Thursday

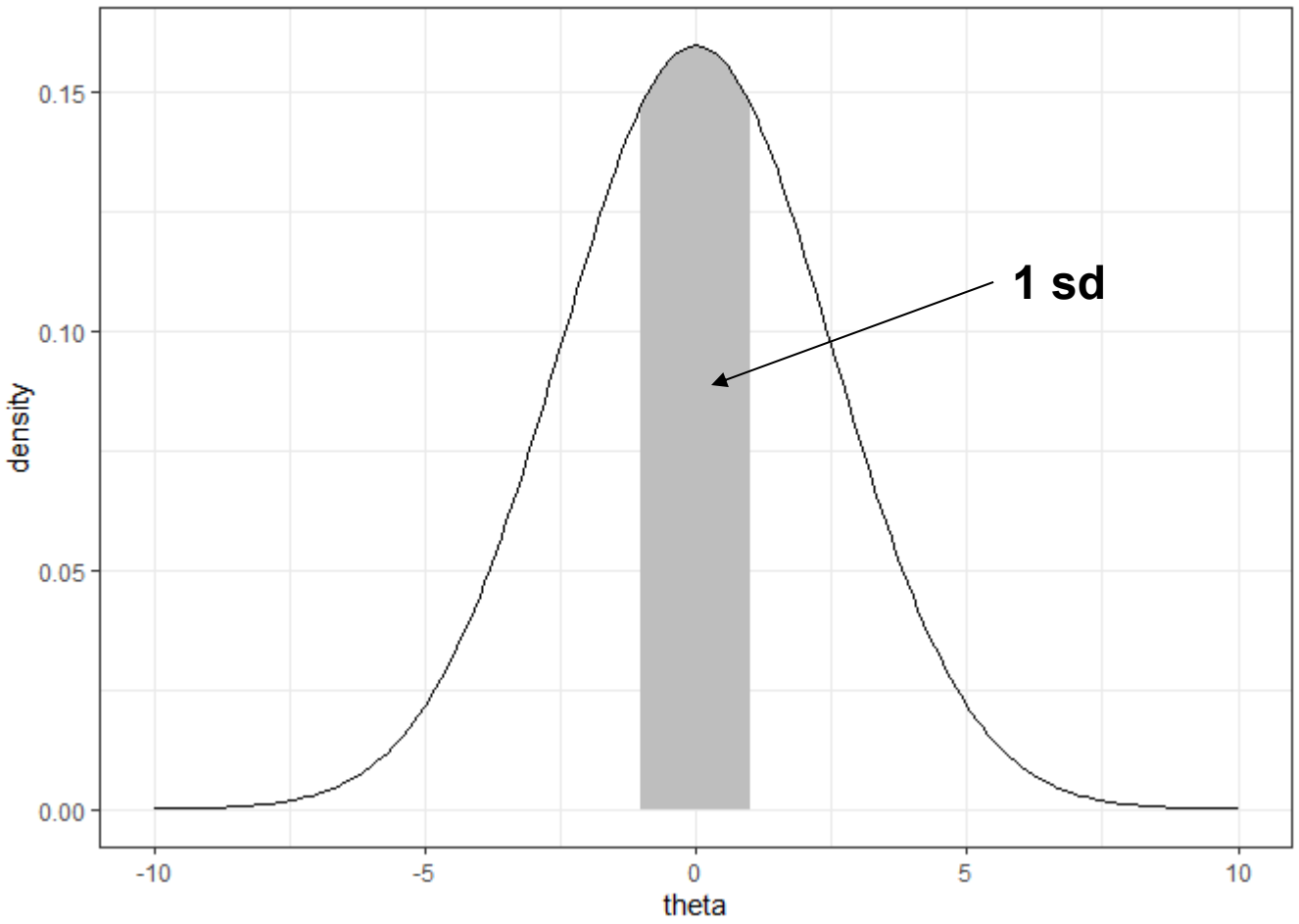
- Individual projects
  - see my initial comment in your repo
  - add more details if necessary (see HW)
  - individual meetings (5 mins)

Today

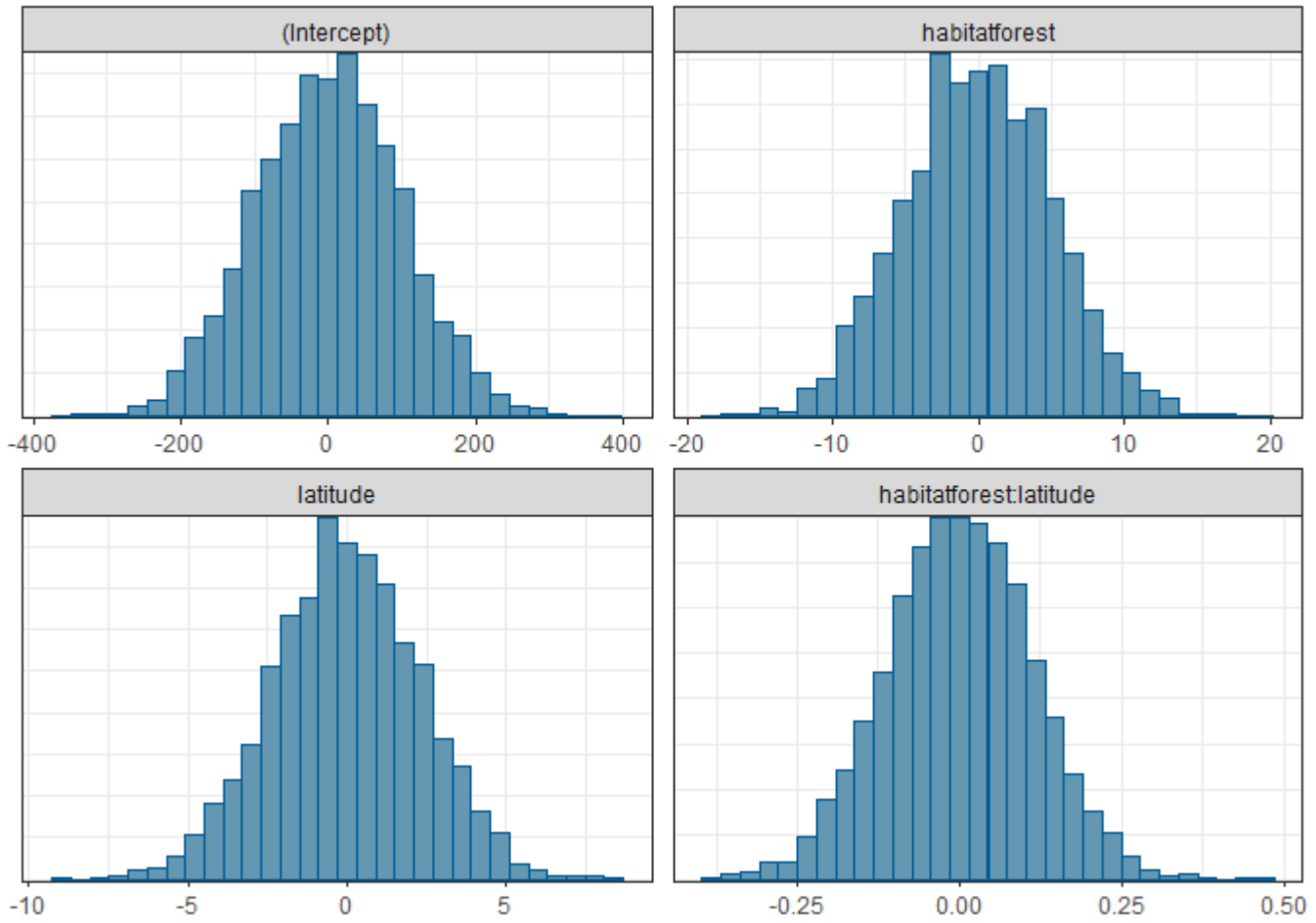
- Priors
- Bayesian model check
- Multilevel models (radon case study)
  - scales and groups
  - study design
  - models and code

Priors

Normal(0, 2.5) weakly informative



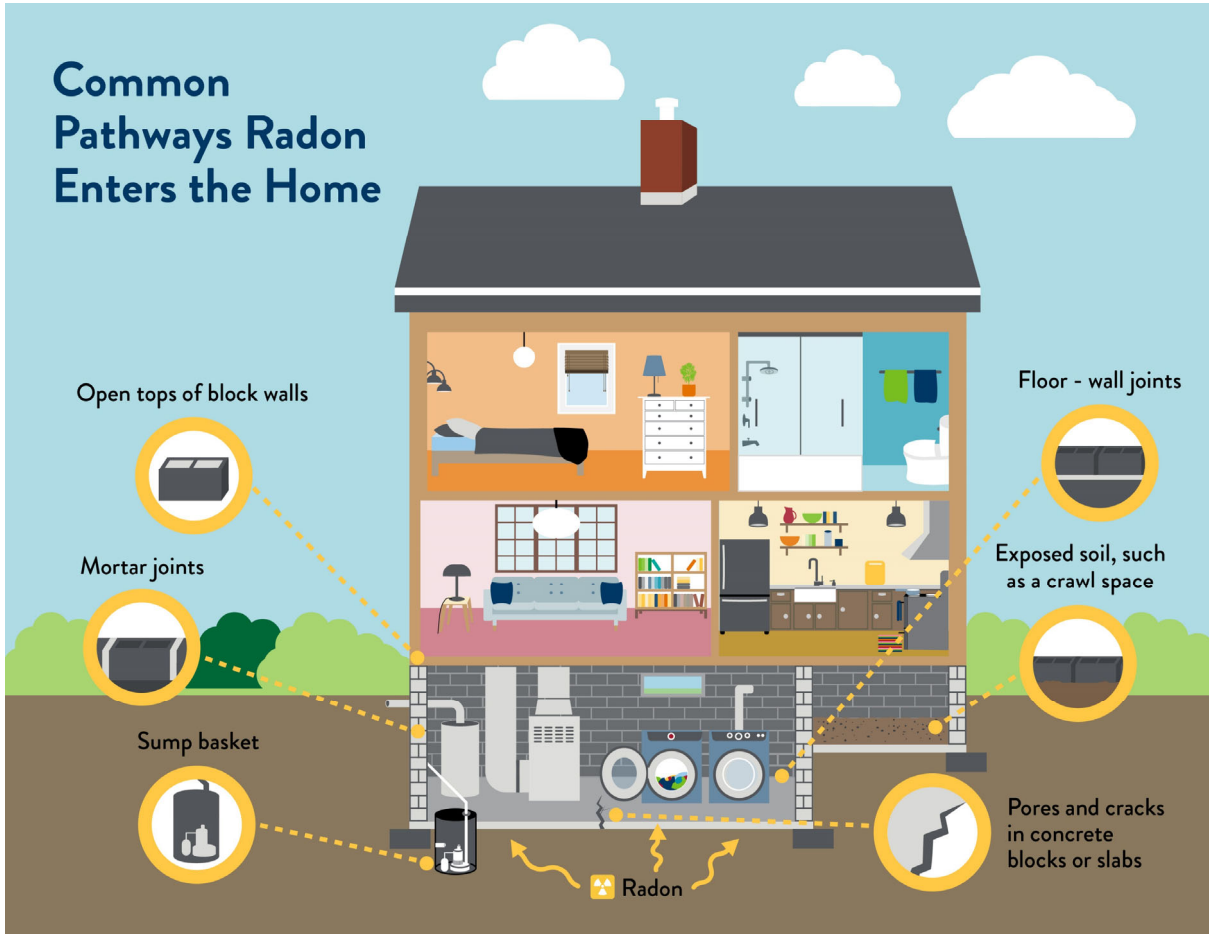
Prior predictive distribution



Bayesian model check

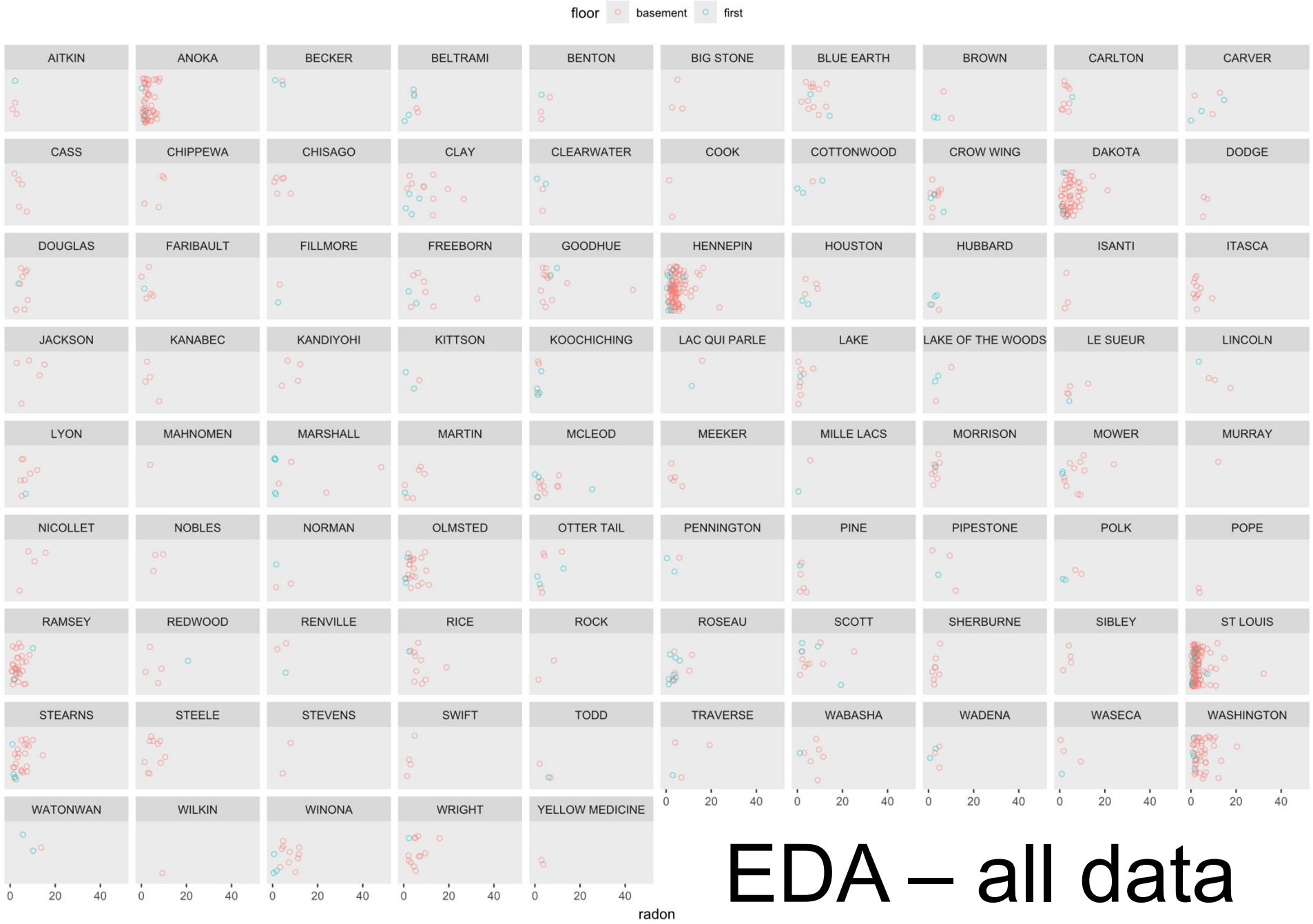
See script

Radon case study



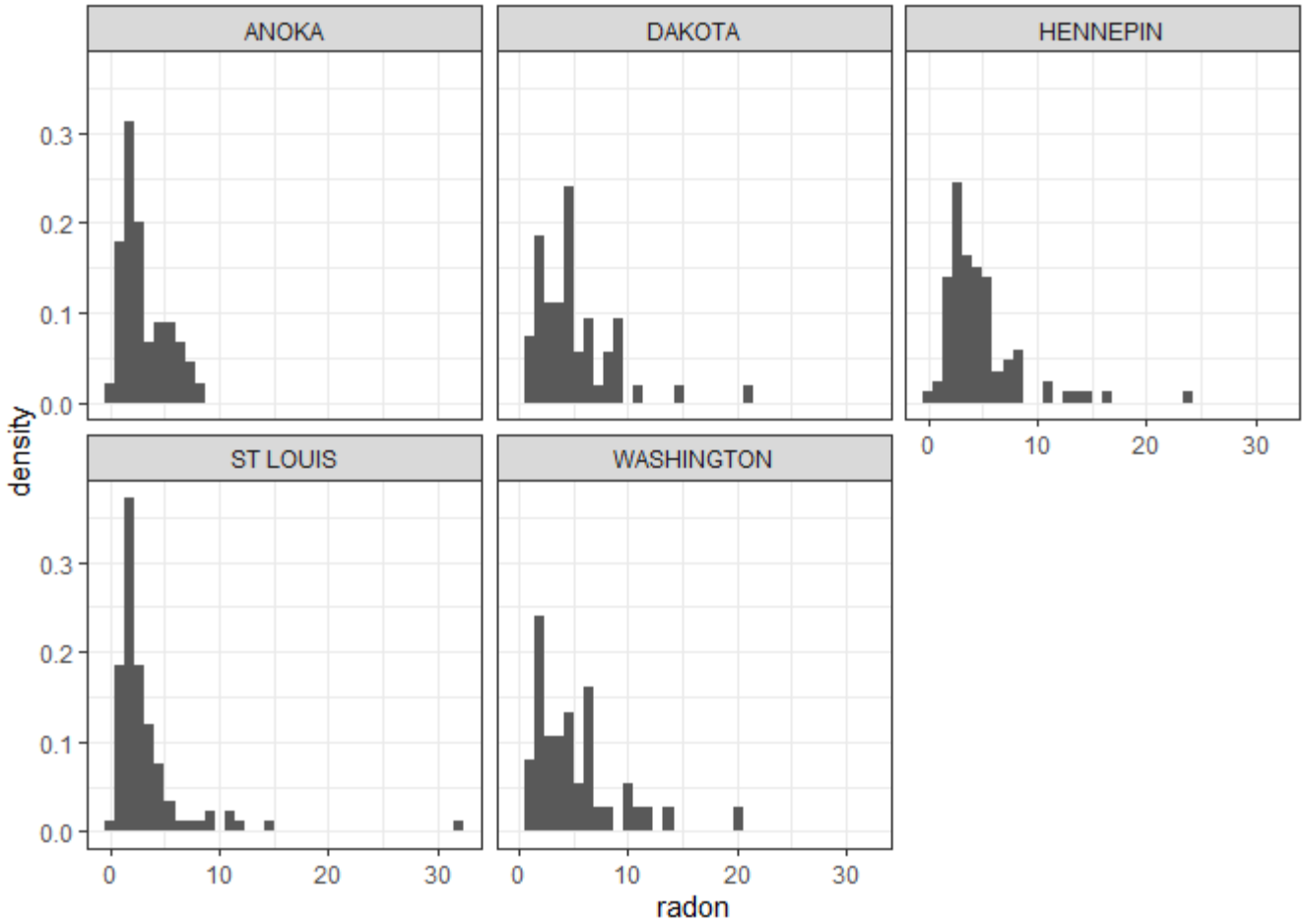
Expect:
basement more susceptible

https://www.health.state.mn.us/communities/environment/air/radon/index.html

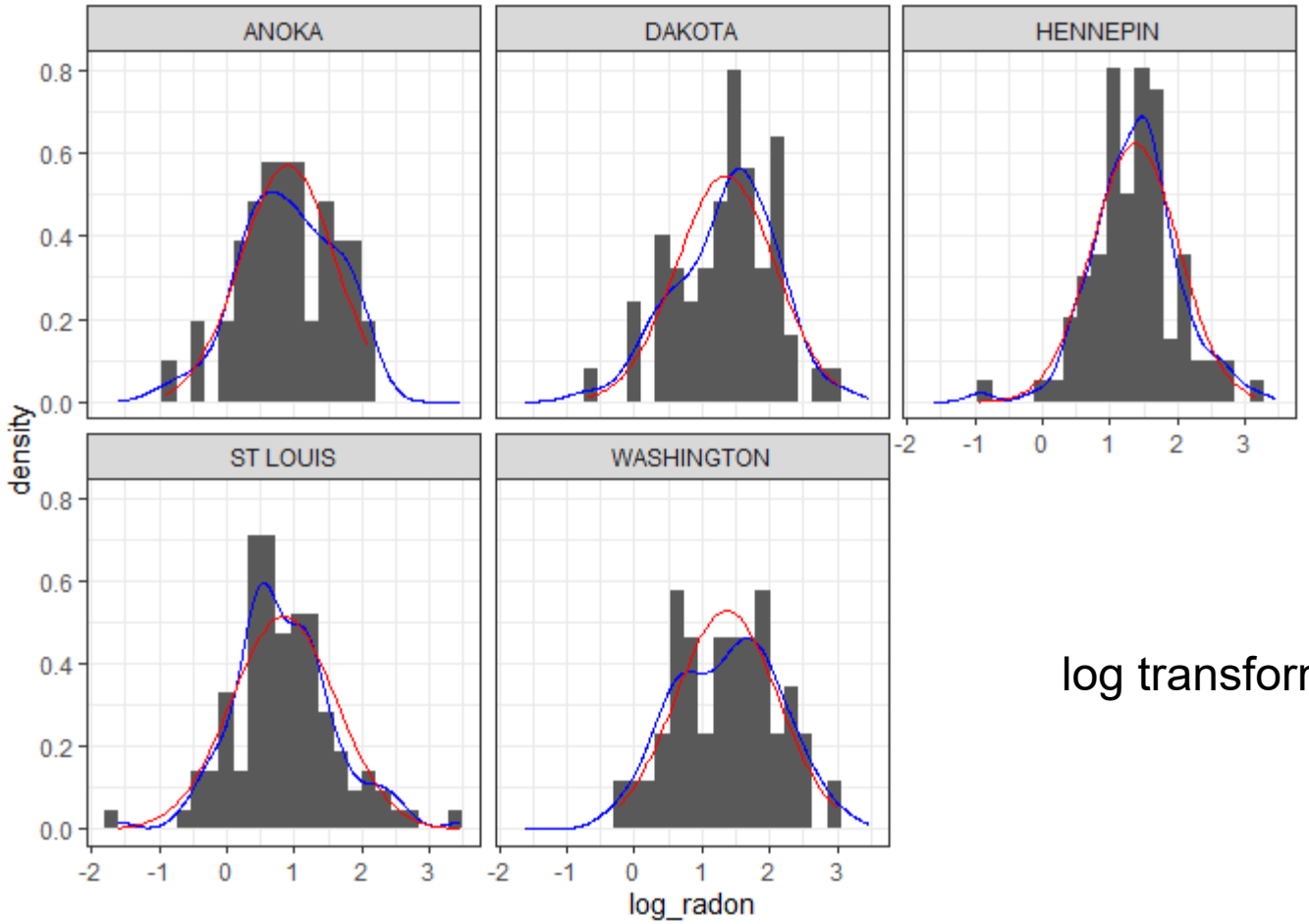

floor
basement
first
AITKIN
ANOKA
BECKER
BELTRAMI
BENTON
BIG STONE
BLUE EARTH
BROWN
CARLTON
CARVER
CASS
CHIPPEWA
CHISAGO
CLAY
CLEARWATER
COOK
COTTONWOOD
CROW WING
DAKOTA
DODGE
DOUGLAS
FARIBAULT
FILLMORE
FREEBORN
GOODHUE
HENNEPIN
HOUSTON
HUBBARD
ISANTI
ITASCA
JACKSON
KANABEC
KANDIYOHI
KITTSON
KOOCHICHING
LAC QUI PARLE
LAKE
LAKE OF THE WOODS
LE SUEUR
LINCOLN
LYON
MAHNOMEN
MARSHALL
MARTIN
MCLEOD
MEEKER
MILLE LACS
MORRISON
MOWER
MURRAY
NICOLLET
NOBLES
NORMAN
OLMSTED
OTTER TAIL
PENNINGTON
PINE
PIPESTONE
POLK
POPE
RAMSEY
REDWOOD
RENVILLE
RICE
ROCK
ROSEAU
SCOTT
SHERBURNE
SIBLEY
ST LOUIS
STEARNS
STEELE
STEVENS
SWIFT
TODD
TRAVERSE
WABASHA
WADENA
WASECA
WASHINGTON
WATONWAN
WILKIN
WINONA
WRIGHT
YELLOW MEDICINE
0
20
40
radon

EDA – all data

EDA – informal modeling

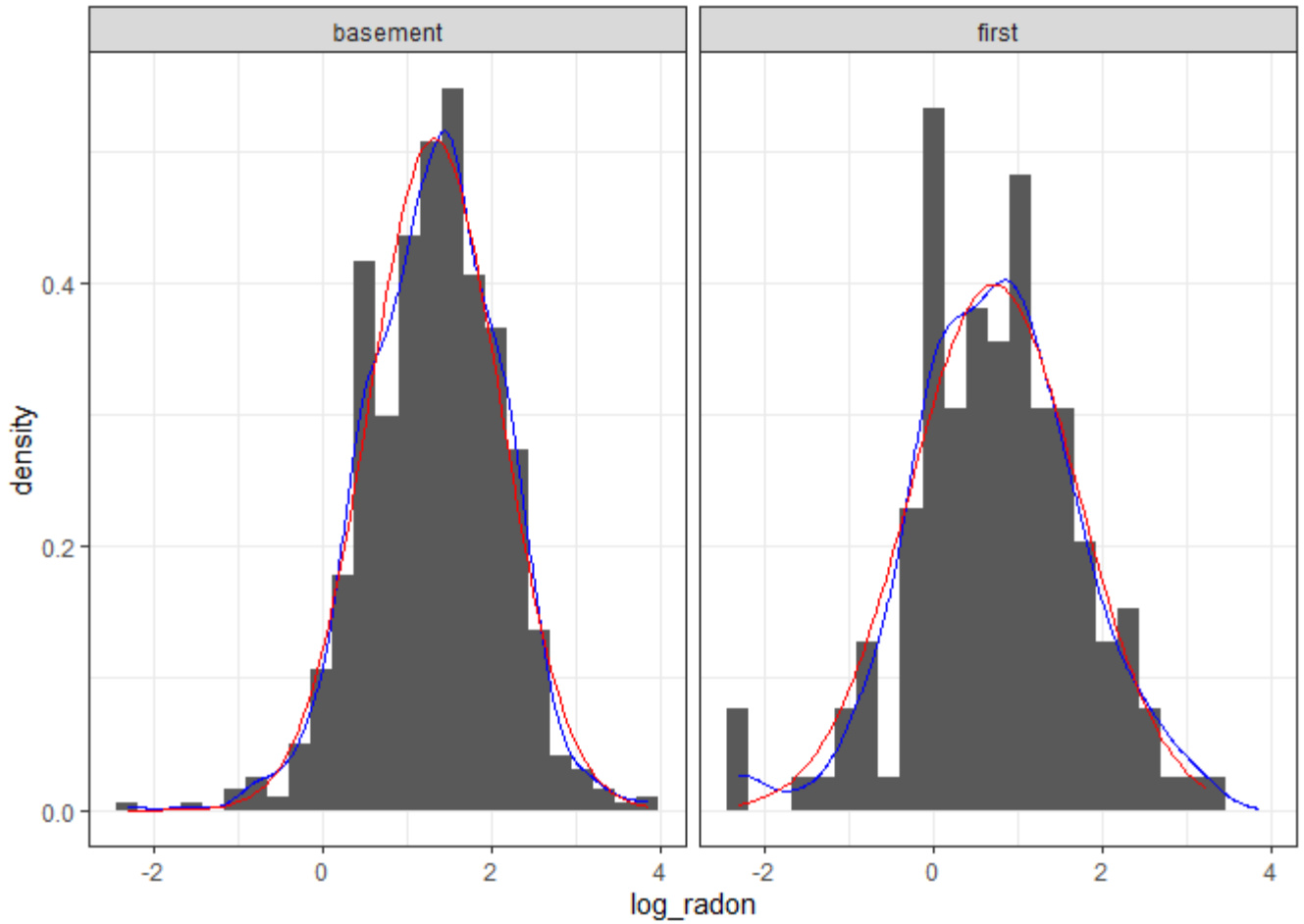


EDA – informal modeling



log transform

EDA – informal modeling



Art of multilevel models

- Sketch data design
- Sketch the data generating process
- Math equations
- Linear model syntax

Scales in radon data

- ?
- also: scope of inference

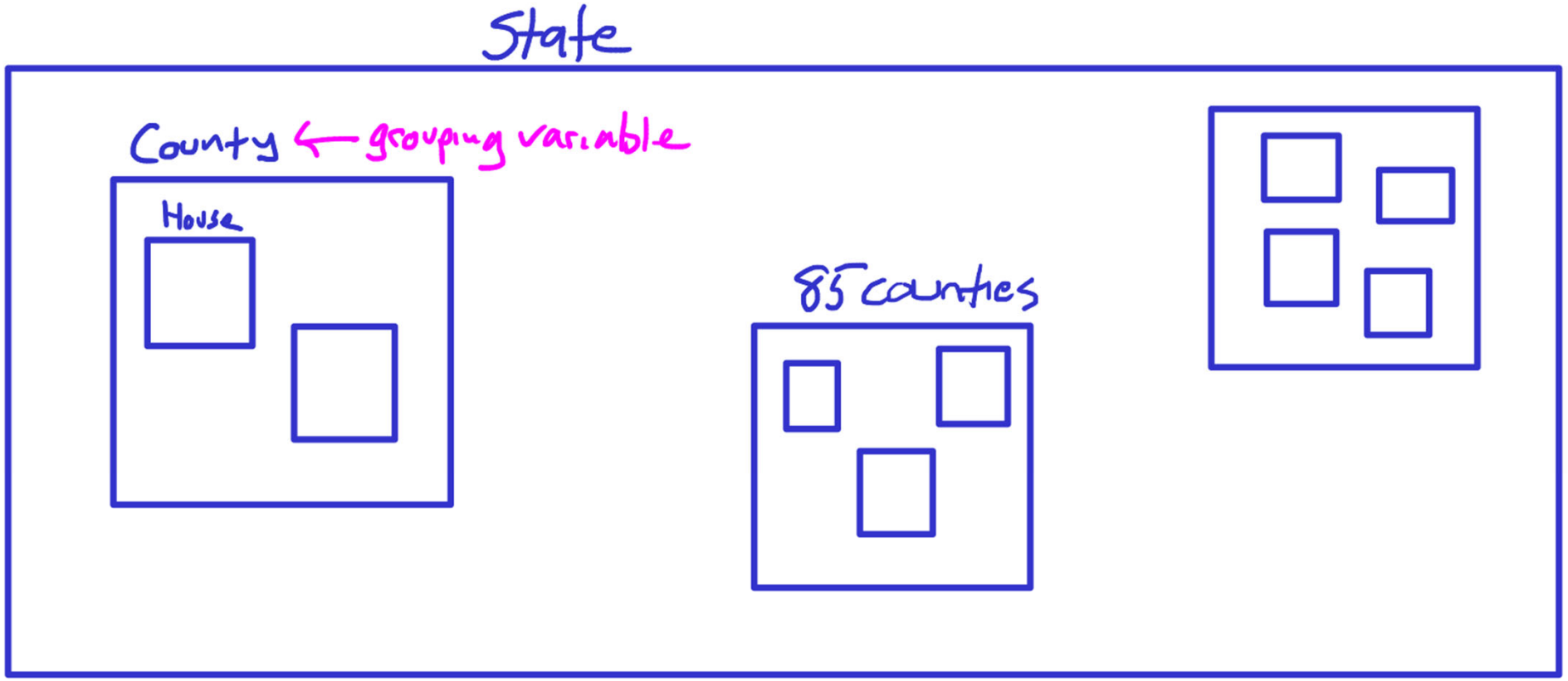
Sketch

Fixed vs random

- Floor: fixed effect
  - deterministic model for parameters
  - shared characteristic
- County: random effect
  - stochastic model for parameters
  - model by group (parameters vary by group)
  - model accounts for correlation among houses within counties

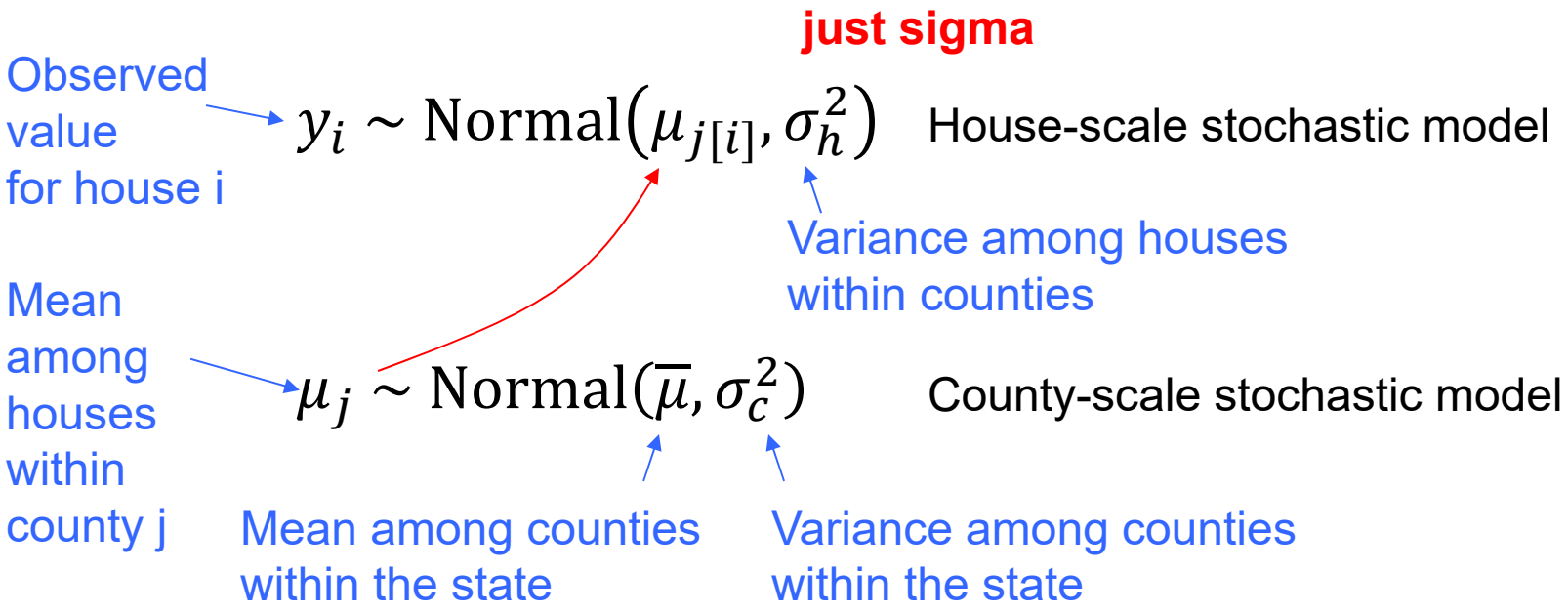
We could alternatively model county as a fixed effect: model parameters would be deterministic

Model 1: random effects



Writing model 1

Basic multilevel model, no predictors



$j[i]$ is the county (j) of house i

Simulate model 1

- algorithm

```
set mu_bar
set sigma_c
set sigma_h
set n_c #number of counties
set n_h #number of houses

for j in 1:n_c
    mu_j = normal(mu_bar, sigma_c)

for i in 1:n_h
        y_i = normal(mu_j[i], sigma_h)
```

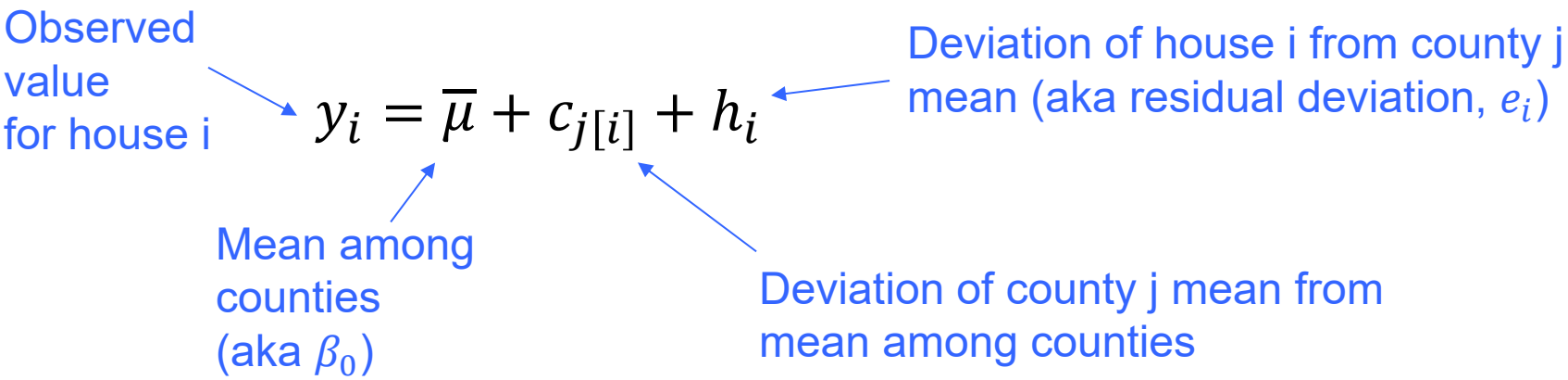
Data story:

First, county determines background radiation level

Then, house determines inside radiation level

Alt parameterization

Additive decomposition



$c_j \sim \text{Normal}(0, \sigma_c^2)$ — Variance among counties within state — County-scale stochastic model

$h_i \sim \text{Normal}(0, \sigma_h^2)$ — Variance among houses within counties — House-scale stochastic model

Sim alt parameterization

- algorithm

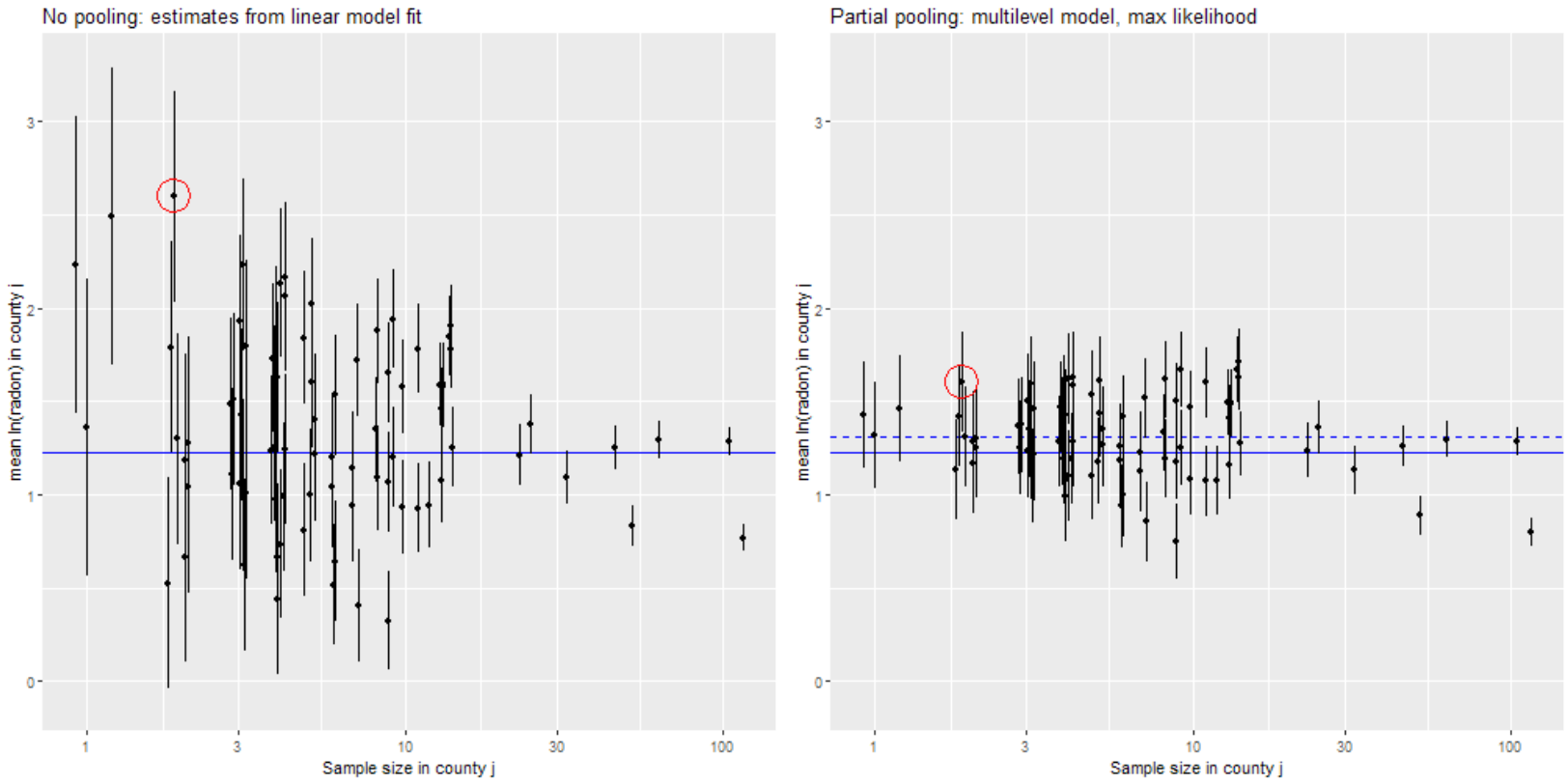
```
set mu_bar
set sigma_c
set sigma_h
set n_c #number of counties
set n_h #number of houses

for j in 1:n_c
    c_j = normal(0, sigma_c)

for i in 1:n_h
    h_i = normal(0, sigma_h)
    y_i = mu_bar + c_j[i] + h_i
```

Linear model syntax

```
log_radon ~ 1 + (1|county)
```

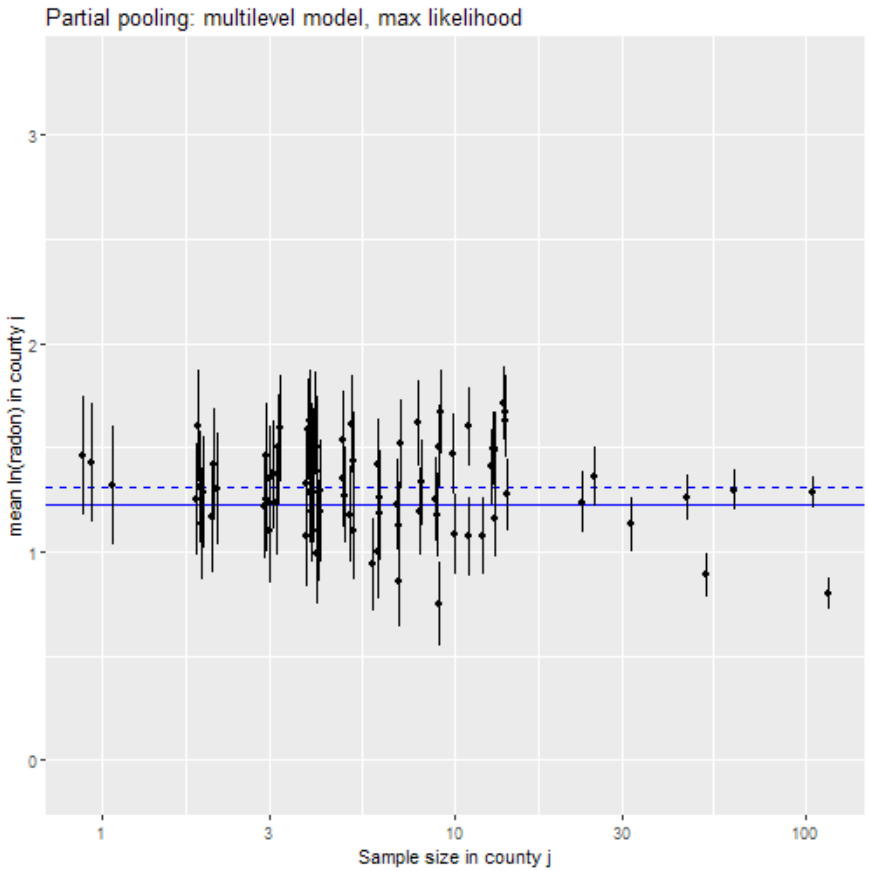
We considered code for the radon example.
See 11_3_radon_multilevel_2.md

Multilevel model - radon

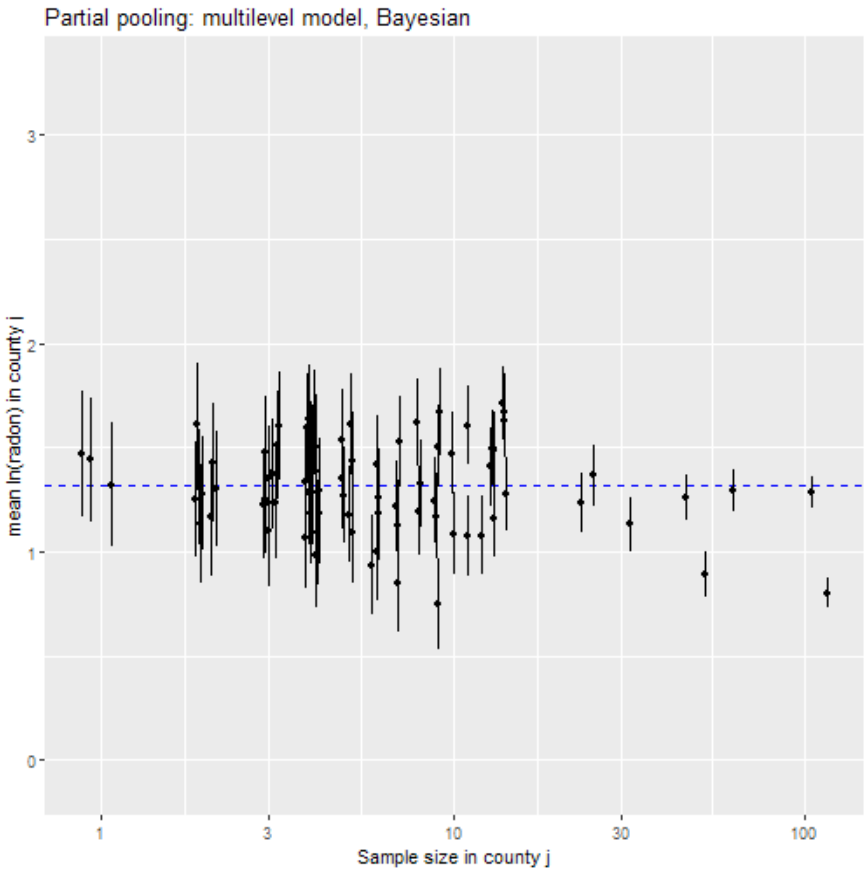


Shrinkage

Multilevel model - radon

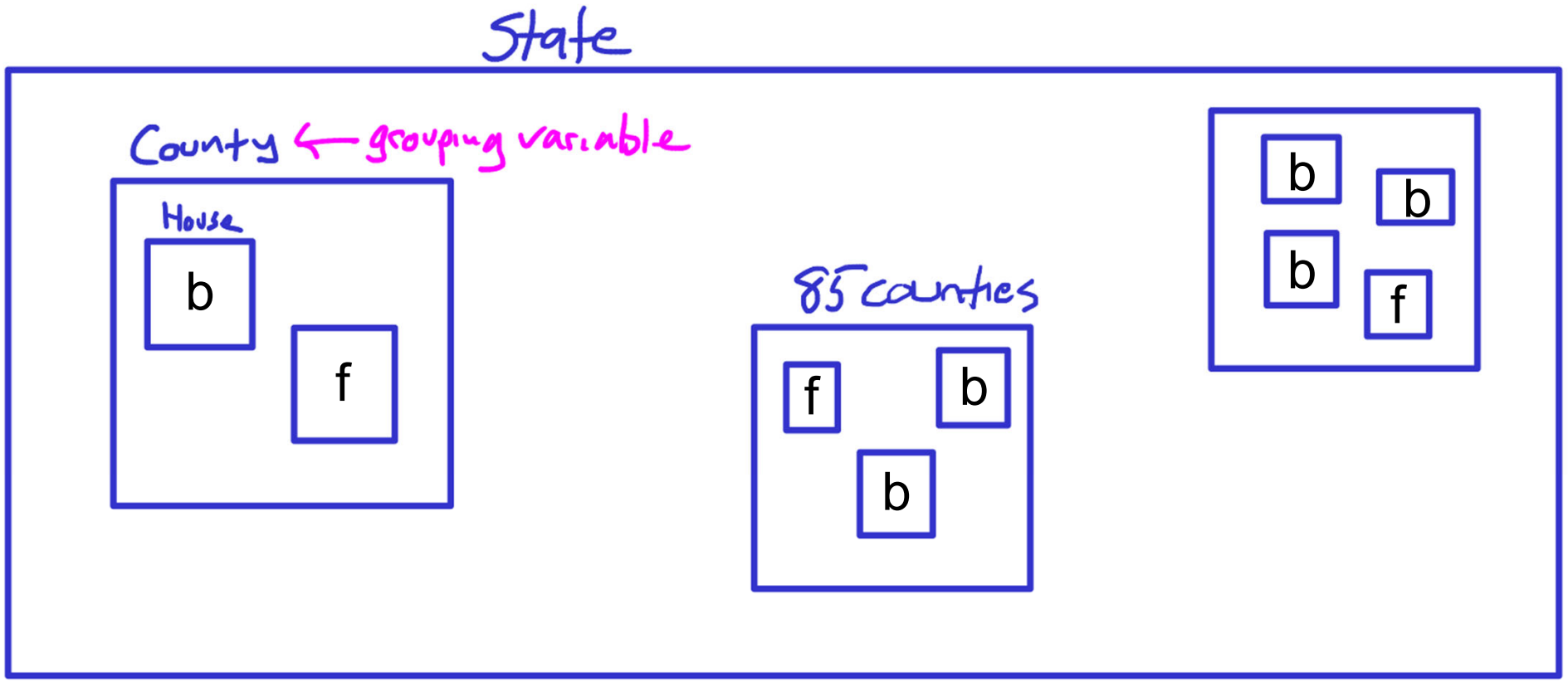


Frequentist



Bayesian

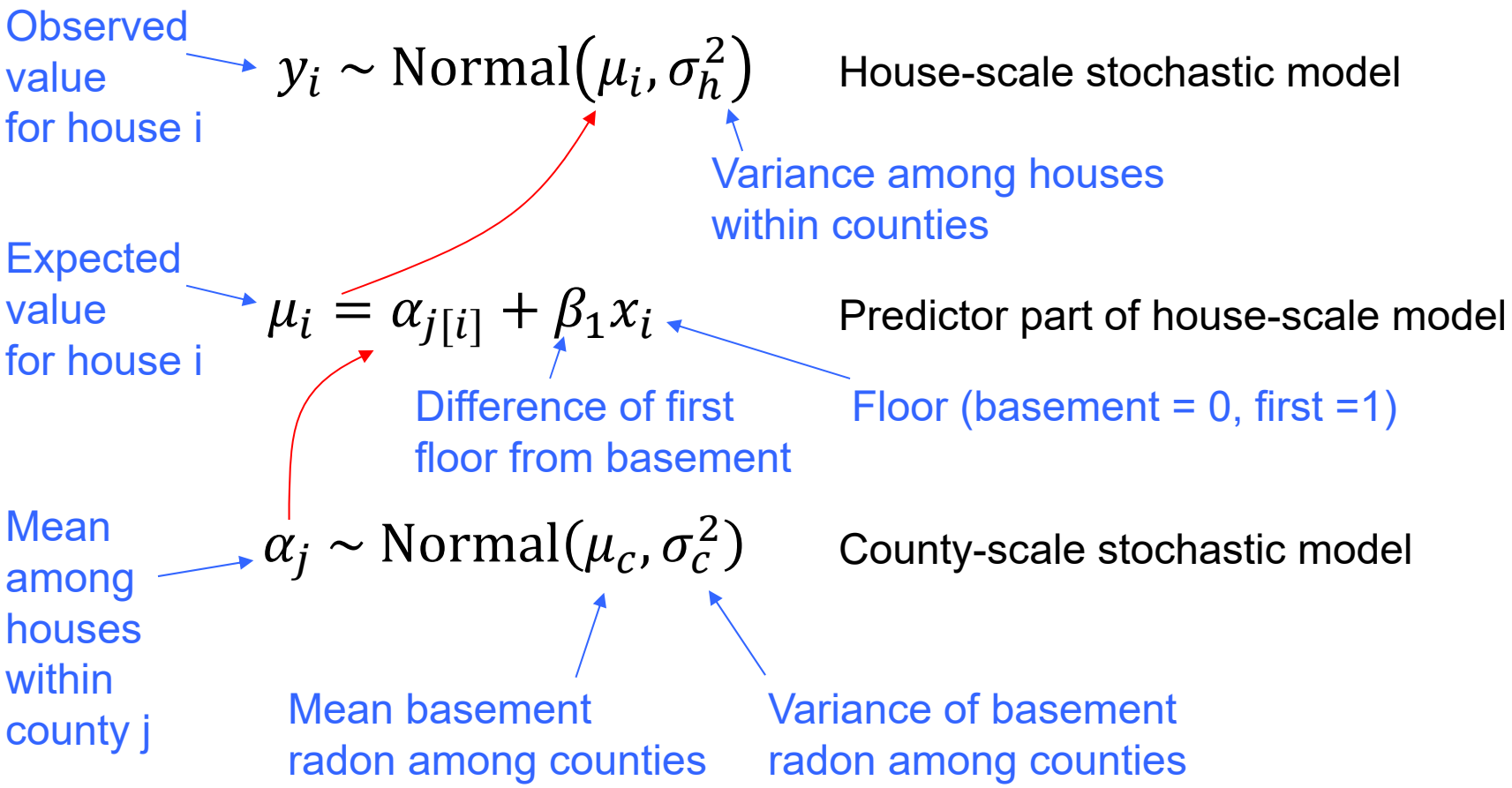
Model 2: 1 predictor



Predictor (fixed effect) is at house scale

Writing model 2

Multilevel model, with 1 predictor at house scale



Model 2: Alt parameterization

Multilevel model, with 1 predictor at house scale

Difference of first floor from basement

Floor (basement = 0, first =1)

$$y_i = \beta_0 + \beta_1 x_1 + c_{j[i]} + e_i$$

Deviation of house i from county j mean

Mean basement radon among counties

Deviation of county j mean from mean among counties

$$c_j \sim \text{Normal}(0, \sigma_c^2)$$

Variance among counties within state

County-scale stochastic model

$$e_i \sim \text{Normal}(0, \sigma_e^2)$$

Variance among houses within counties

House-scale stochastic model

Linear model syntax

```
log_radon ~ floor + (1|county)
```

Equivalent:

```
log_radon ~ 1 + floor + (1|county)
```

Model with one house-scale predictor

See script