For each analysis problem

- Sketch data design (i.e. structure)
- Math equations for model
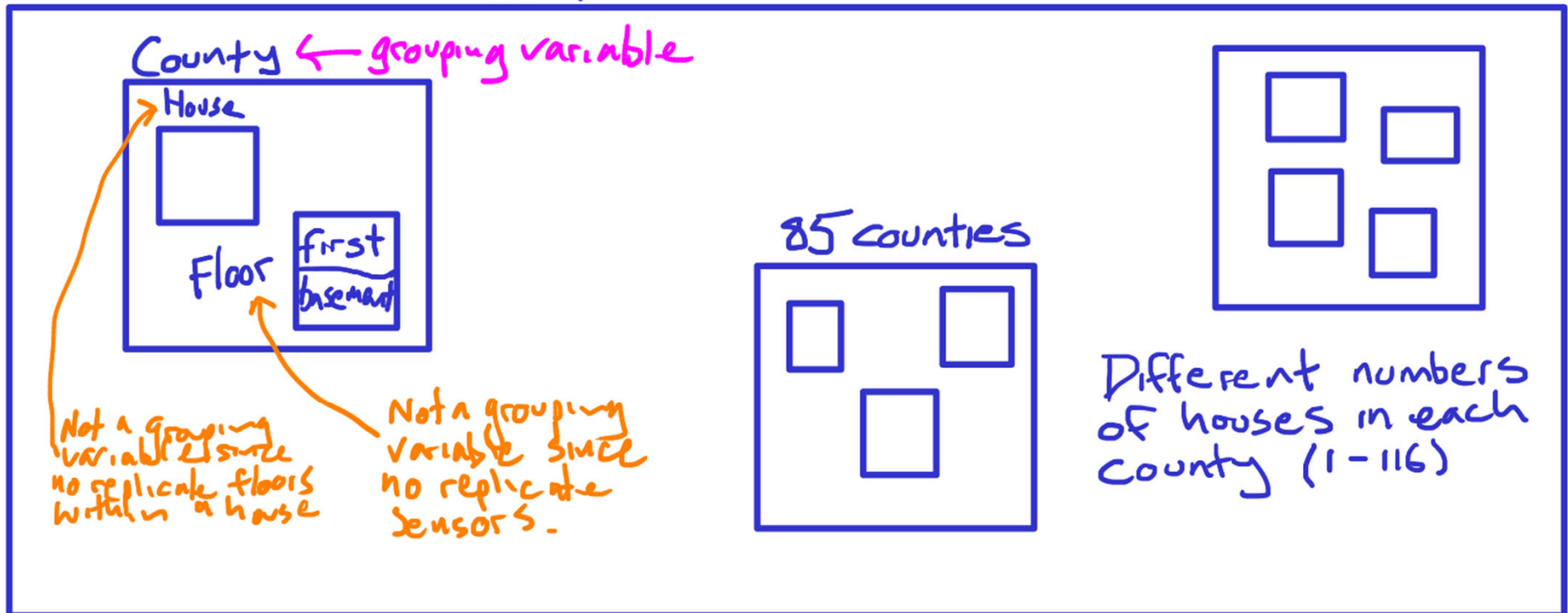- Linear model syntax

Grouping scales in radon data

- You identified these potential grouping scales:

- State, county, house, floor, radon sensor

- Hierarchical structure: nested
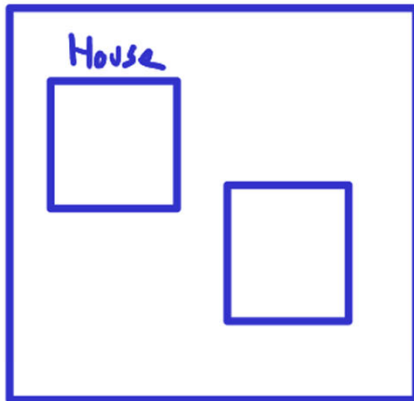
Sketch

Fixed vs random

- Floor: fixed effect
  - deterministic model for parameters
  - shared characteristic

- County: random effect
  - stochastic model for parameters
  - model by group (parameters vary by group)
  - model accounts for correlation among houses within counties

We could alternatively model county as a fixed effect: model parameters would be deterministic
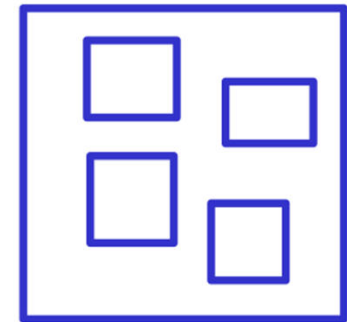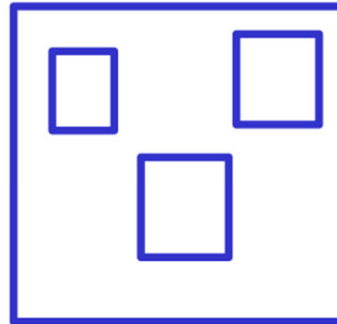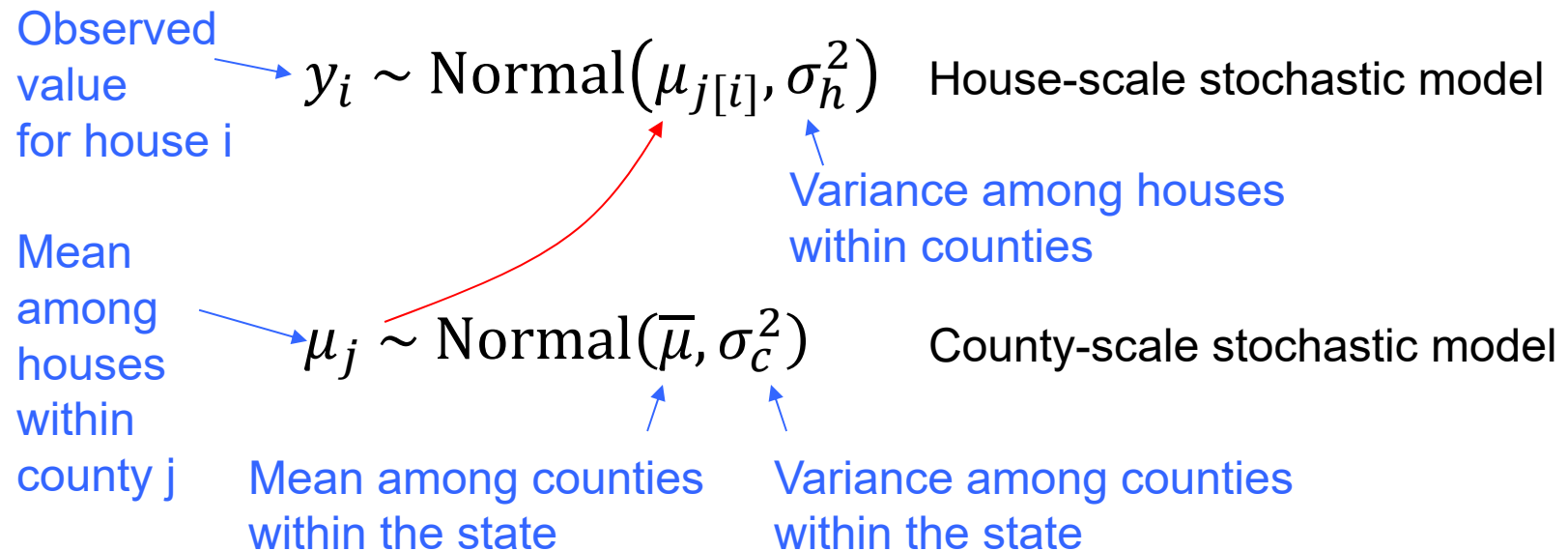
Model 1: random effects

Writing model 1

Basic multilevel model, no predictors

Observed value for house i

$$y_i \sim \text{Normal}\left(\mu_{j[i]}, \sigma_h^2\right)$$ House-scale stochastic model

Variance among houses within counties

Mean among houses within county j

$$\mu_j \sim \text{Normal}(\overline{\mu}, \sigma_c^2)$$ County-scale stochastic model

Mean among counties within the state

Variance among counties within the state

$j[i]$ is the county (j) of house i

Alt parameterization

Additive decomposition

Observed value for house i

Deviation of house i from county j mean

$$y_i = \bar{\mu} + c_{j[i]} + h_i$$

Mean among counties $= \beta_0$

Deviation of county j mean from mean among counties

$$c_j \sim \text{Normal}(0, \sigma_c^2)$$ Variance among counties within state — County-scale stochastic model

$$h_i \sim \text{Normal}(0, \sigma_h^2)$$ Variance among houses within counties — House-scale stochastic model
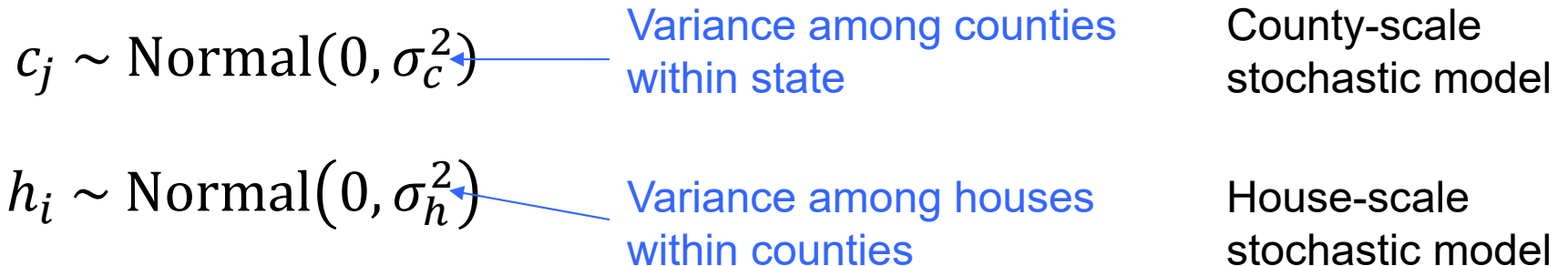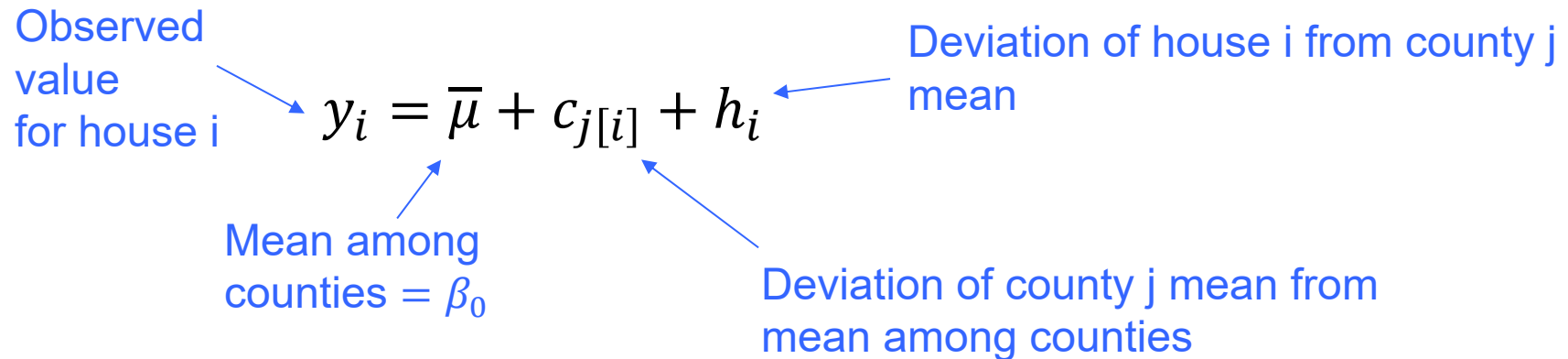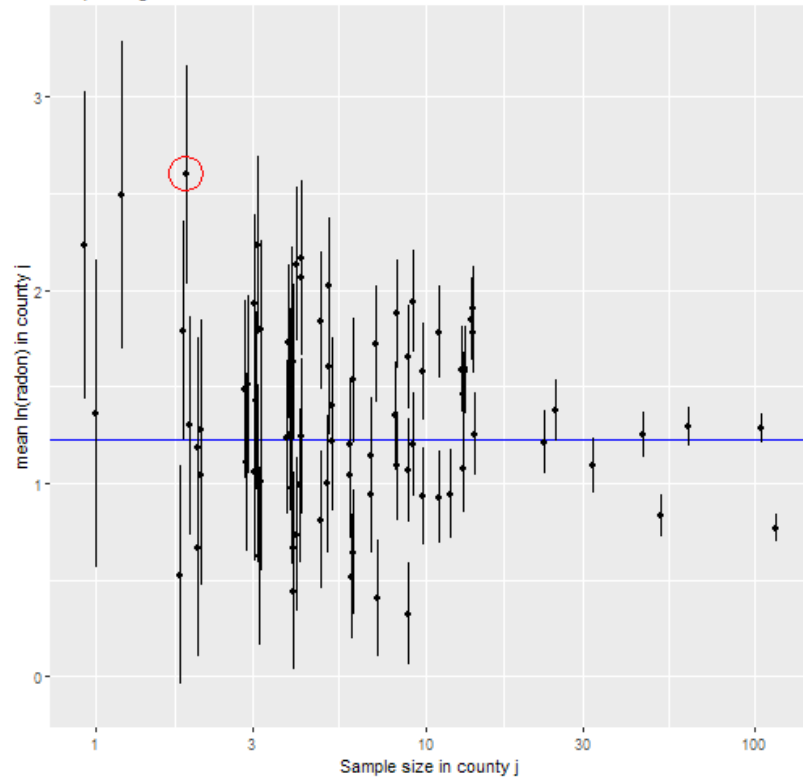
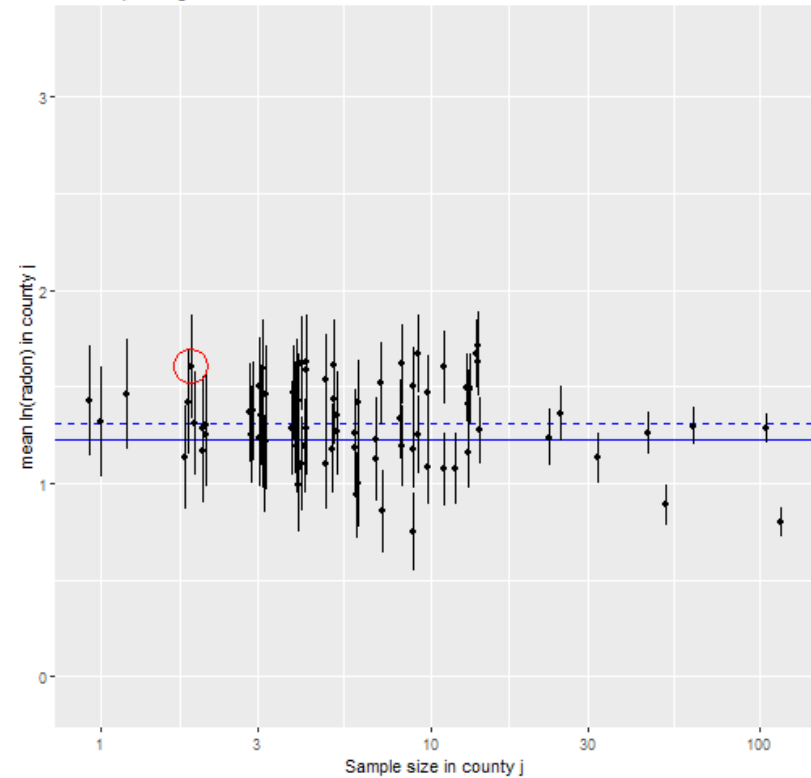# Linear model syntax

```
log_radon ~ 1 + (1|county)
```

We considered code for the radon example.
See 10_3_radon_multilevel.md

Multilevel model - radon

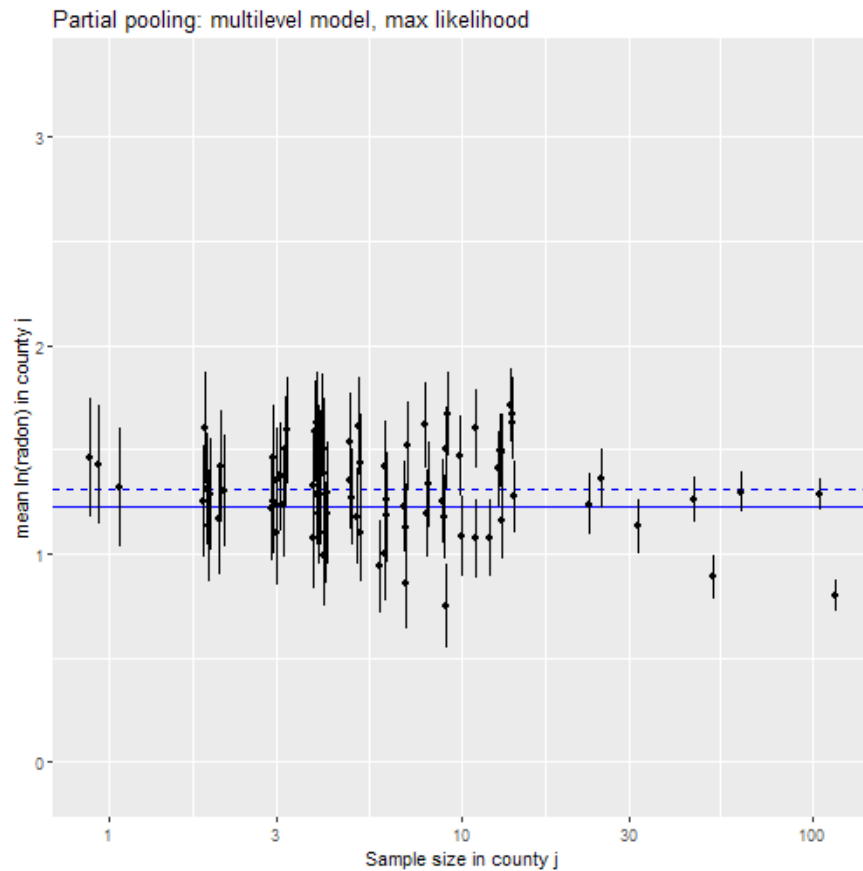No pooling: estimates from linear model fit

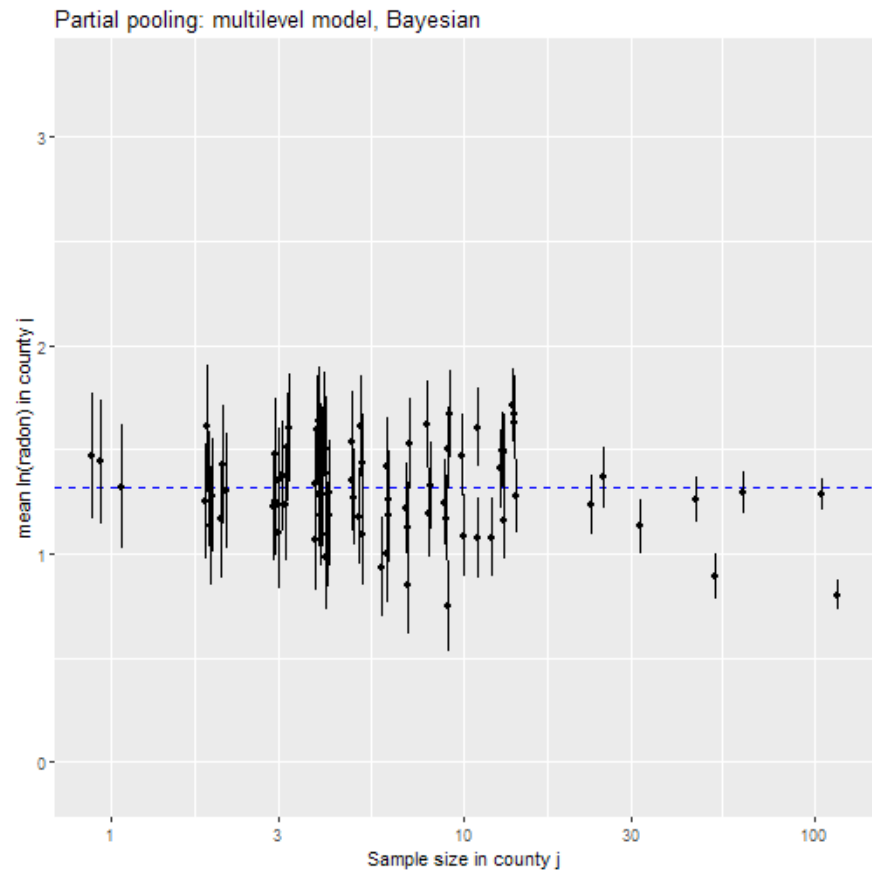Partial pooling: multilevel model, max likelihood

Shrinkage

Multilevel model - radon

Frequentist

Bayesian