Today

- Individual projects
  - see my initial comment in your repo
  - Wednesday individual meetings (5 mins)
- Radon
  - scales and groups
  - study design
  - models and code

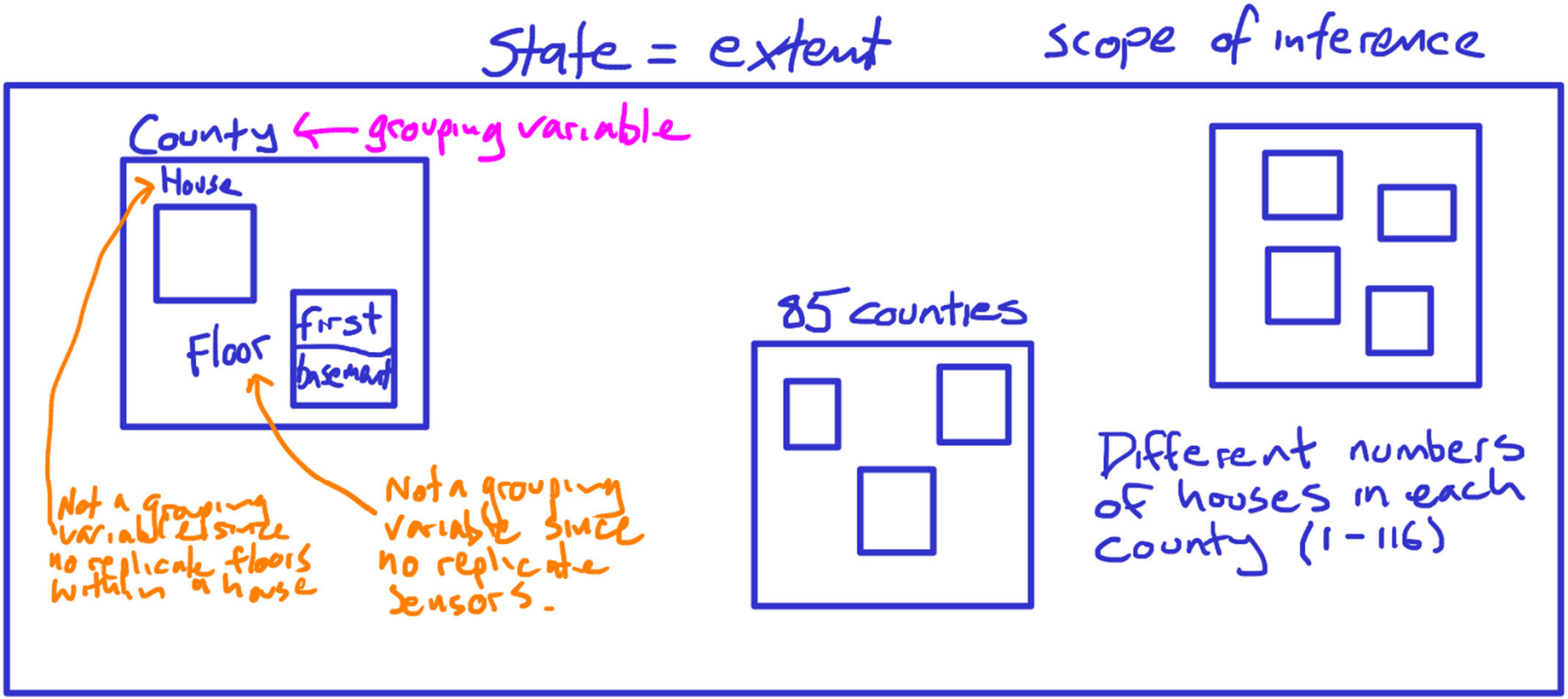
For each analysis problem

- Sketch data design
- Math equations
- Linear model syntax

Scales in radon data

- You identified these scales:
- State, county, house, floor, radon sensor, time?, sample (unit)
- Hierarchical structure: nested
- State: scope of inference
- Sample (unit) is smallest scale
- Potential groups: state, county, house, floor, sensor ... but 1 sample per house so house is smallest (unit scale)

Sketch

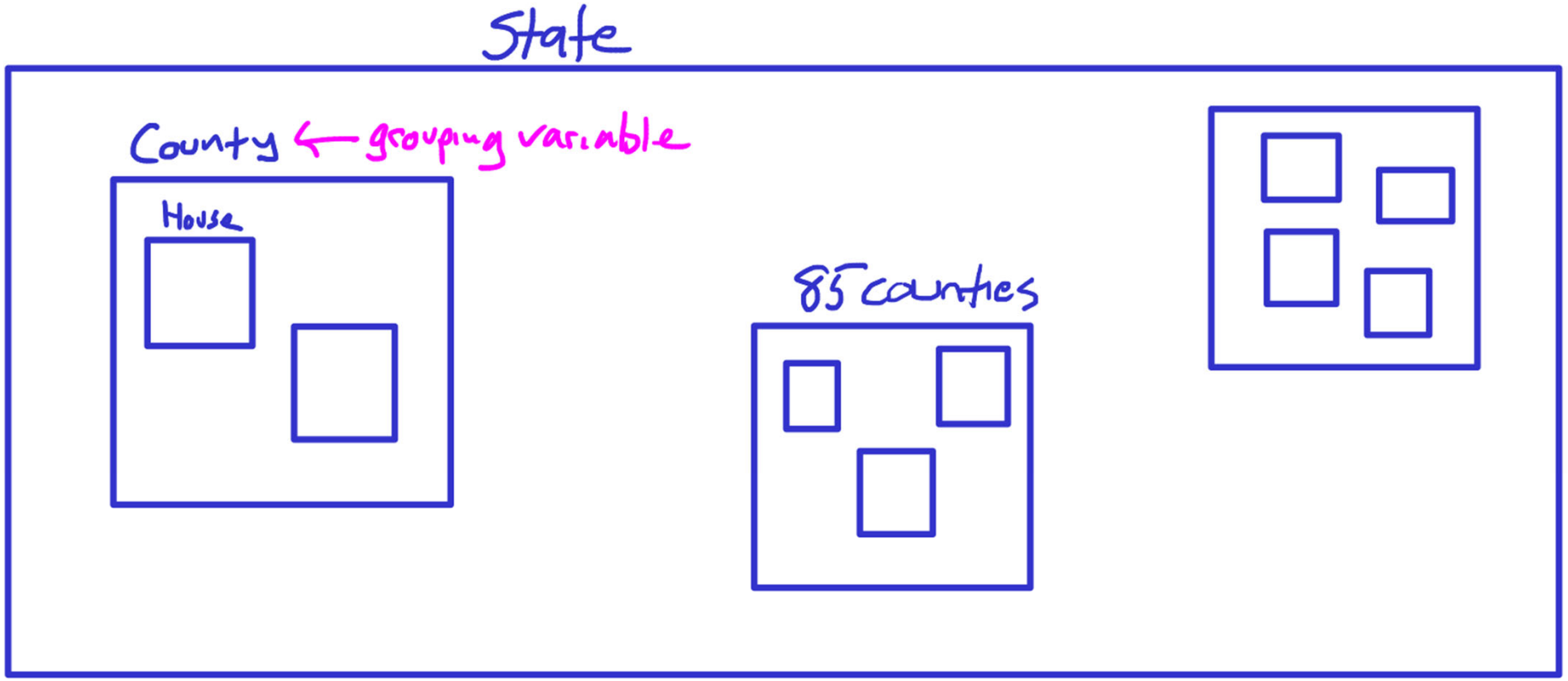

State = extent
scope of inference
County ← grouping variable
House
Floor
first
basement
Not a grouping variable since no replicate floors within a house
Not a grouping variable since no replicate sensors.
85 counties
Different numbers of houses in each county (1-116)

Fixed vs random

- Floor: fixed effect
  - deterministic model for parameters
  - shared characteristic
- County: random effect
  - stochastic model for parameters
  - model by group (parameters vary by group)
  - model accounts for correlation among houses within counties

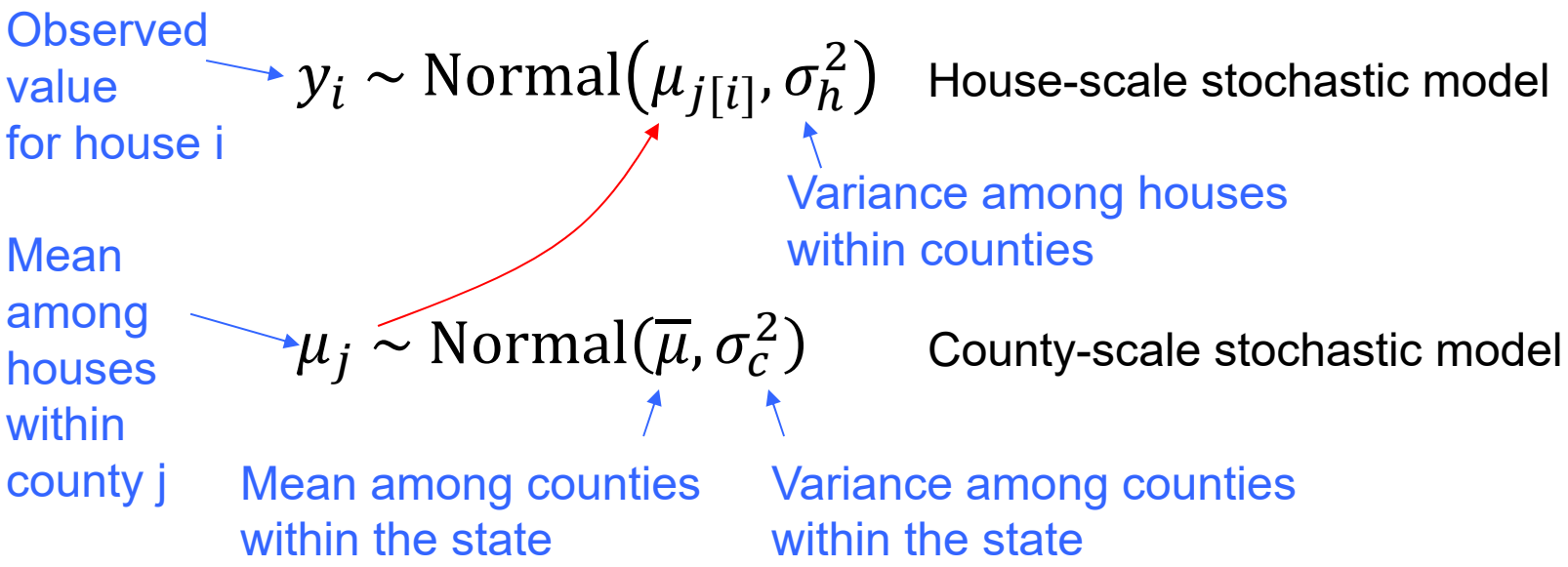
We could alternatively model county as a fixed effect: model parameters would be deterministic

Model 1: random effects



Writing model 1

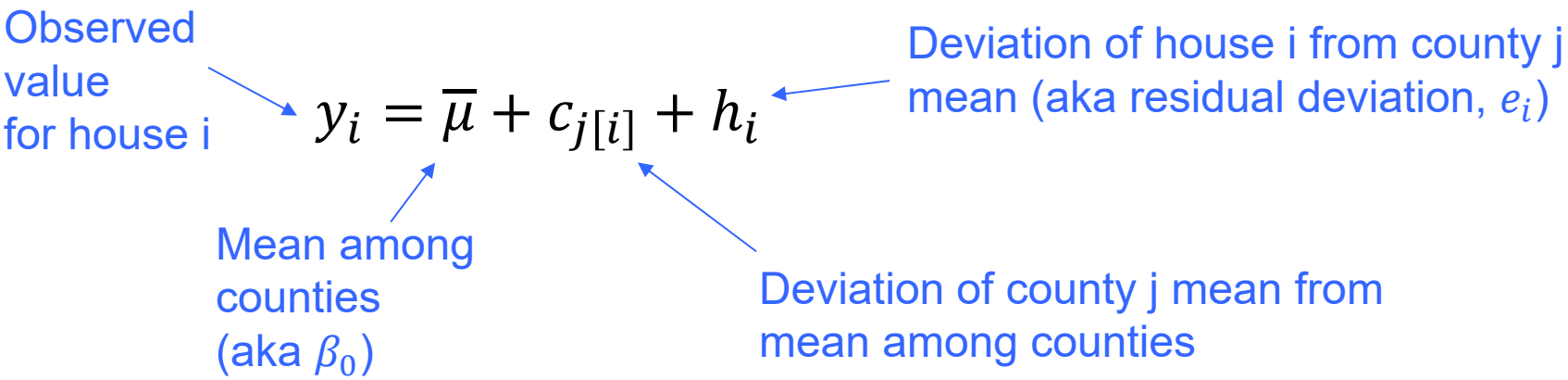
Basic multilevel model, no predictors



$j[i]$ is the county (j) of house i

Alt parameterization

Additive decomposition



$c_j \sim \text{Normal}(0, \sigma_c^2)$ — Variance among counties within state — County-scale stochastic model

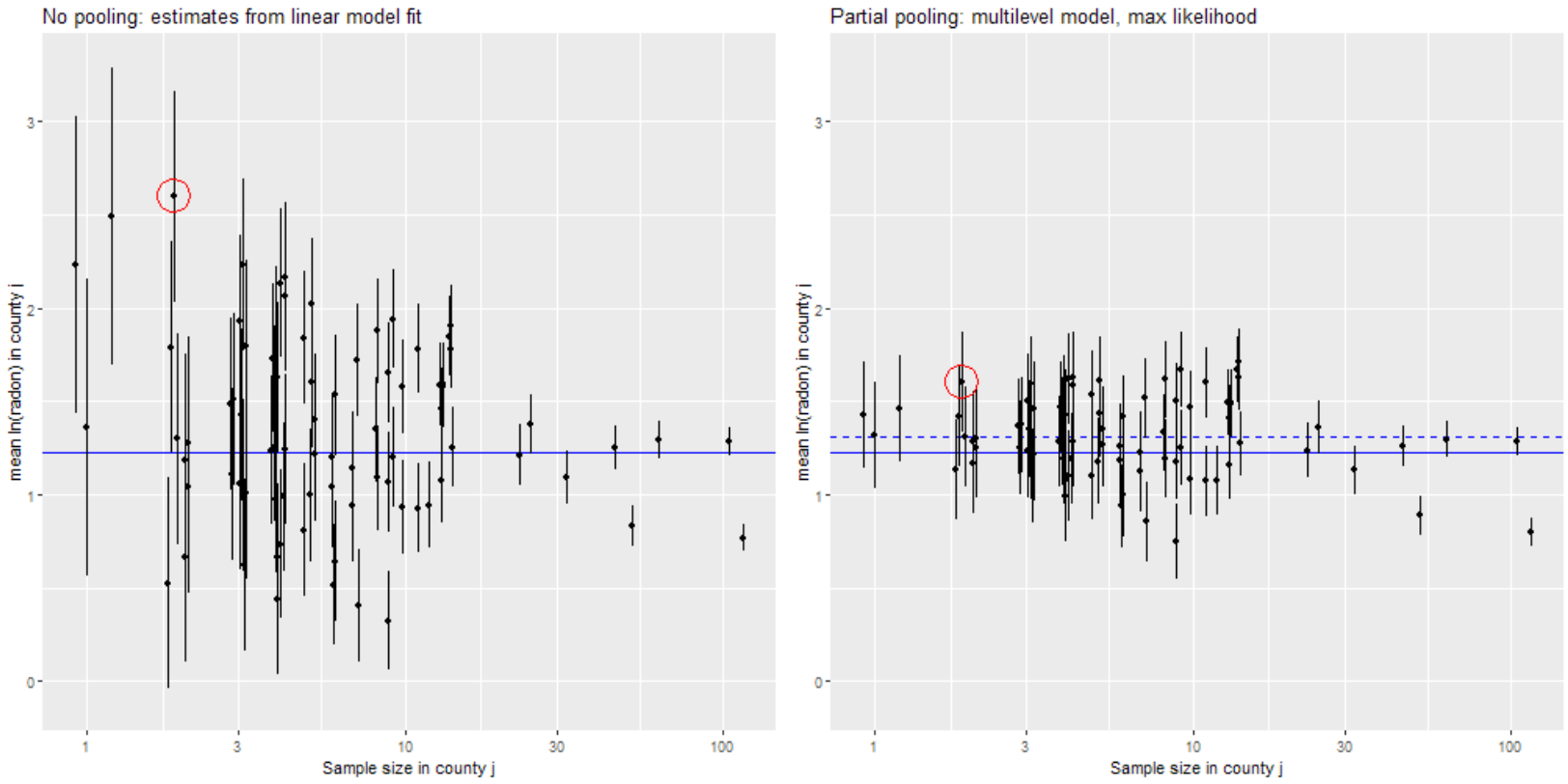
$h_i \sim \text{Normal}(0, \sigma_h^2)$ — Variance among houses within counties — House-scale stochastic model

Linear model syntax

```
log_radon ~ 1 + (1|county)
```

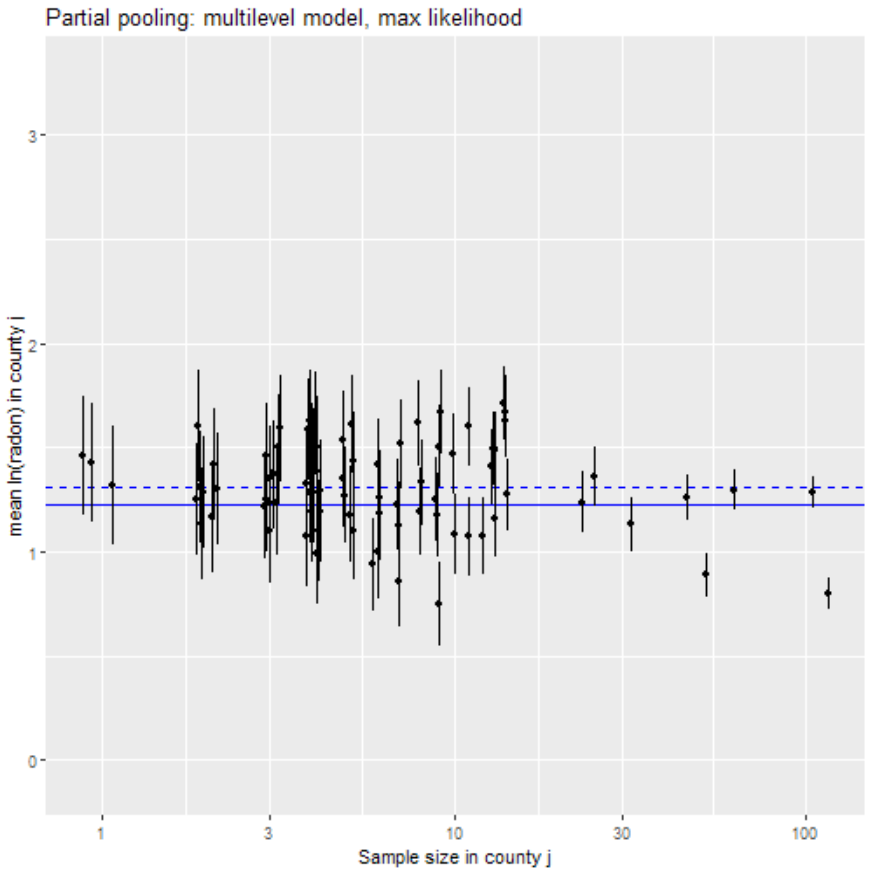
We considered code for the radon example.
See 11_3_radon_multilevel_2.md

Multilevel model - radon

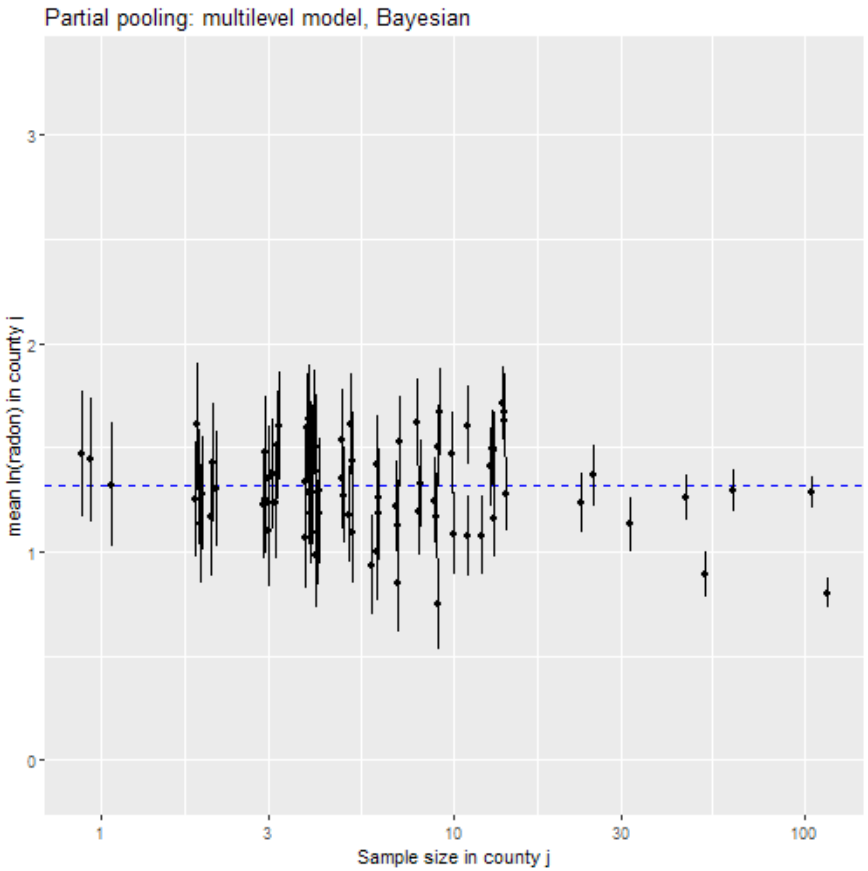


Shrinkage

Multilevel model - radon

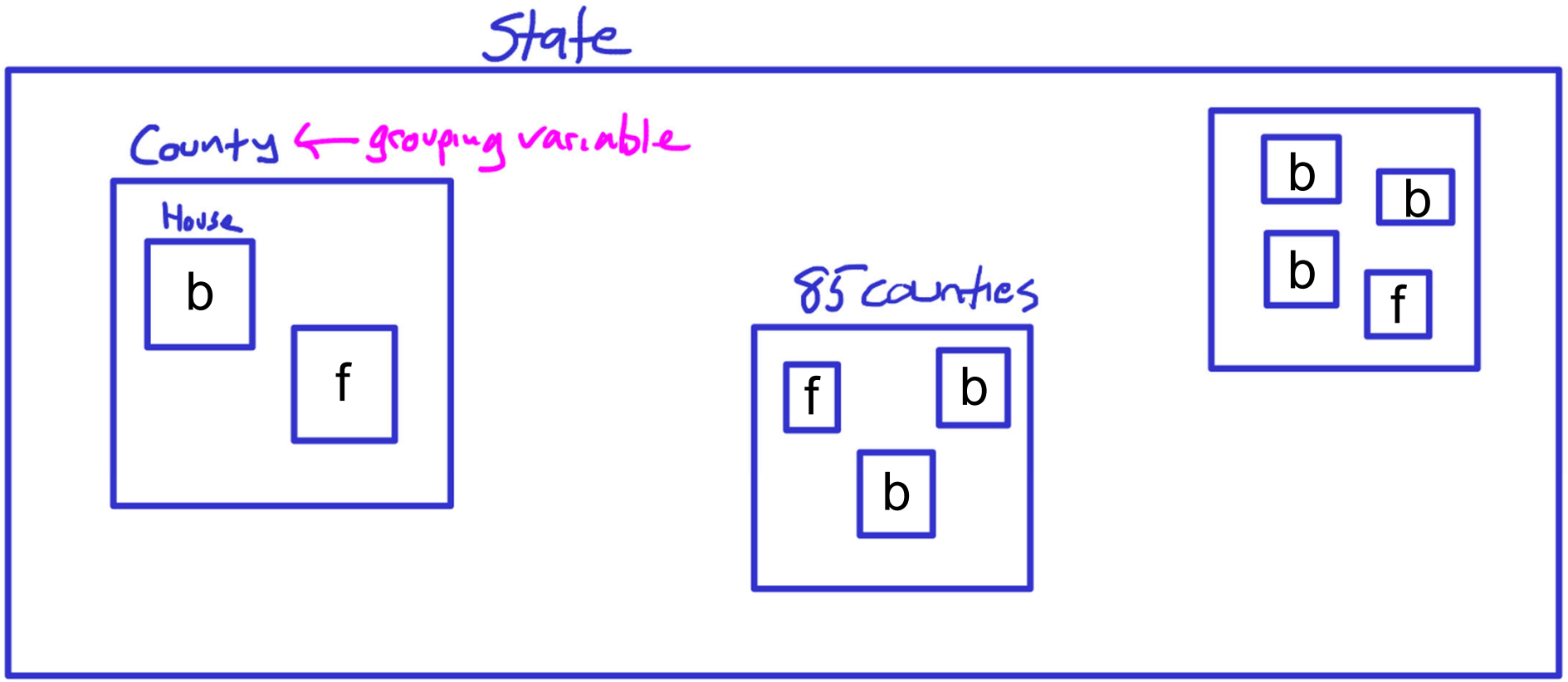


Frequentist



Bayesian

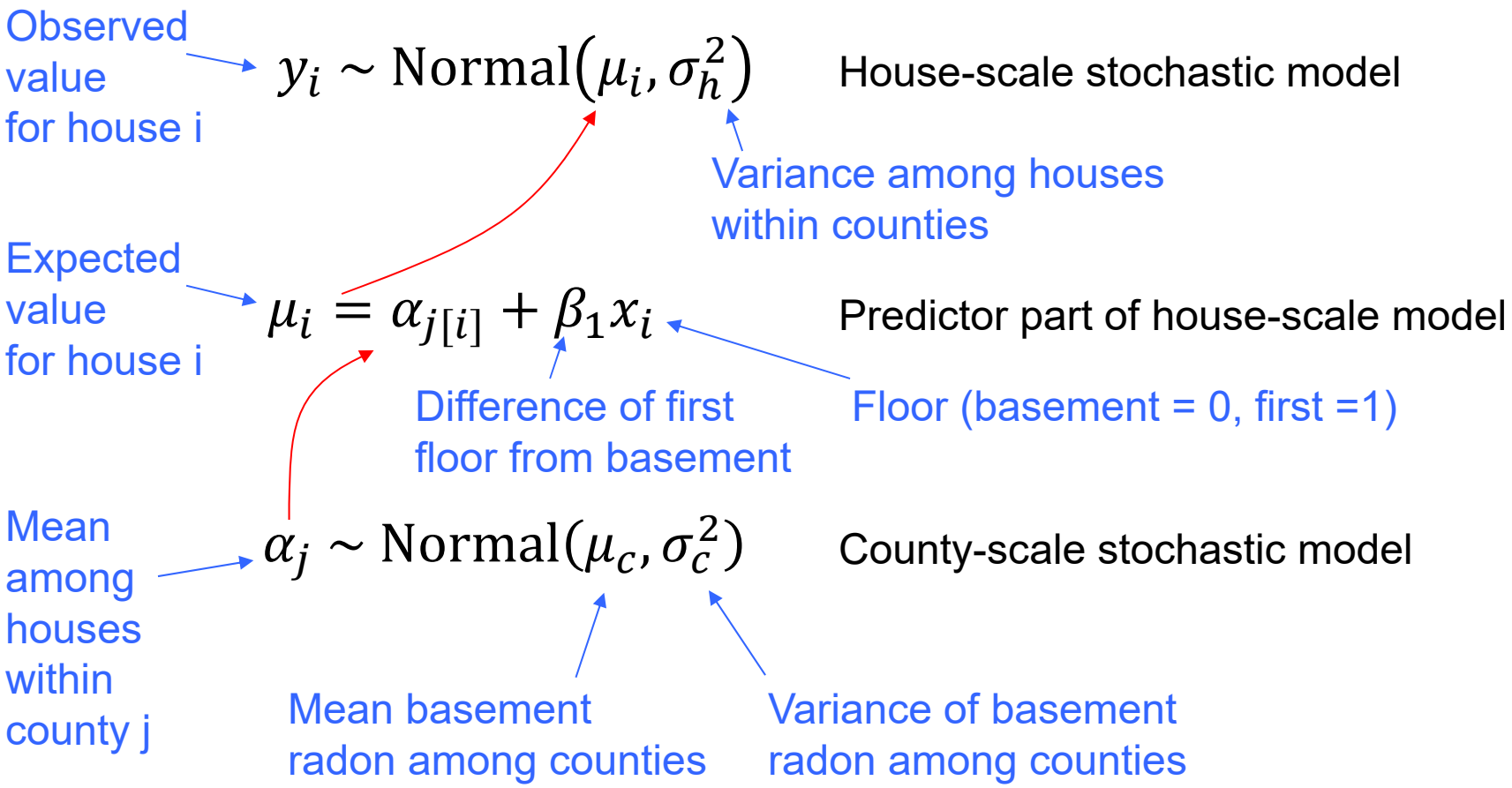
Model 2: 1 predictor



Predictor (fixed effect) is at house scale

Writing model 2

Multilevel model, with 1 predictor at house scale



Model 2: Alt parameterization

Multilevel model, with 1 predictor at house scale

Difference of first floor from basement

Floor (basement = 0, first =1)

$$y_i = \beta_0 + \beta_1 x_1 + c_{j[i]} + e_i$$

Deviation of house i from county j mean

Mean basement radon among counties

Deviation of county j mean from mean among counties

$$c_j \sim \text{Normal}(0, \sigma_c^2)$$

Variance among counties within state

County-scale stochastic model

$$e_i \sim \text{Normal}(0, \sigma_e^2)$$

Variance among houses within counties

House-scale stochastic model

Linear model syntax

```
log_radon ~ floor + (1|county)
```

Equivalent:

```
log_radon ~ 1 + floor + (1|county)
```

Model with one house-scale predictor

See 12_2_radon_multilevel_3.R
12_2_radon_multilevel_3.md