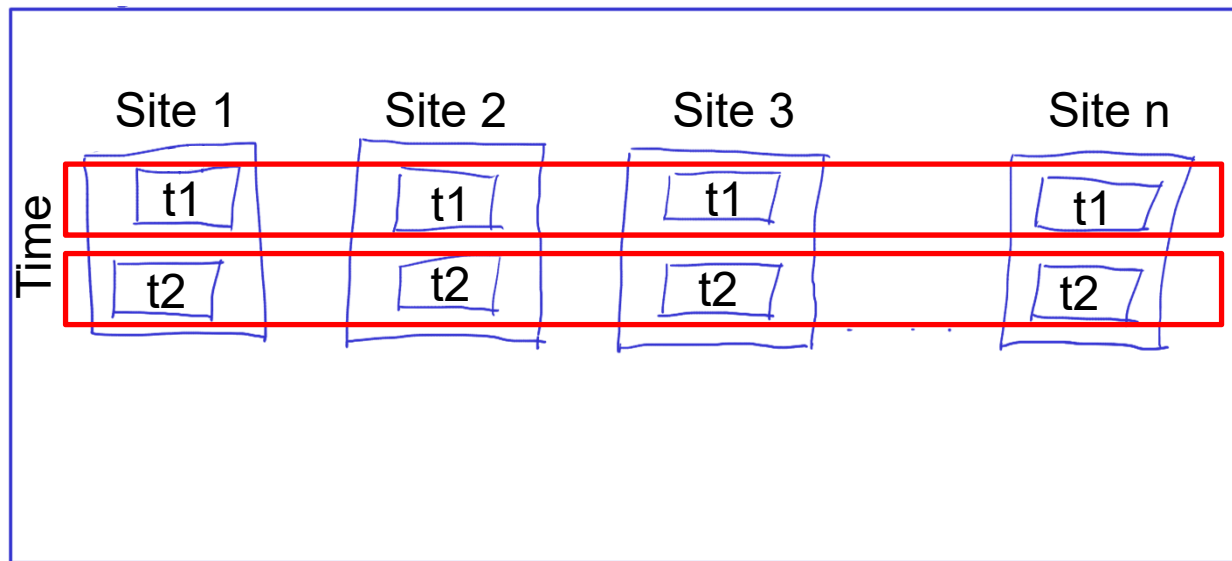


# Space and time

- Space-time scales of variation



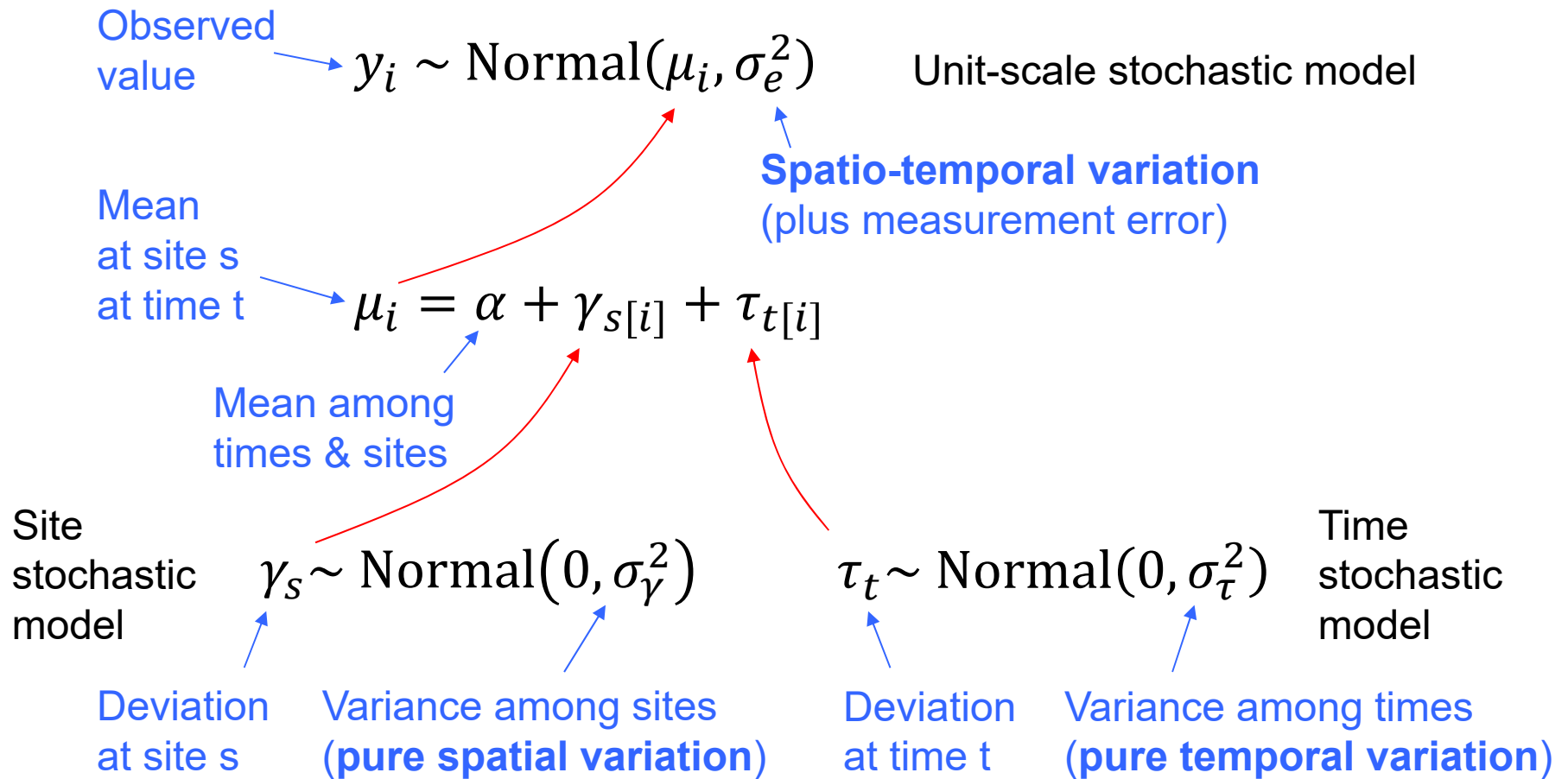
Samples from the same site are likely correlated  
Samples from the same time are likely correlated

Sites (space) have samples at multiple times.

Site (random effect) groups the times together at the same site

Time (random effect) groups the sites together at the same time

# Space-time scales of variation

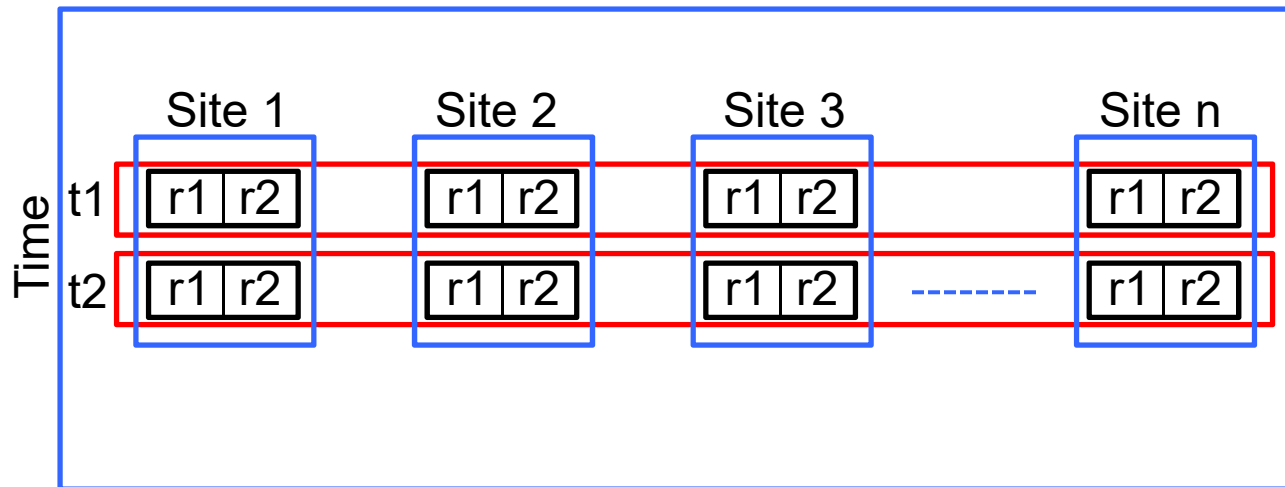


R code: `stan_lmer(y ~ (1|site) + (1|time))`

# Space and time

- Space-time scales of variation

With replicate samples at each site at each time



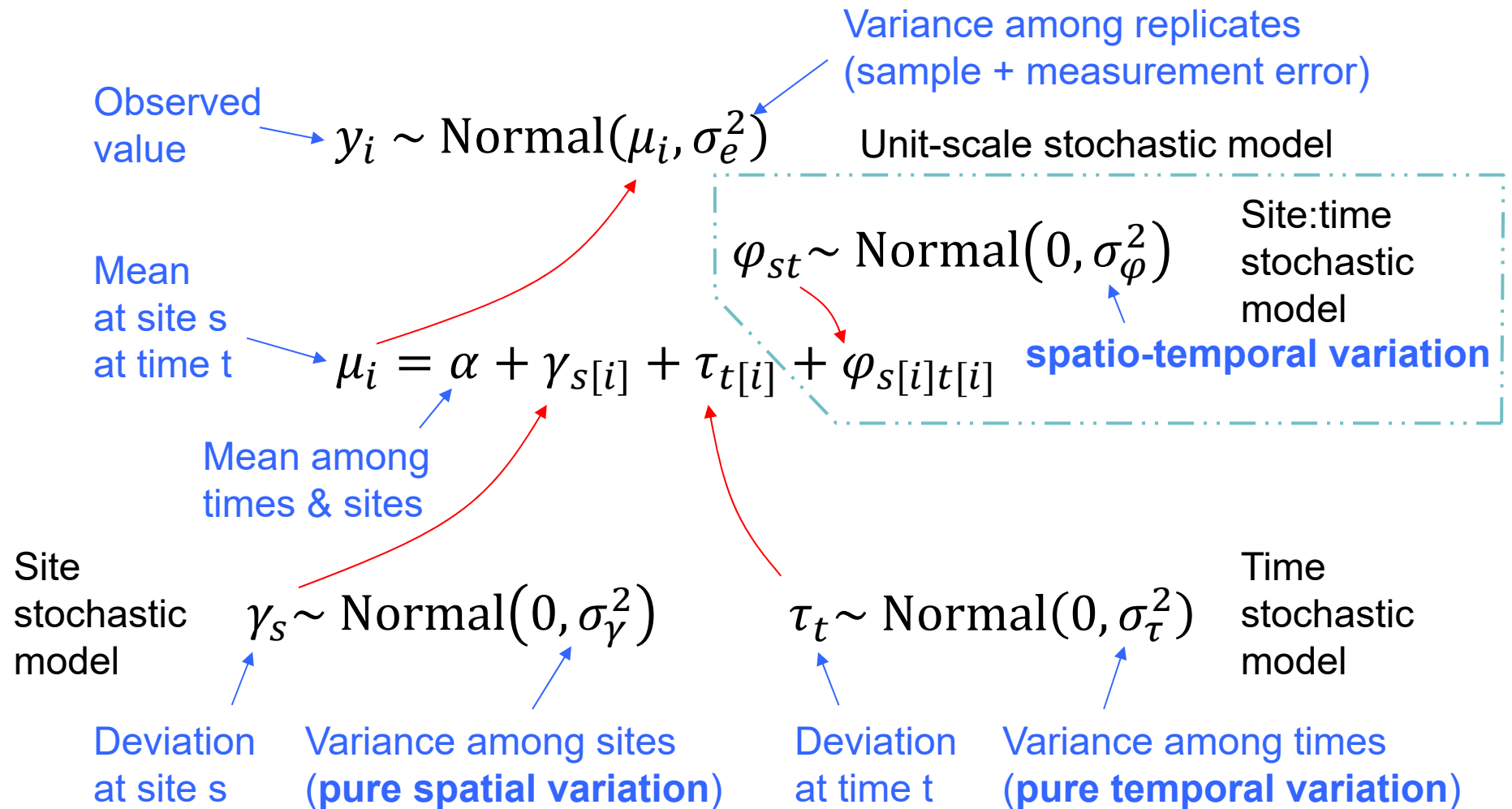
Site (random effect)  
groups the times  
together at the same site

Time (random effect)  
groups the sites together  
at the same time

Site:time (random effect)  
groups the replicates  
together at the same site  
and time

Samples from the same site are likely correlated  
Samples from the same time are likely correlated  
Samples from the same site-time are likely correlated

# Space-time scales of variation

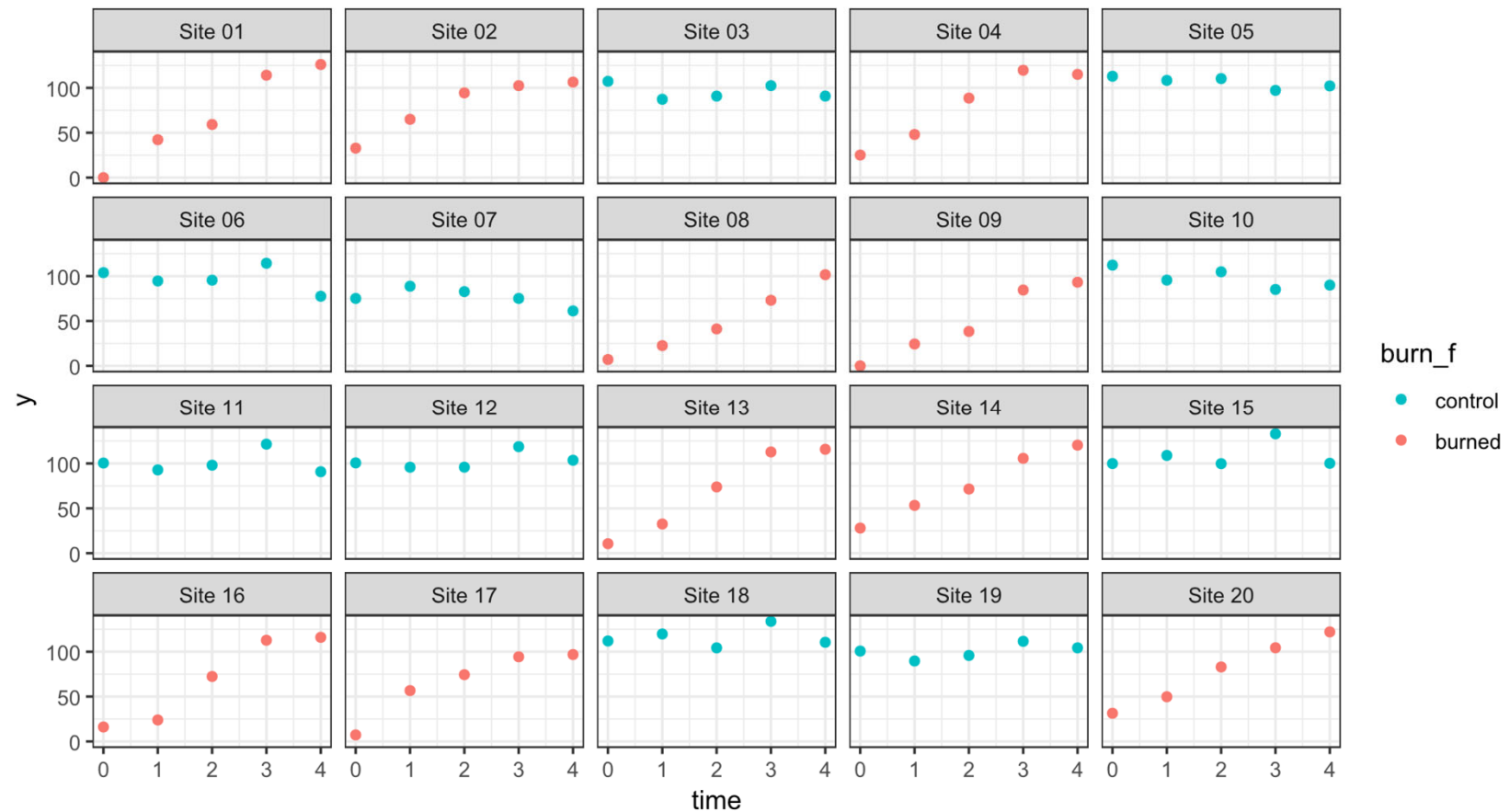


R code: `stan_lmer(y ~ (1|site) + (1|time) + (1|site:time))`

# Space and time

- Mixed treatment and grouping variables
- Repeated measurements
  - at the same location or same individual
  - space groups times together
- Space or time can be both fixed (modeling trends) and random (modeling remaining variation) in the same model

# Repeated measurements



Burn treatment is applied randomly to sites  
Space (site) groups the times together

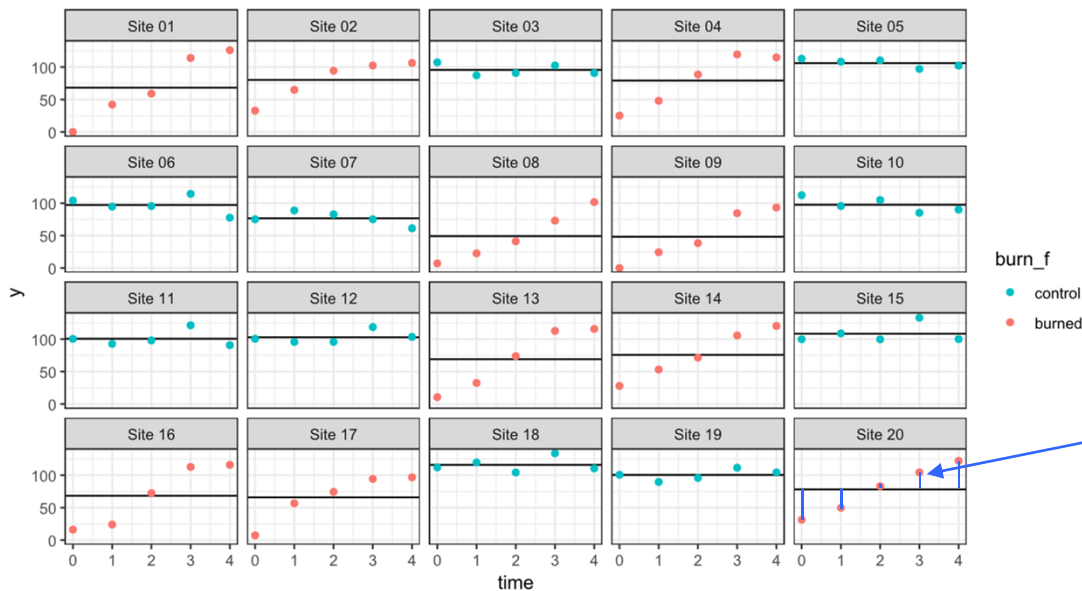
# Repeated measurements

Model with **burn** as fixed effect + **site** as random effect

$$y_i = \beta_0 + \beta_1 x_{1,i} + \gamma_{s[i]} + e_i$$

Control      Burn effect      Space (site) deviations      Time + space:time + error

R code: `stan_lmer(y ~ burn + (1|site))`



$$\gamma_s \sim \text{Norm}(0, \sigma_\gamma^2)$$

$$e_i \sim \text{Norm}(0, \sigma_e^2)$$

$e_i$  deviations include  
temporal + spatiotemporal  
variation + error

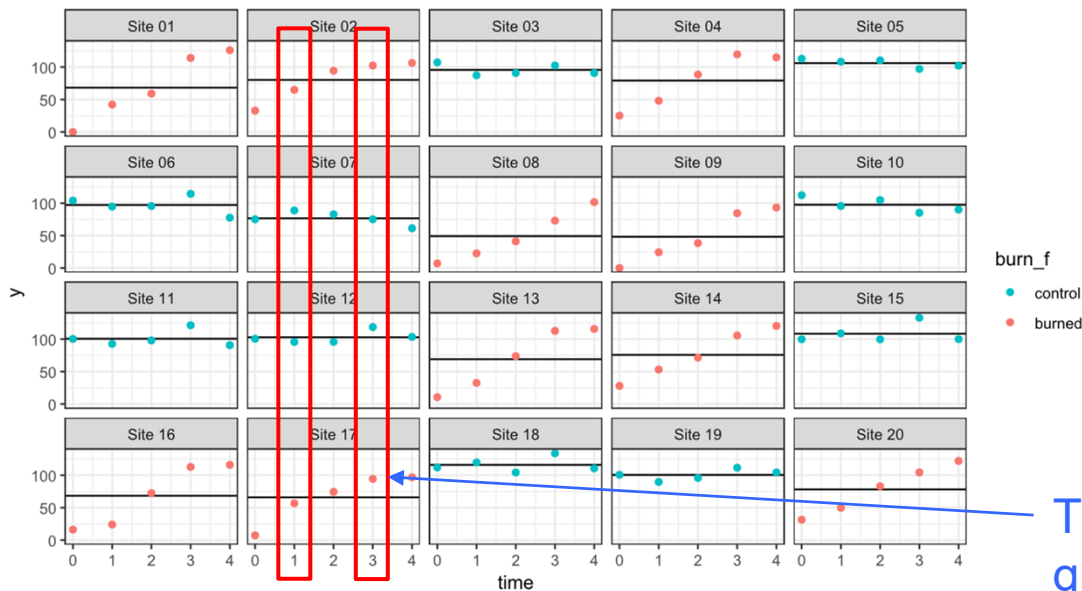
# Repeated measurements

Model with **burn** as fixed effect + **site** and **time** as random effects

$$y_i = \beta_0 + \beta_1 x_{1,i} + \gamma_{s[i]} + \tau_{t[i]} + e_i$$

Control      Burn effect      Space deviations      Time deviations      Space:time + error

R code: `stan_lmer(y ~ burn + (1|site) + (1|time))`



$$\gamma_s \sim \text{Norm}(0, \sigma_\gamma^2)$$

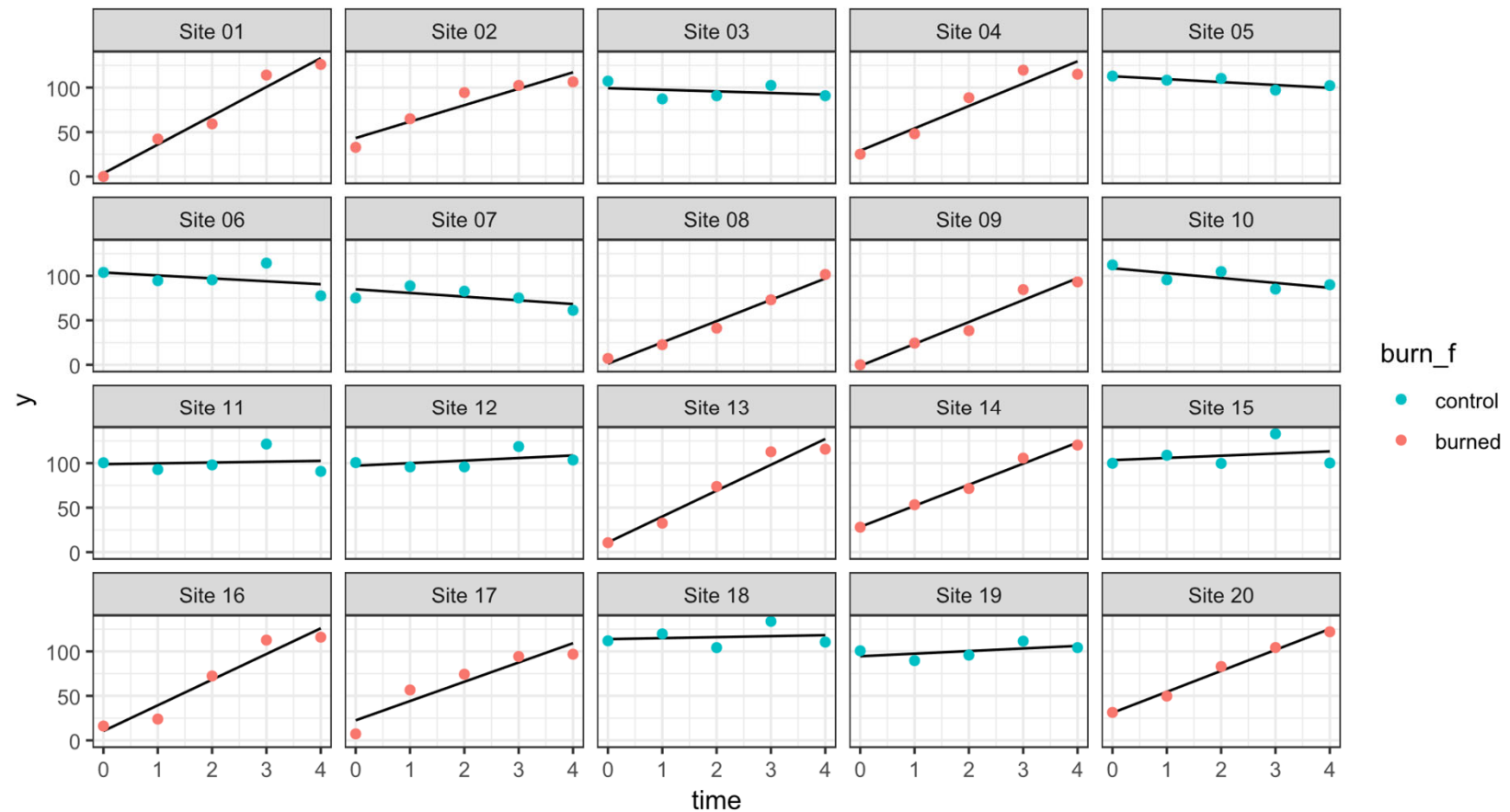
$$\tau_t \sim \text{Norm}(0, \sigma_\tau^2)$$

$$e_i \sim \text{Norm}(0, \sigma_e^2)$$

Time random effect  
groups the sites together



# Repeated measurements



We actually have trends over time

# Repeated measurements

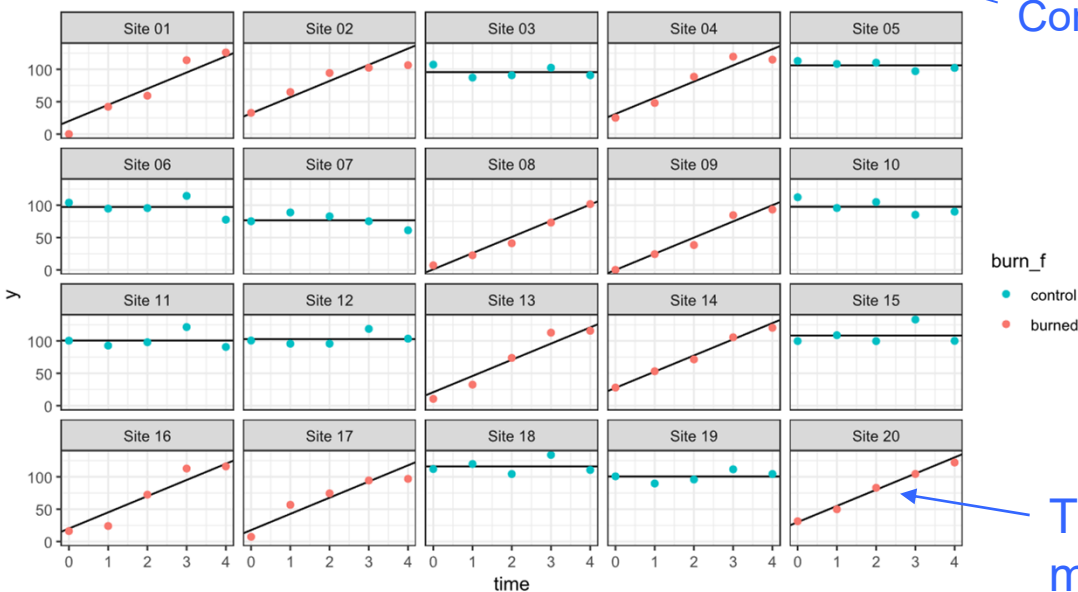
Model with **time** and **burn** as fixed effects + **site** and **time** as random effects

$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \beta_3 x_{1,i} x_{2,i} + \gamma_{s[i]} + \tau_{t[i]} + e_i$$

Control      Time trend for control      Burn effect on intercept      Burn effect on slope      Space deviations      Time deviations      Space:time + error

R code: `stan_lmer(y ~ time * burn + (1|site) + (1|time))`

Space random effect is varying intercepts



Continuous

Factor

$$\gamma_s \sim \text{Norm}(0, \sigma_\gamma^2)$$

$$\tau_t \sim \text{Norm}(0, \sigma_\tau^2)$$

$$e_i \sim \text{Norm}(0, \sigma_e^2)$$

Time fixed effect is modeling the trend

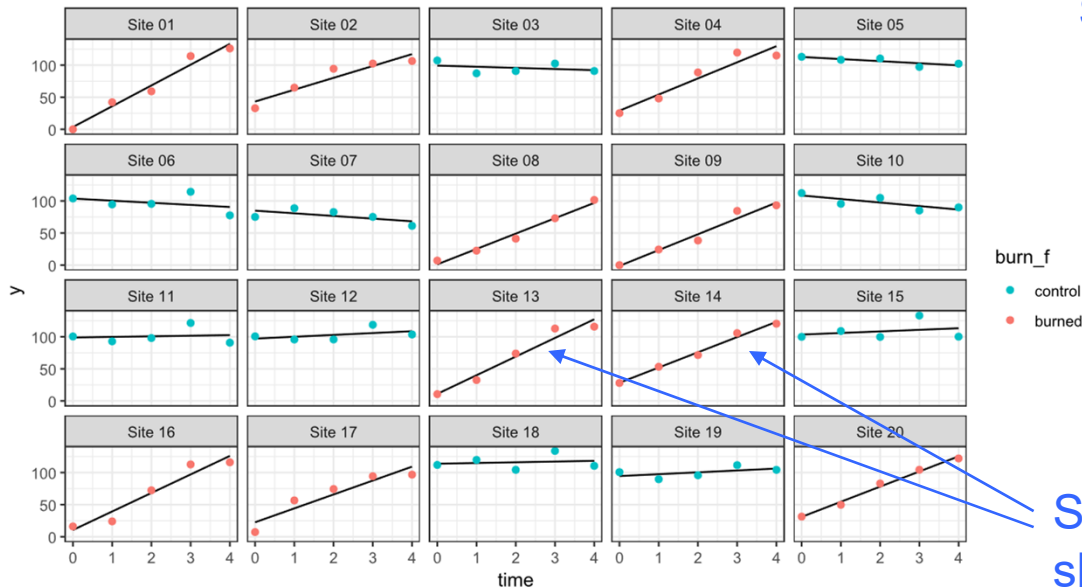
# Repeated measurements

Model with **time** and **burn** as fixed effects + **site** and **time** as random effects  
+ **random slope** for **time**

$$y_i = \beta_0 + (\beta_1 + b_{s[i]})x_{1,i} + \beta_2 x_{2,i} + \beta_3 x_{1,i}x_{2,i} + \gamma_{s[i]} + \tau_{t[i]} + e_i$$

Time trend for control
Space (slope) deviations
Space (intercept) deviations

R code: `stan_lmer(y ~ time * burn + (1+time|site) + (1|time))`



Slope of y vs time varies by site

$$\gamma_s \sim \text{Norm}(0, \sigma_\gamma^2)$$

$$b_s \sim \text{Norm}(0, \sigma_b^2)$$

$$\tau_t \sim \text{Norm}(0, \sigma_\tau^2)$$

$$e_i \sim \text{Norm}(0, \sigma_e^2)$$

Slope random effect allows slope to vary by site