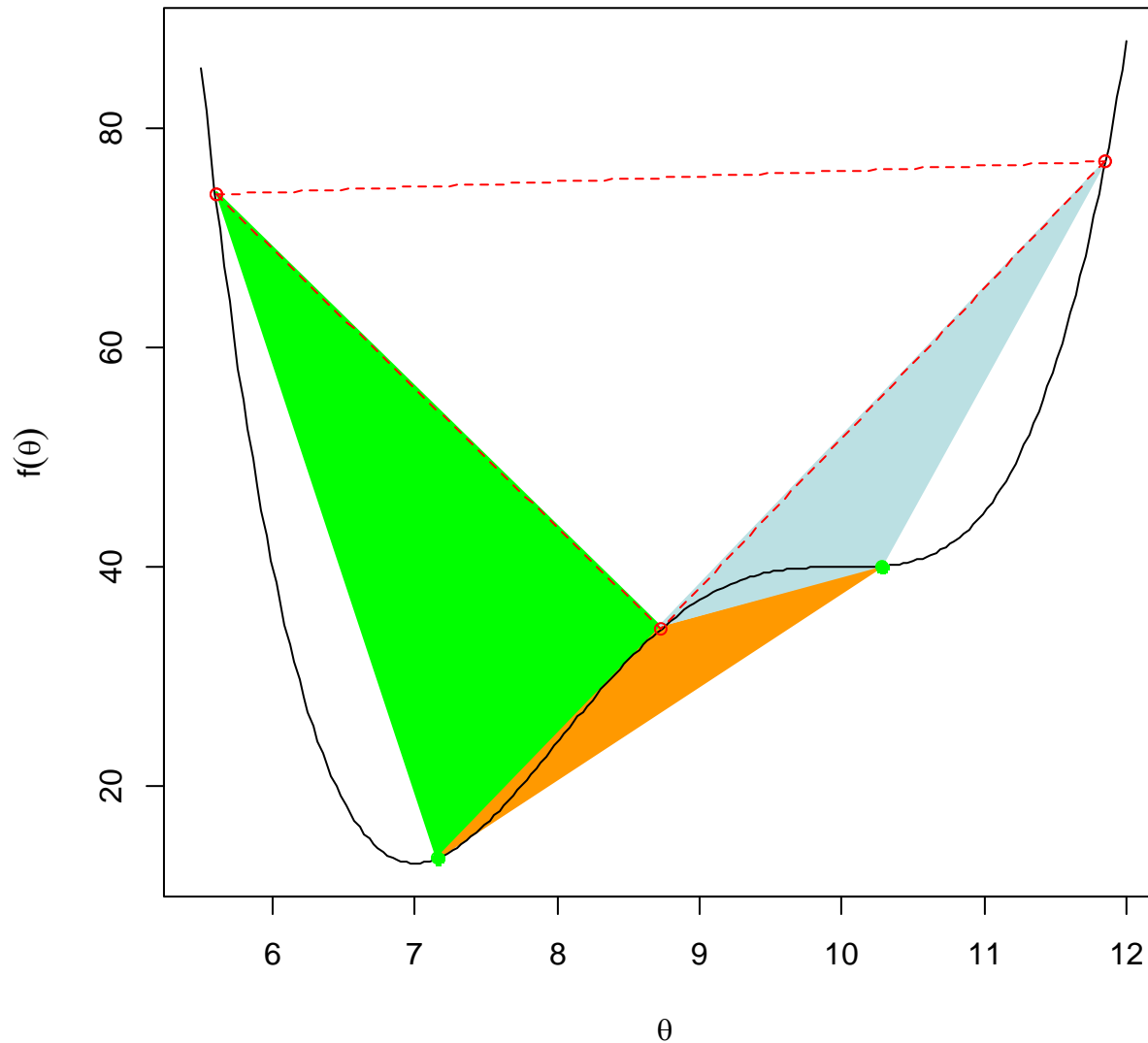
Bisect

Make triangle

Bisect lower sides

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Bisection  
algorithm

Start with 2 points

Bisect

Make triangle

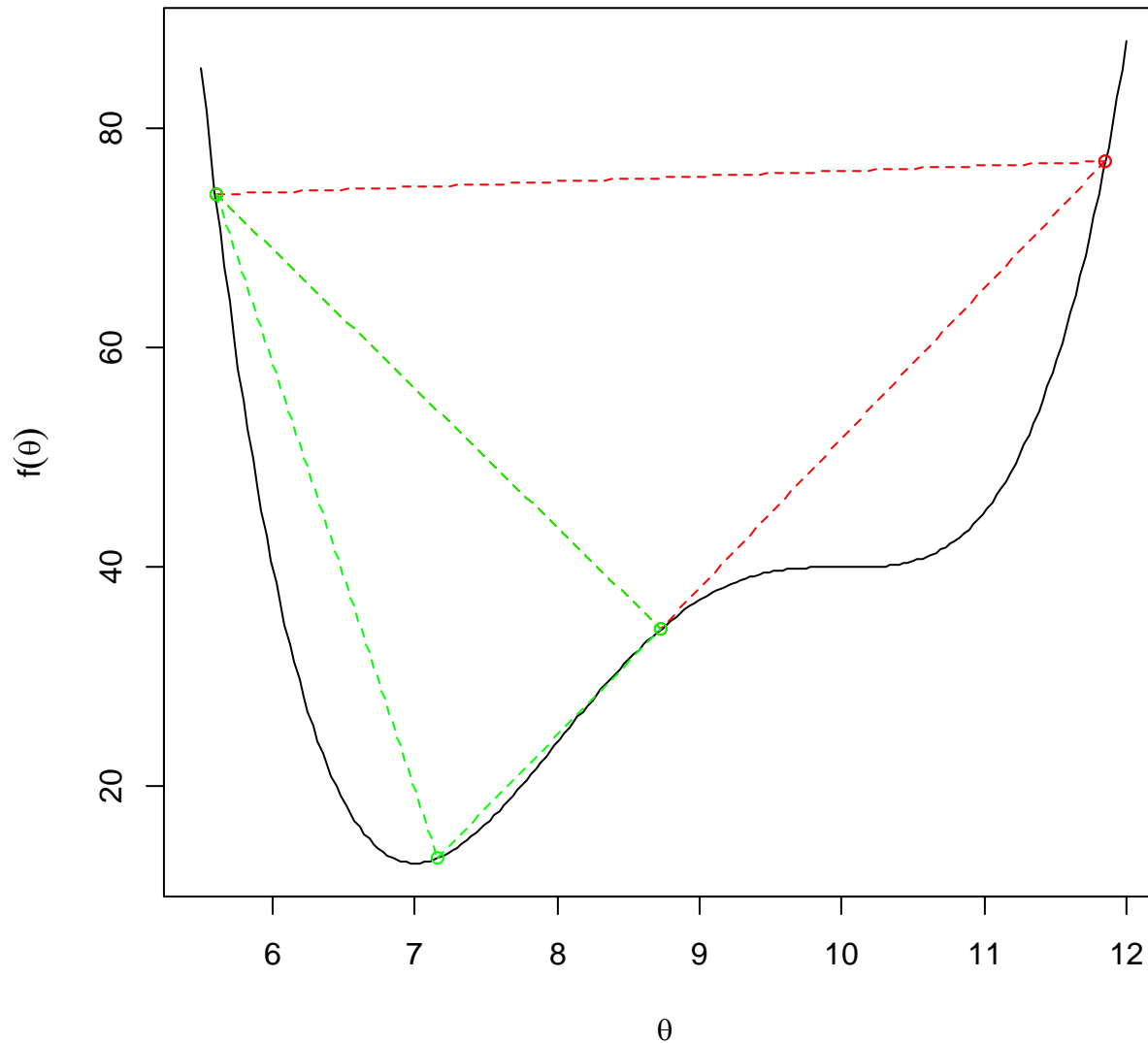
Bisect lower sides

Make lowest triangle



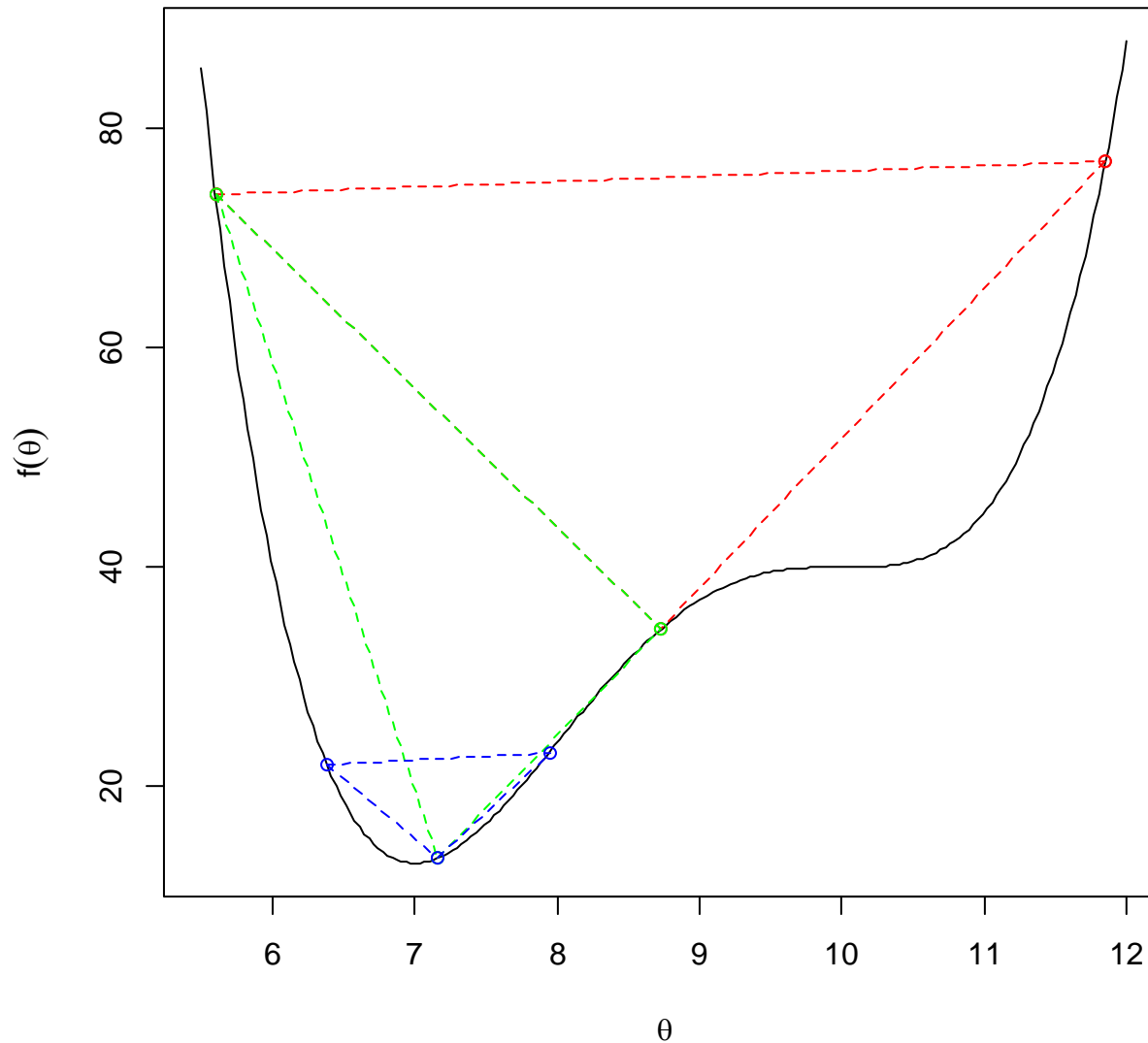
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Bisection  
algorithm

Start with 2 points

Bisect

Make triangle

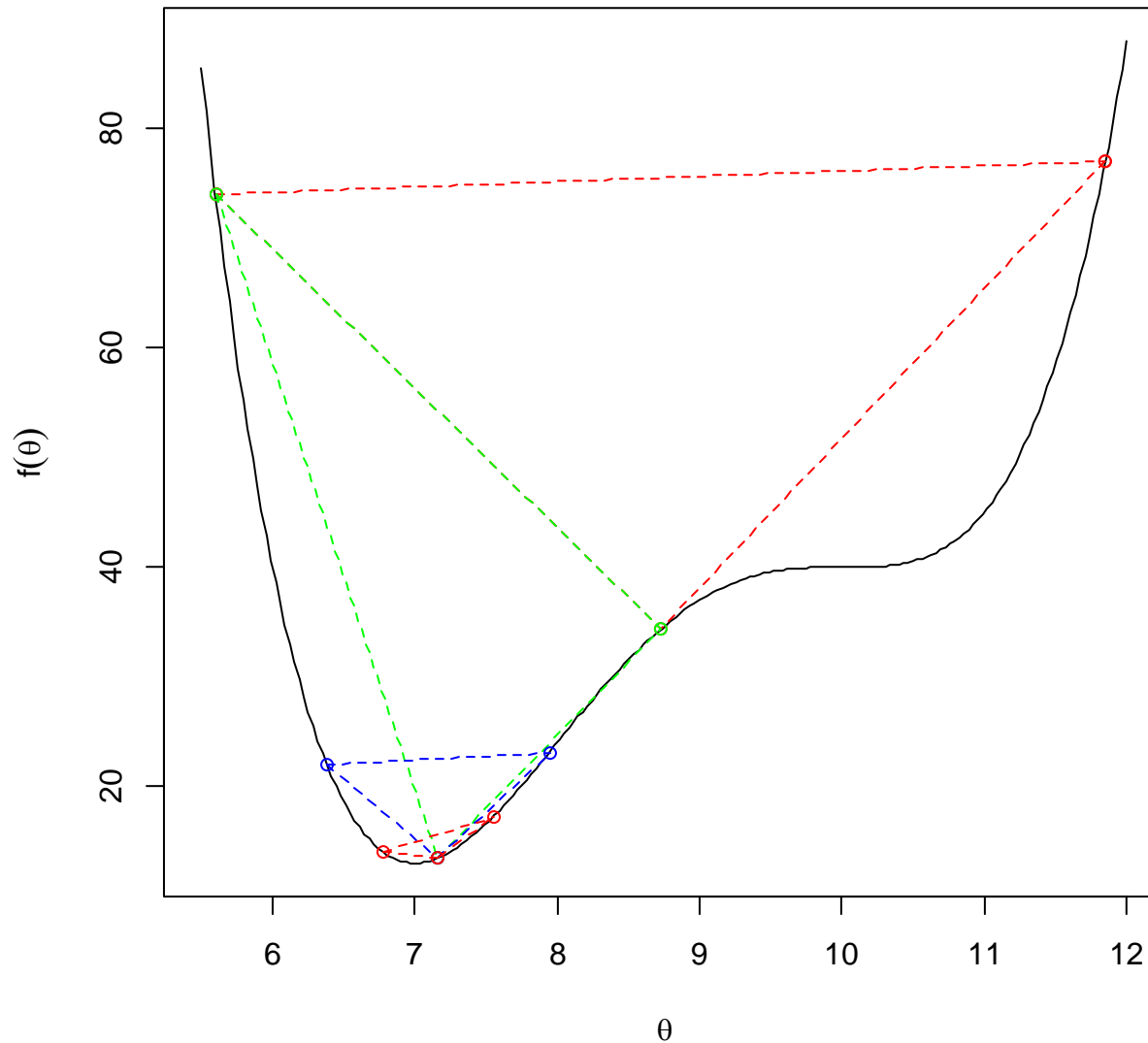
Bisect lower sides

Make lowest triangle

Keep repeating

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Optimize  $\theta$ : find  $\theta$  such that  $f(\theta)$  is minimum



Bisection  
algorithm

Start with 2 points

Bisect

Make triangle

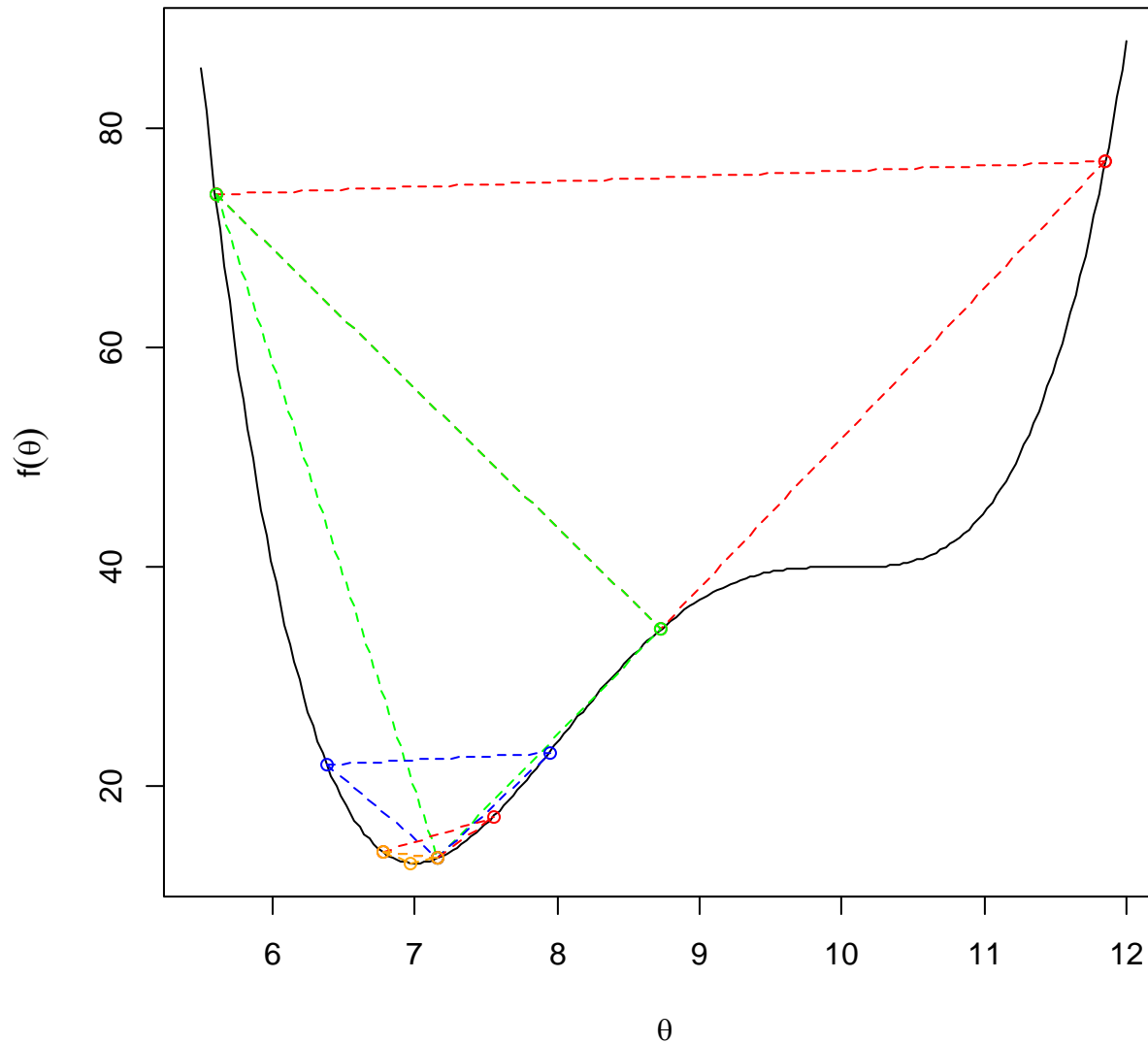
Bisect lower sides

Make lowest triangle

Keep repeating

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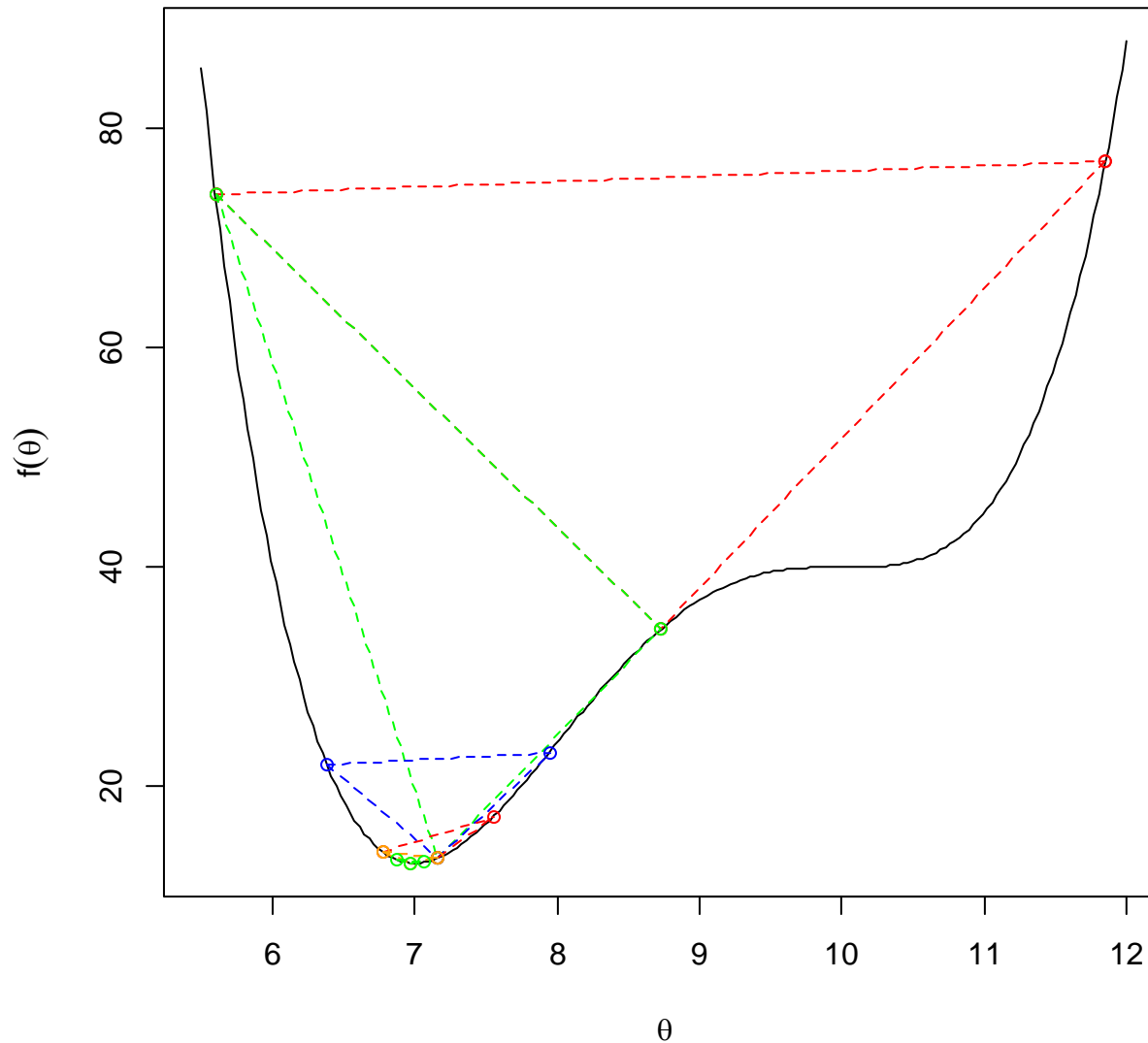


Bisection  
algorithm

Start with 2 points  
Bisect  
Make triangle  
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Make lowest triangle  
Keep repeating

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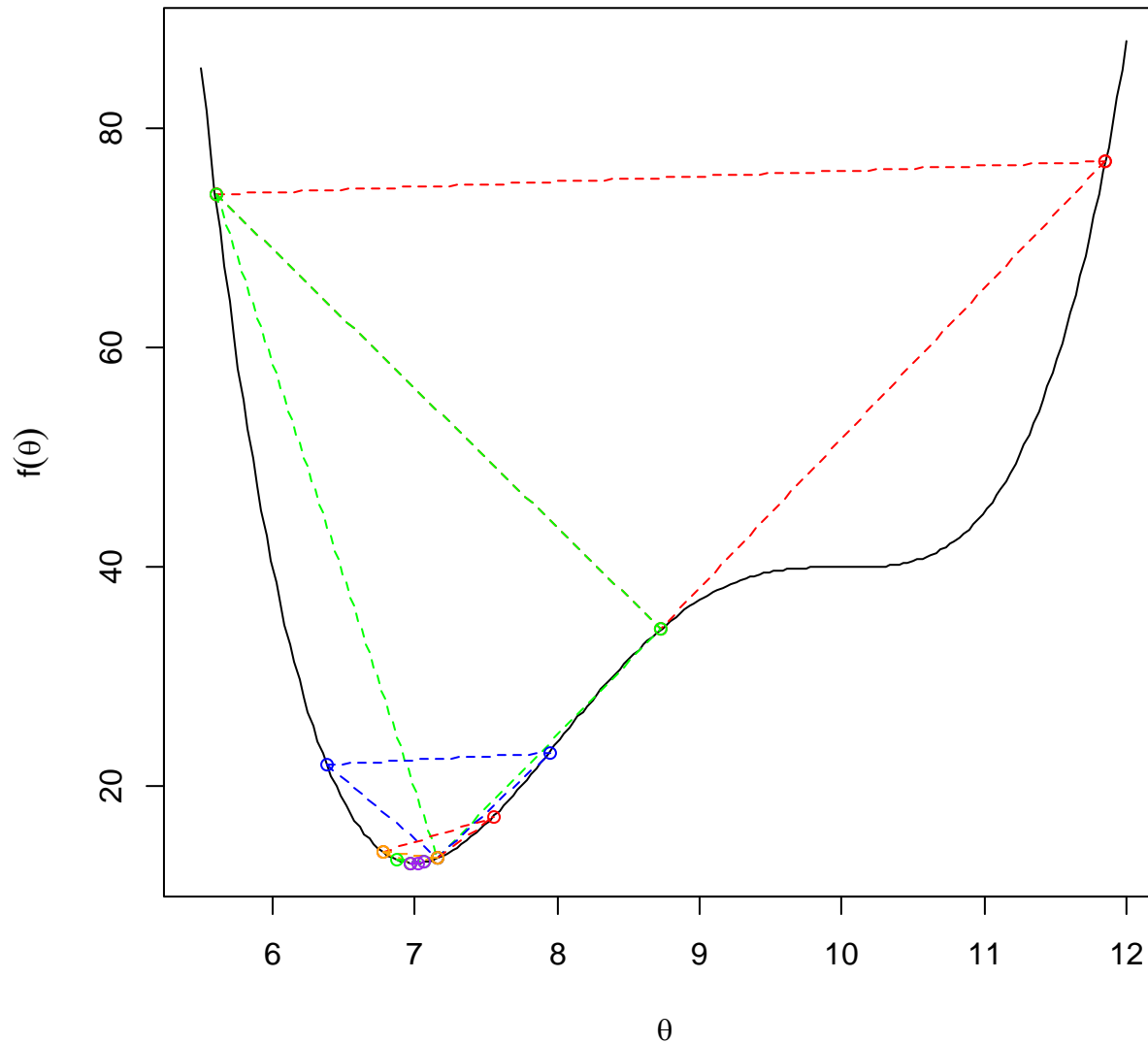


Bisection  
algorithm

Start with 2 points  
Bisect  
Make triangle  
Bisect lower sides  
Make lowest triangle  
Keep repeating

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Bisection  
algorithm

Start with 2 points

Bisect

Make triangle

Bisect lower sides

Make lowest triangle

Keep repeating

# Nelder-Mead algorithm

- Has a similar flavor to the bisection algorithm
- Instead of triangles, has a generalized shape called a **simplex** that tumbles down into the basin
- Quite robust. Good option to try first in many optimization problems.

# Descent algorithms

- Not guaranteed to find the global optimum (might find a local optimum)
- Might not converge (might not get all the way to the optimum)



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## **What does R do for linear regression?**

lm(y~x) solves a system of linear equations using linear algebra

Mathematical theory shows what to do (QR decomposition)

Numerical algorithm is needed to do it (householder algorithm)

Fast, guaranteed to find the minimum SSQ

Only works for SSQ: limited to ordinary linear regression

# Optimization methods

- Bolker (2008). Ecological Models and Data in R. Princeton University Press.
- Chapter 7