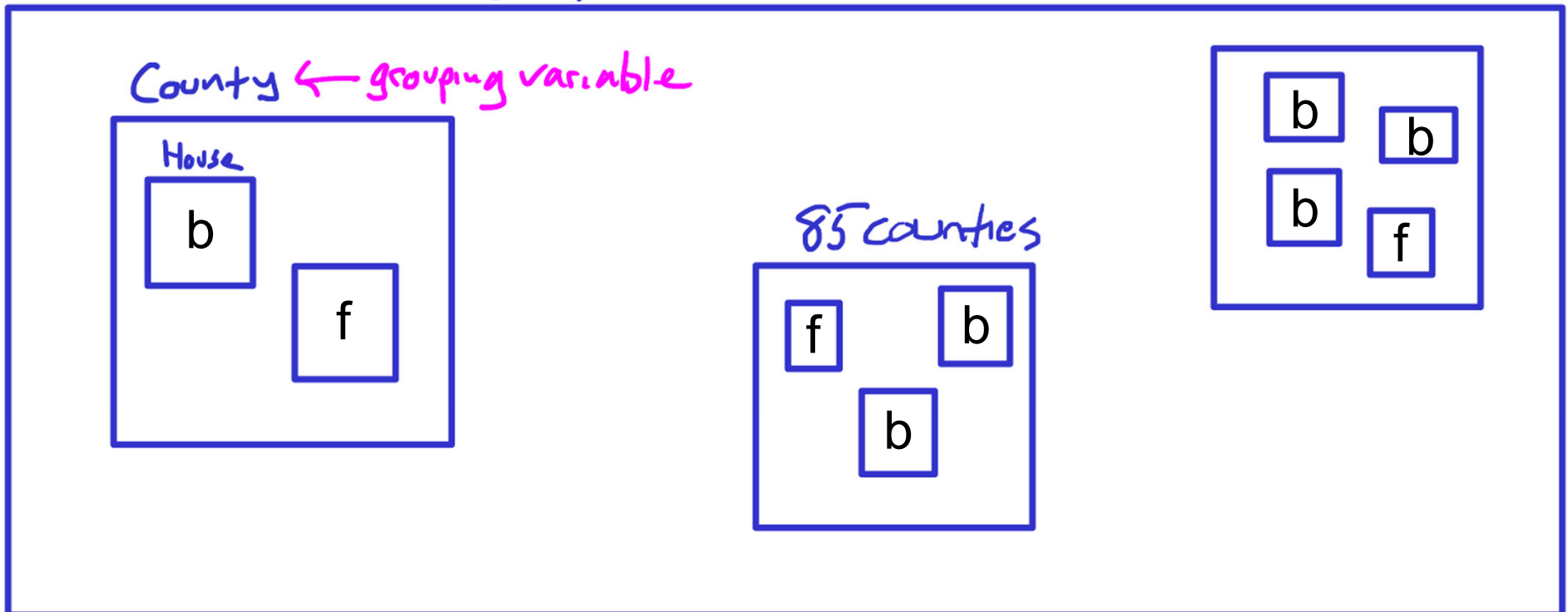


# Today

- Radon (30 mins)
  - finish model 2 (1 predictor)
  - model 3 (2 predictors at different scales)
- Individual project meetings (4-5 PM)
  - 6 mins each. See email schedule.
  - outcome: ready to finalize proposal

# Model 2: 1 predictor

*State*



Predictor (fixed effect) is at house scale

**Model with one house-scale predictor**

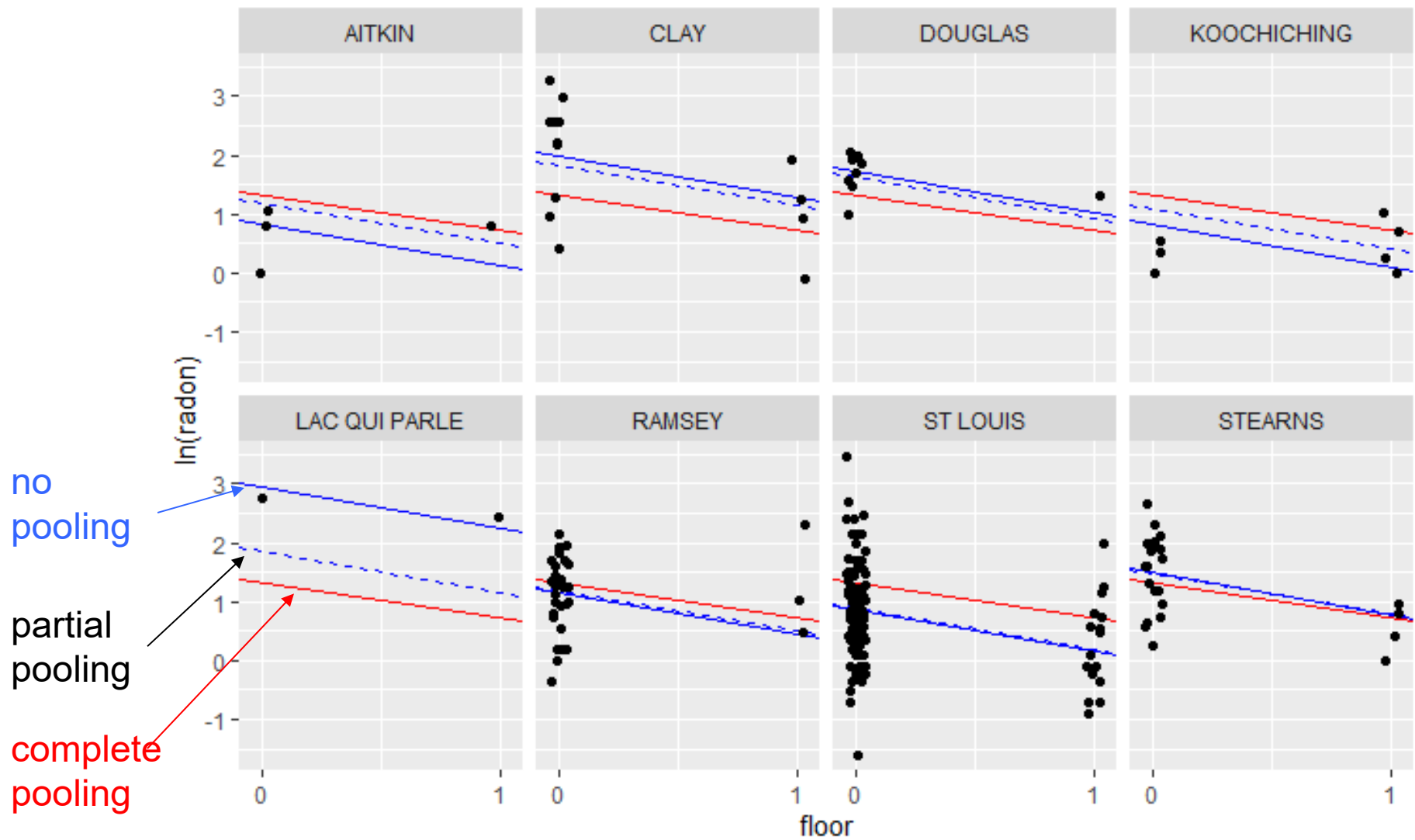
**See 12\_2\_radon\_multilevel\_3.R  
12\_2\_radon\_multilevel\_3.md**

```
#` Examine the parameter estimates:  
summary(ppfit)
```

```
## Linear mixed model fit by maximum likelihood ['lmerMod']  
## Formula: log_radon ~ floor_x + (1 | county)  
## Data: radon_dat  
##  
##      AIC      BIC   logLik deviance df.resid  
## 2171.7 2190.9 -1081.8 2163.7 915  
##  
## Scaled residuals:  
##      Min       1Q   Median       3Q      Max  
## -4.4071 -0.6164  0.0056  0.6398  3.4288  
##  
## Random effects:  
## Groups   Name      Variance Std.Dev.  
## county   (Intercept) 0.1053  0.3245  
## Residual                0.5703  0.7552  
## Number of obs: 919, groups: county, 85  
##  
## Fixed effects:  
##              Estimate Std. Error t value  
## (Intercept)  1.46116    0.05124  28.516  
## floor_x     -0.69264    0.07036  -9.844  
##  
## Correlation of Fixed Effects:  
##      (Intr)  
## floor_x -0.290
```

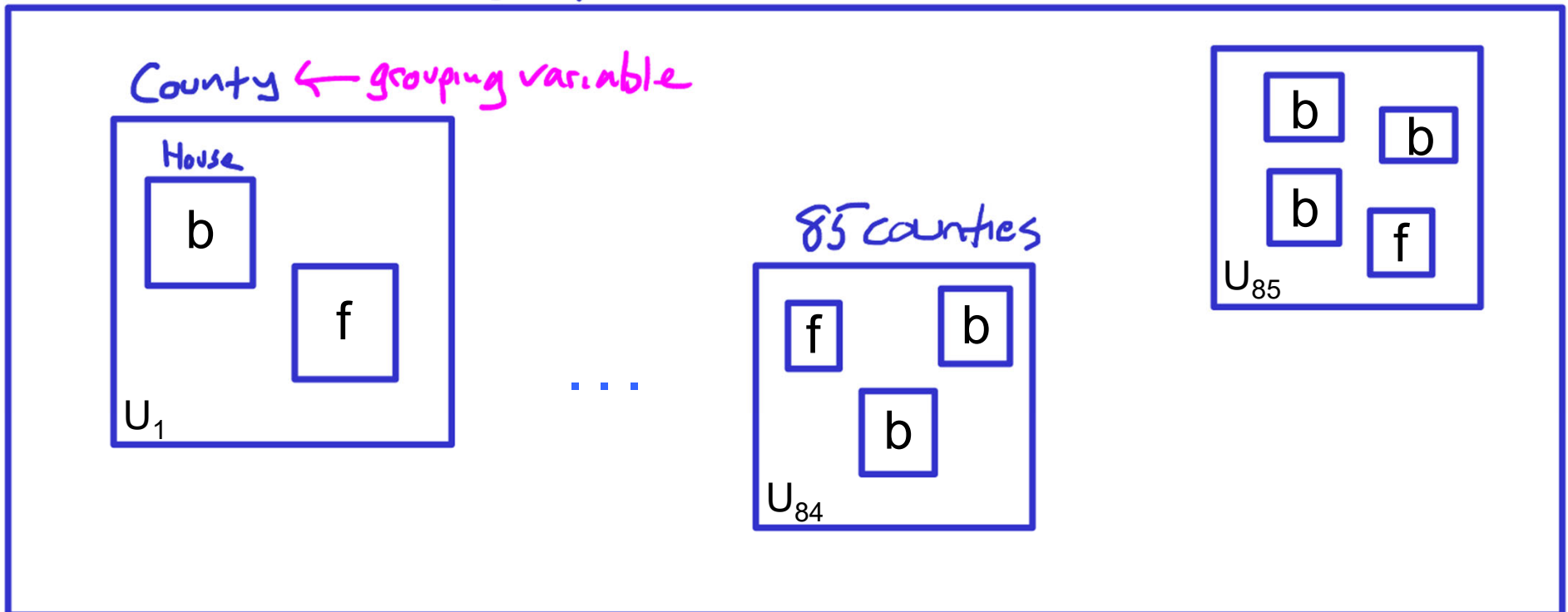
= sqrt(Variance)  
i.e. this is not  
uncertainty of  
the Variance  
estimate

Partial pooling (dashed): multilevel model, max likelihood estimates



# Model 3: 2 predictors

*State*



Predictors (fixed effects):

floor: house scale, categorical (b, f)

uranium: county scale, continuous +

# Writing model 2

Multilevel model, with 1 predictor at house scale

$$y_i \sim \text{Normal}(\mu_i, \sigma_h^2)$$

House-scale stochastic model

$$\mu_i = \alpha_{j[i]} + \beta_1 x_i$$

Predictor part of house-scale model

$$\alpha_j \sim \text{Normal}(\mu_c, \sigma_c^2)$$

County-scale stochastic model

# Writing model 3

Multilevel model, with 2 predictors at different scales

$$y_i \sim \text{Normal}(\mu_i, \sigma_h^2)$$

House-scale stochastic model

$$\mu_i = \alpha_{j[i]} + \beta_1 x_i$$

Predictor part of house-scale model

$$\alpha_j \sim \text{Normal}(\mu_\alpha, \sigma_\alpha^2)$$

County-scale stochastic model

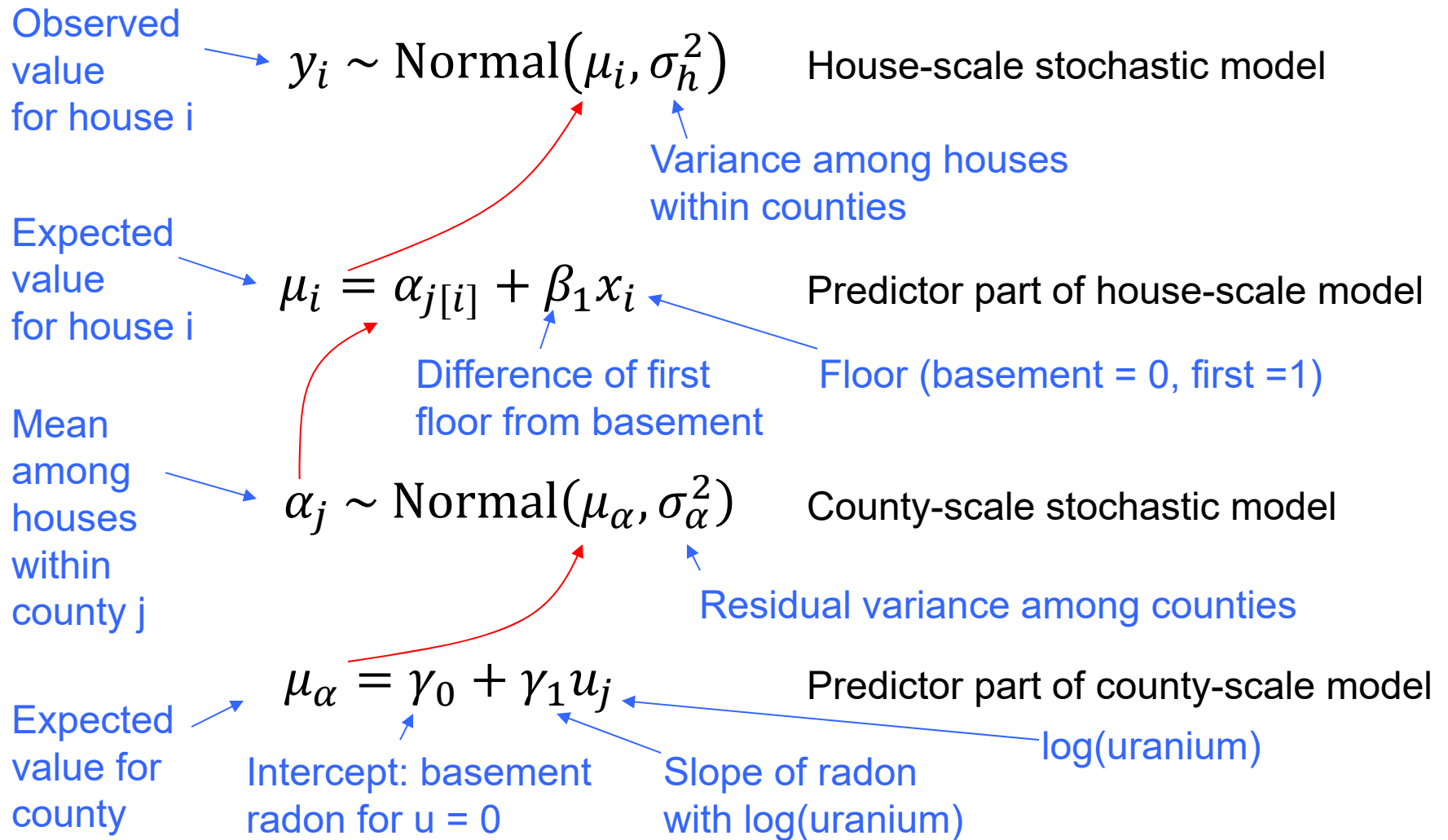
$$\mu_\alpha = \gamma_0 + \gamma_1 u_j$$

Predictor part of county-scale model



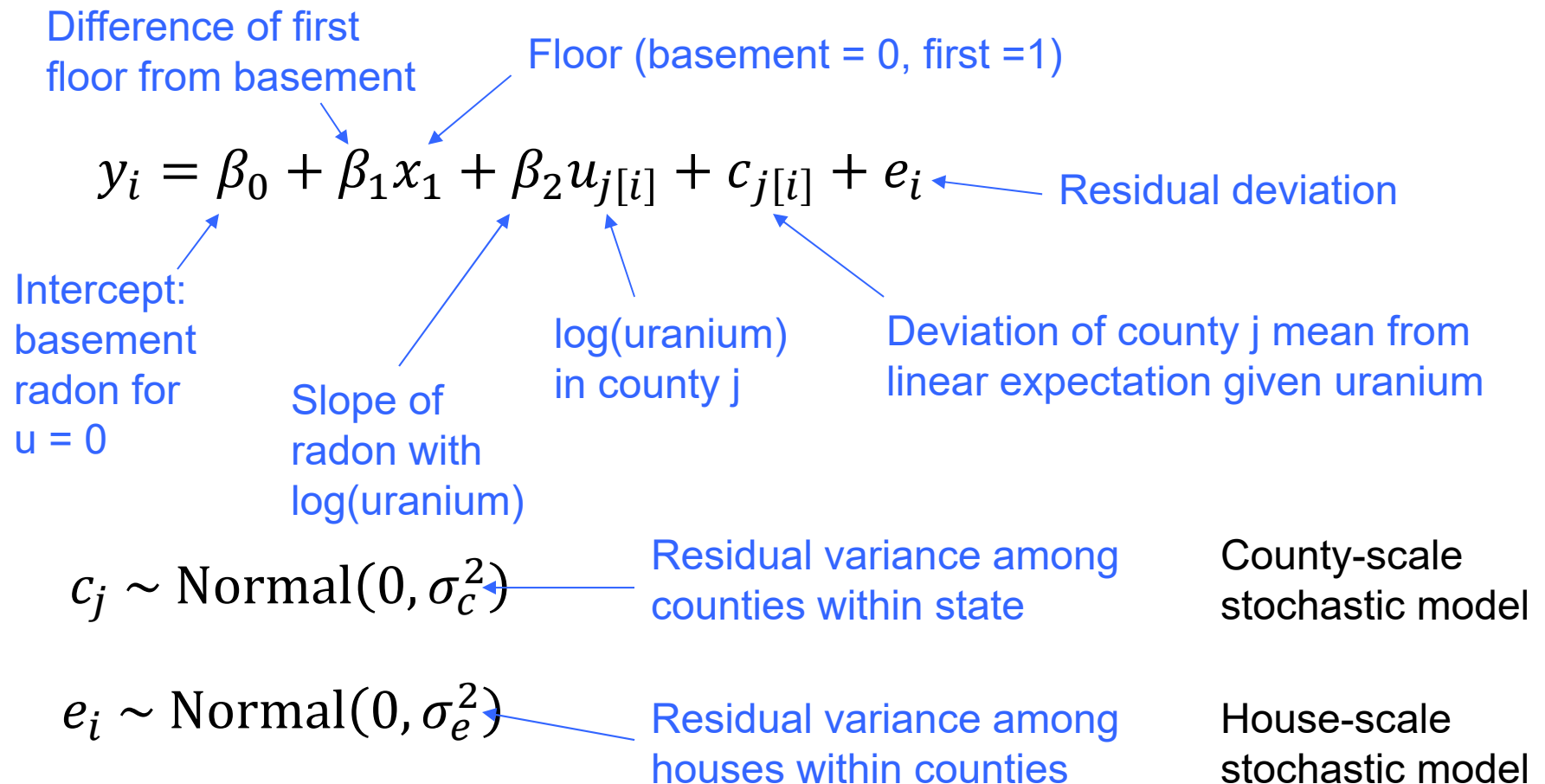
# Writing model 3

Multilevel model, with 2 predictors at different scales



# Alternative parameterization

Multilevel model, with 2 predictors at different levels



# Linear model syntax

```
log_radon ~ floor + logu + (1|county)
```

**Model with two predictors, each at different levels**

**House level predictor: floor (basement, first floor)**

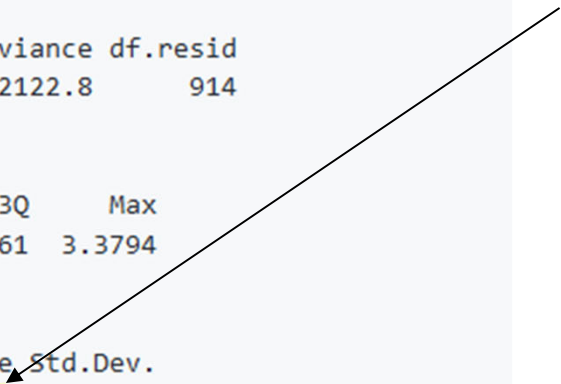
**County level predictor: uranium concentration in substrate**

**See 12\_3\_radon\_multilevel\_4.R  
12\_3\_radon\_multilevel\_4.md**

```
summary(ppfit)
```

```
## Linear mixed model fit by maximum likelihood ['lmerMod']
## Formula: log_radon ~ floor_x + logu + (1 | county)
## Data: radon_dat
##
##      AIC      BIC   logLik deviance df.resid
## 2132.8  2156.9 -1061.4  2122.8     914
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.9976 -0.6163  0.0307  0.6561  3.3794
##
## Random effects:
## Groups   Name      Variance Std.Dev.
## county   (Intercept) 0.02127  0.1458
## Residual                0.57499  0.7583
## Number of obs: 919, groups: county, 85
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  1.46427    0.03714  39.421
## floor_x      -0.66644    0.06865  -9.708
## logu         0.72320    0.08965   8.067
##
## Correlation of Fixed Effects:
##              (Intr) floor_x
## floor_x     -0.361
## logu        0.154 -0.011
```

Now a lot less  
because  
uranium  
accounts for it



```
ppfit_bayes <- stan_lmer(log_radon ~ floor_x + logu + (1|county), data=radon_dat)
```

```
print(summary(ppfit_bayes)[,c("mean","sd","n_eff","Rhat")], digits=3)
```

##	mean	sd	n_eff	Rhat
## (Intercept)	1.46e+00	0.0390	2510	1.000
## floor_x	-6.68e-01	0.0700	5342	0.999
## logu	7.22e-01	0.0942	2140	1.000
## b[(Intercept) county:AITKIN]	-2.46e-02	0.1465	4445	1.001
## b[(Intercept) county:ANOKA]	1.00e-02	0.1027	2583	1.000
## b[(Intercept) county:BECKER]	1.33e-02	0.1519	5190	0.999
## b[(Intercept) county:BELTRAMI]	1.12e-01	0.1477	3595	0.999
## b[(Intercept) county:BENTON]	7.06e-02	0.1450	5771	1.000
## b[(Intercept) county:WATONWAN]	1.28e-01	0.1019	2822	1.000
## b[(Intercept) county:WILKIN]	2.57e-02	0.1635	5435	1.000
## b[(Intercept) county:WINONA]	-6.49e-02	0.1304	4673	0.999
## b[(Intercept) county:WRIGHT]	8.30e-02	0.1253	4844	1.000
## b[(Intercept) county:YELLOW_MEDICINE]	-4.40e-02	0.1599	4798	1.000
## sigma	7.59e-01	0.0183	3662	1.000
## Sigma[county:(Intercept),(Intercept)]	2.71e-02	0.0165	1062	1.002
## mean_PPD	1.22e+00	0.0348	4574	1.000
## log-posterior	-1.18e+03	9.4548	787	1.004

Random  
effects

$\sigma$  = s.d.

$\Sigma$  = variance

Two predictors  
 Floor (house scale)  
 Uranium (county scale)

Each point below  
 is the intercept above

Each point is:  
 $\beta_0 + \beta_2 u_j + c_j$

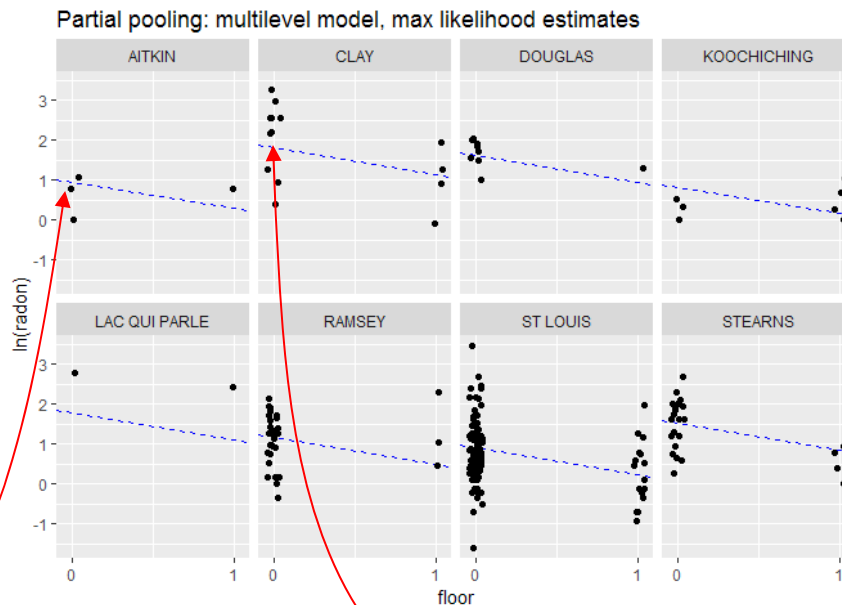


Fig. 12.5

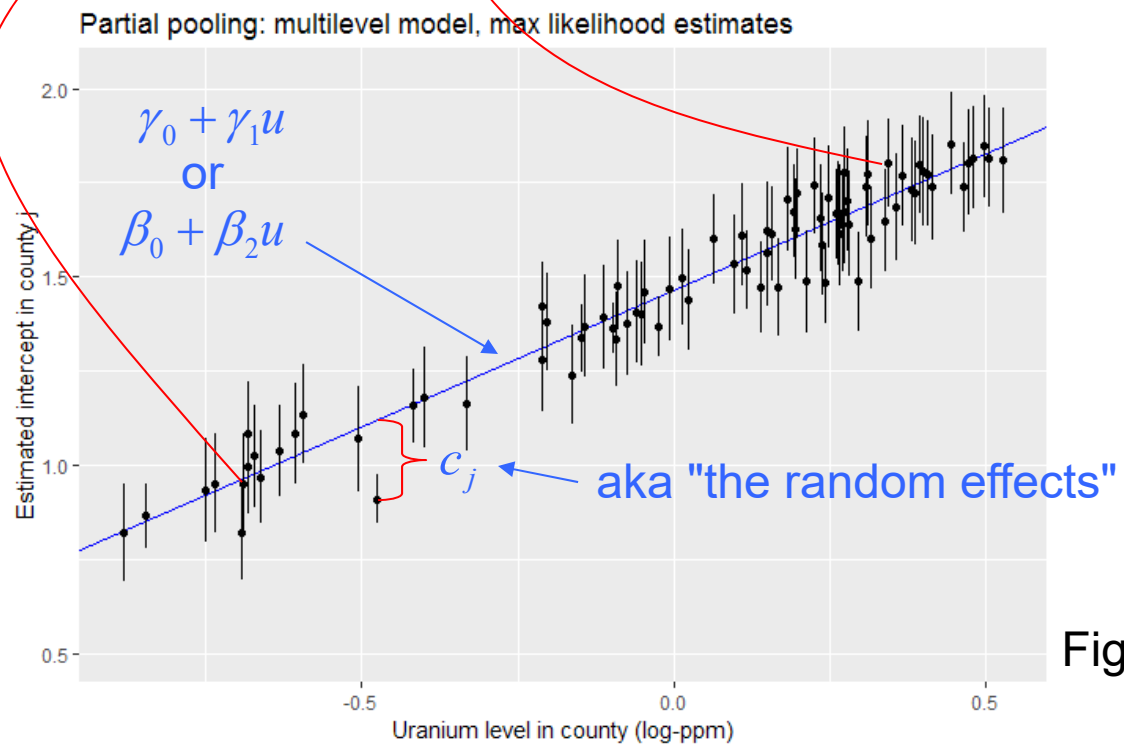
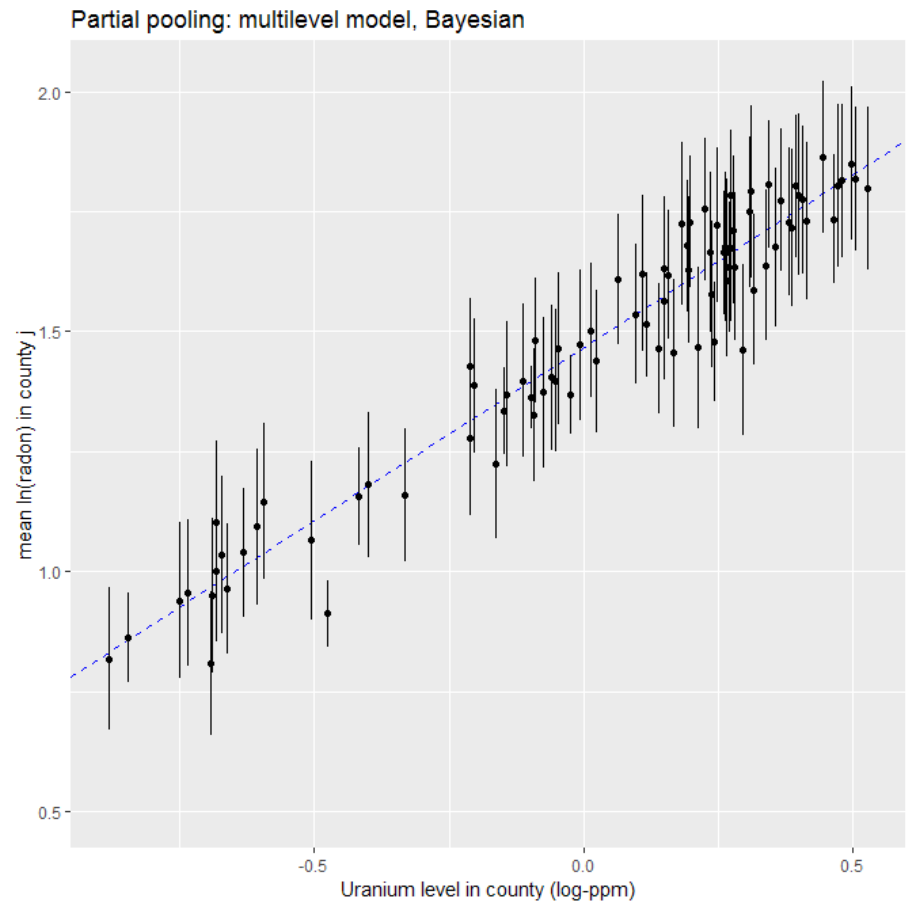
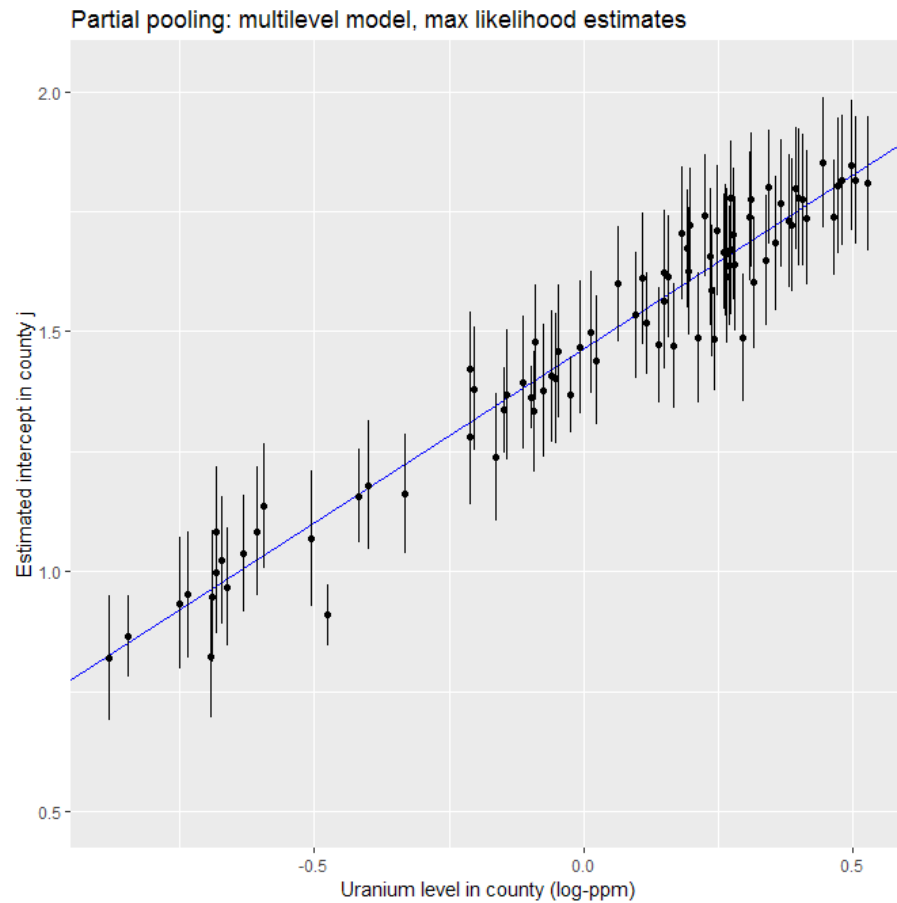


Fig. 12.6

# Max lik vs Bayesian



Some minor differences (e.g. Bayesian intervals a bit wider)