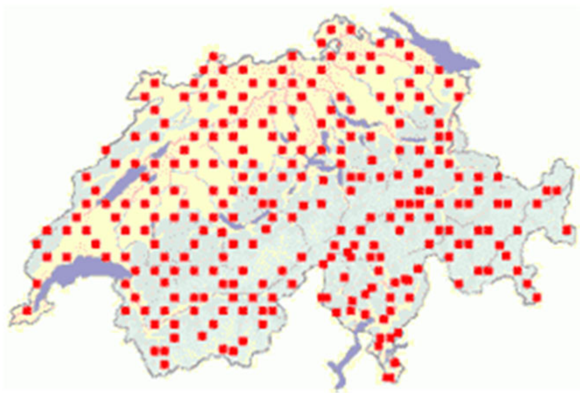
Binomial GLM

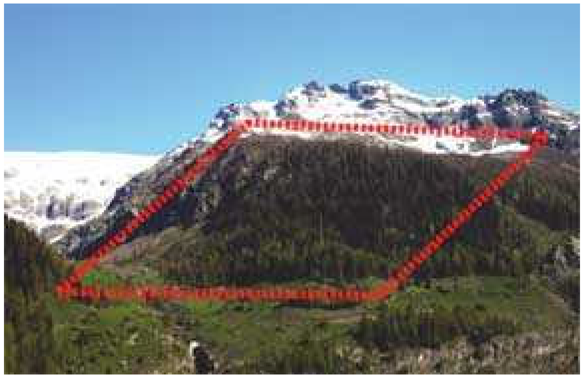
- Swiss BBS example
- Files
  - swissbbs.R
  - switzerland.csv
  - wtmatrix.csv

Binomial data

- Swiss breeding bird survey (www.vogelwarte.ch)
- Skilled observers, 1 km^2 cells



Switzerland; showing survey locations



A 1 km^2 survey cell

- Willow tit territory presence-absence in relation to altitude

see swissbbs.R

Royle JA, Dorazio RM (2008) Hierarchical Modeling and Inference in Ecology. Academic Press, Oxford. p 87.

The model

- Linear predictor; logit link function

Biology/pattern

Logit function

$$\mathrm{logit}(p) = \ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x + \beta_2 x^2$$

Quadratic function allows for hump

Elevation

$$p = \frac{\exp(\mathrm{logit}(p))}{1+\exp(\mathrm{logit}(p))}$$

Antilogit function: backtransforms to give probability *p*

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- Statistical model (biology + error)

$$Y \sim \mathrm{Binomial}(p, N = 1)$$

Number of trials = 1

Y is occurrence

$$L(p) = P(Y = y \mid p) = p^y (1-p)^{1-y}$$

Likelihood for a single data point

1 for present, 0 for absent

Back to basics: likelihood the hard way

1

```
p_pred_quadratic  <- function(b0,b1,b2,elev){
       lp <- b0 + b1 * elev + b2 * elev^2   #logit p
       prob <- exp(lp)/(1+exp(lp))          #antilogit
       return(prob)
}
```

β_2 is orders of magnitude smaller than other parameters. Rescale to get optim to behave better.

2

```
quadratic_nll <- function(p,occ,elev){

       b2 <- p[3] * 1e-06  #Rescale
       ppred <- p_pred_quadratic(b0=p[1],b1=p[2],b2,elev)
       nll <- -sum(dbinom(occ,size=1,prob=ppred,log=TRUE))
       return(nll)
}
```

β_2 = -4e-06

3

```
par <- c(-5,0.02,-4) #Starting values
optim(par,quadratic_nll,occ=occ,elev=elev)
```

Hump hypothesis test

- H_1: Hump

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x + \beta_2 x^2$$

- H_0: No hump

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x$$

Nested models since we can set β_2 to zero and get H_0.

- Likelihood ratio test

P = 2.7e-11

R code

- See
- swissbbs.R
- swissbbs.md