# Today

- Inference algorithms intro
- Frequentist inference algorithms
  - sampling distribution algorithm
  - coverage algorithm (confidence intervals)

# Inference algorithms

★ ★ ★ ★ Crucial (consider later) ★ ★ ★ ★

Scope and veracity of inference depends on study design

# Statistical inference

- Judge the accuracy of an estimation or prediction algorithm
  - Efron & Hastie 2016
- Reliability
- Uncertainty

ISO definition of accuracy: the closeness of a measurement to the true value
Two components: bias, variance

# Different inference problems

**Estimation**
Infer a property of a population (e.g. mean) from a sample

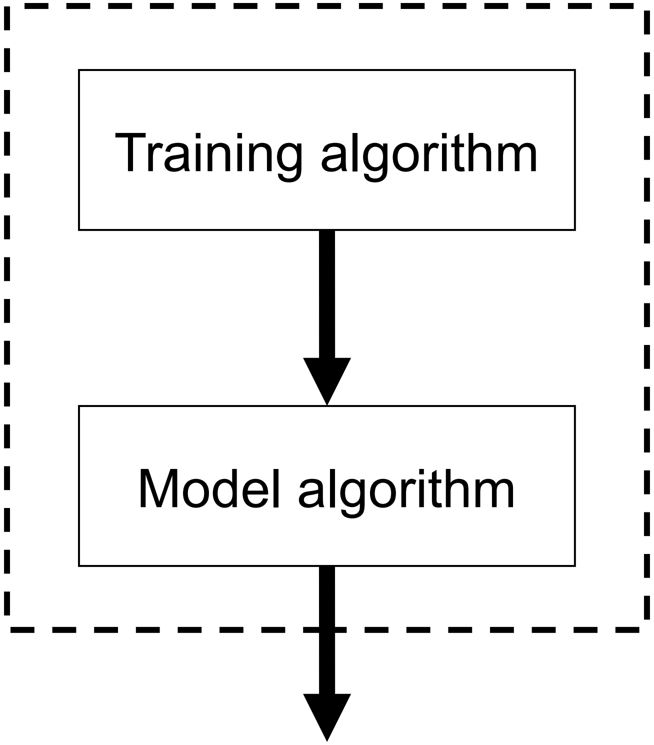
**Model comparison (weigh evidence)**
Infer the data generating process from among a set of candidate data-generating processes

**Hypothesis test (association)**
Infer that y is associated with x

**Causation (by experiment or observation)**
Infer that x causes y
Infer the size of an effect due to an intervention (estimation)
Infer that an intervention had an effect (H-test)

**Prediction**
Predict the value of a new observation or population state (extrapolation or interpolation)
Predict the population state in the future (forecast/extrapolation)

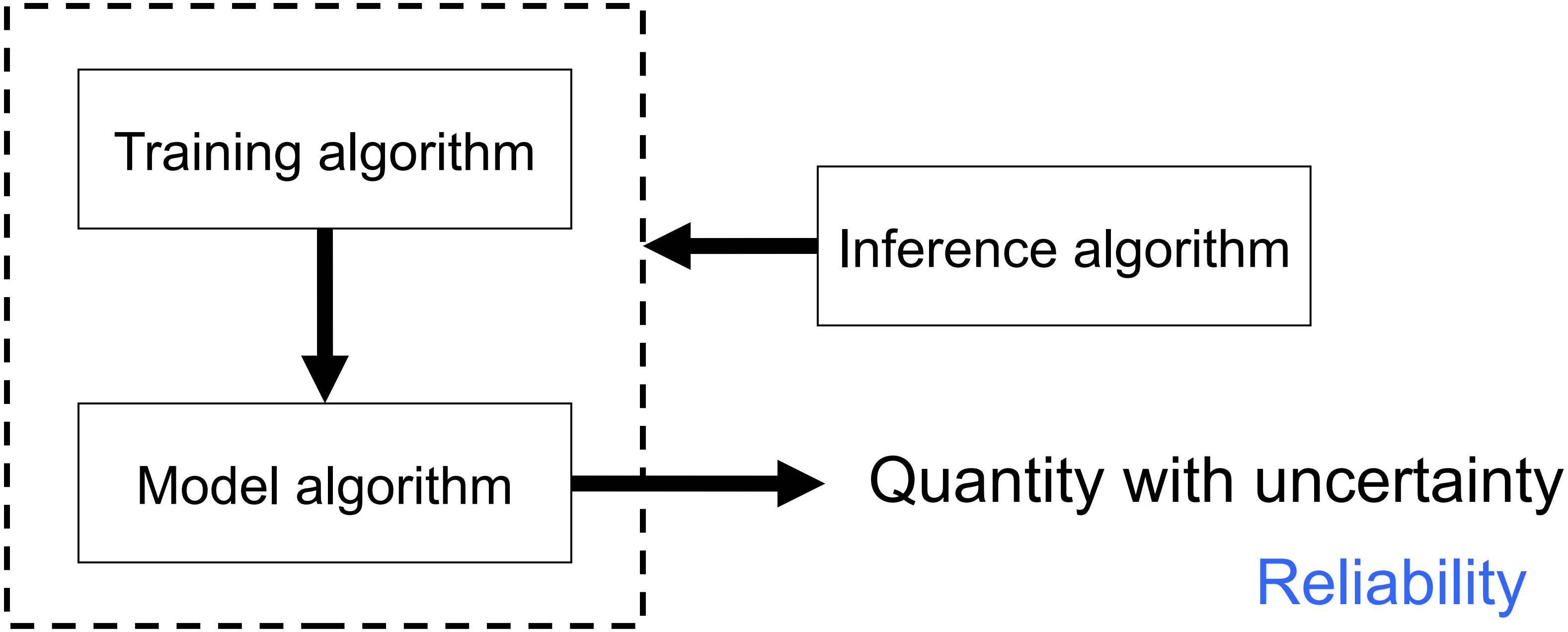
# Algorithms in data science



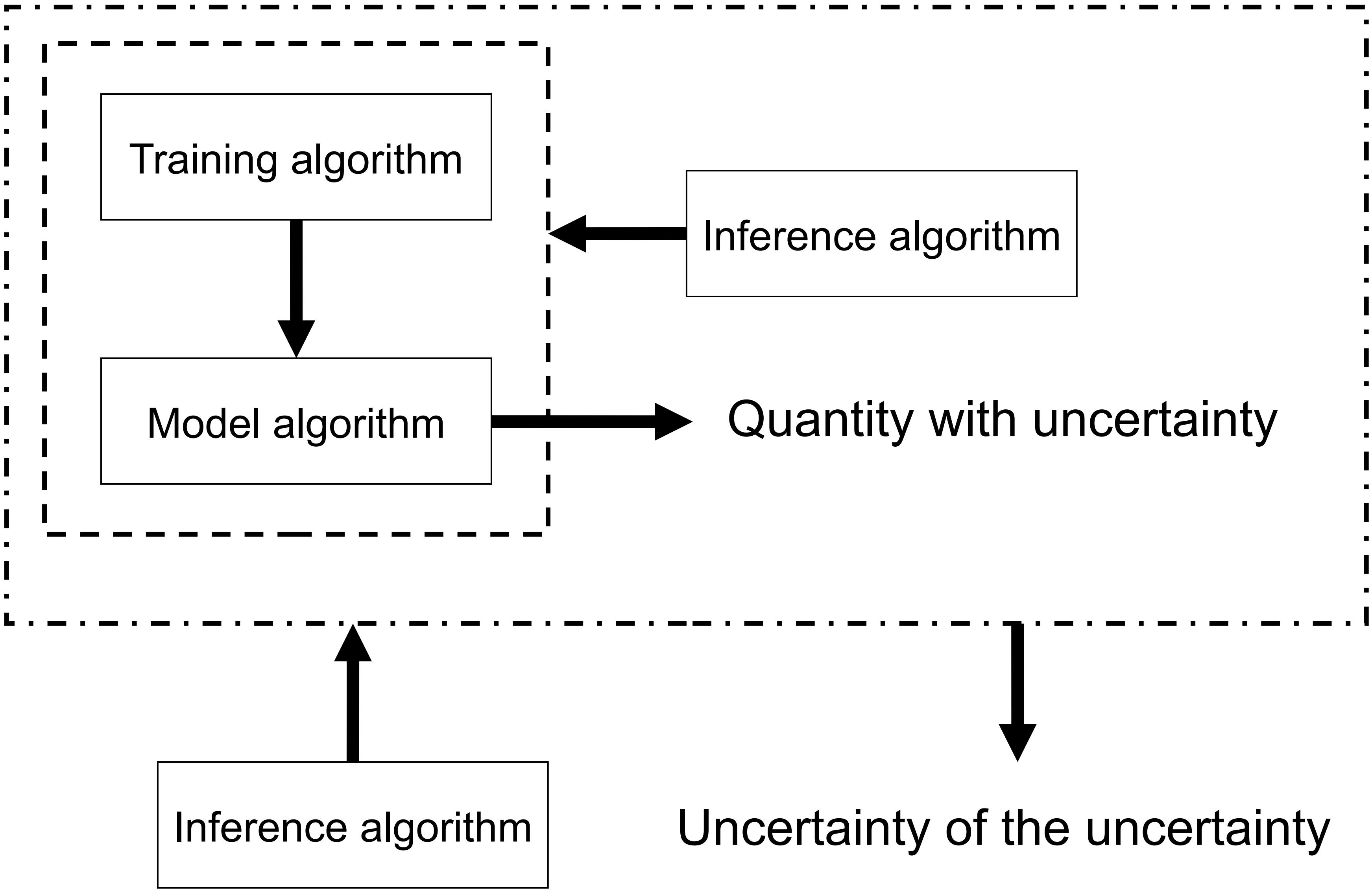

Quantity

"Dumb" - doesn't say about reliability

# Algorithms in data science



# Algorithms in data science



# Algorithms in data science

- Inference algorithm
  - looking back: considering all the ways data could have happened
  - looking forward: predicting new data and testing against them

These are two big ideas in data science

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- Inference algorithm
  - looking back: considering all the ways data could have happened
    - frequentist (sampling distribution)
    - likelihood (probability accounting)
    - Bayesian (likelihood + belief updating)
  - looking forward: predicting new data and testing against them
    -

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# Algorithms in data science

- Inference algorithm
  - looking back: considering all the ways data could have happened
    - frequentist (sampling distribution)
    - likelihood (probability accounting)
    - Bayesian (likelihood + belief updating)
  - looking forward: predicting new data and testing against them
    - cross validation, AIC, machine learning

These are two big ideas in data science

# Frequentist inference

- Frequentist probability = long-run frequency
  - e.g. tossing a coin

$$P\left(heads\right)=\lim_{n\to\infty}\frac{\sum heads}{n}$$

# Sample vs population statistic

- Population statistic
  - e.g. mean weight
  - there is a true value
  - “fixed” not random
- Sample statistic
  - e.g. mean of sample
  - random variable

# Sampling distribution

- Frequentist notion of looking back: considering all the ways data could have happened
- Imaginary repeated sampling from the data generating process

# Sampling distribution algorithm

- Data generating process repeated many times
- Each time calc sample statistic

```
repeat very many times
     sample n units from the population
     calculate the sample statistic
plot sampling distribution (histogram) of the sample statistic
```

# Make the algorithm

What is the mean weight of an individual of this species?

repeat very many times
    sample n units from the population
    calculate the sample statistic
plot sampling distribution (histogram) of the sample statistic

Important: population statistic, sample statistic

# Sampling distribution

132 orange-spotted warblers. 1 indicates infected

```
pathogen <-
    c(1,1,1,1,1,0,0,0,0,0,0,0,1,1,1,0,1,1,0,1,1,0,0,0,1,1,0,0,1,1,0,1,0,0,0,0,
      1,1,1,0,1,1,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,1,0,0,1,0,0,0,1,0,0,1,1,
      0,1,0,0,0,1,0,0,0,1,0,0,1,0,0,1,1,0,1,1,0,1,1,0,0,0,0,0,0,0,1,0,1,1,1,0,
      1,0,1,0,0,0,0,0,0,1,1,0,0,0,1,1,1,1,0,0,1)
```

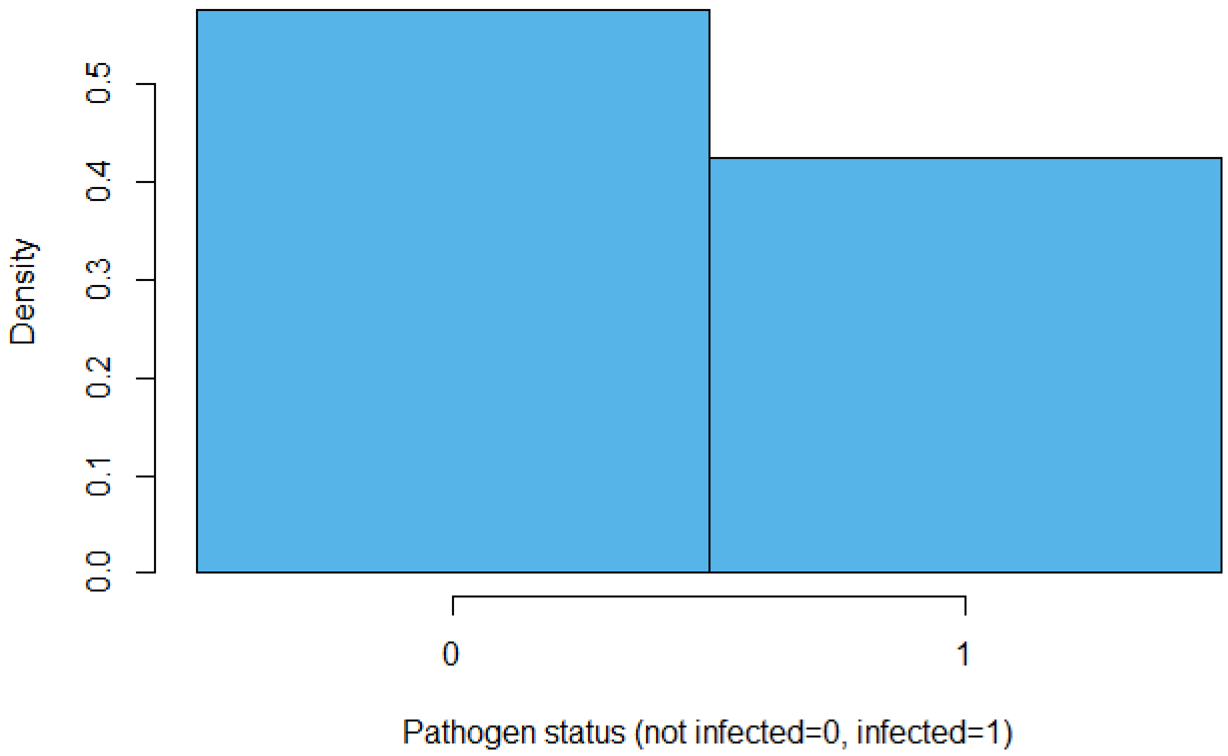
Take a sample:

```
sample(pathogen,10)
```

0 1 0 0 0 0 0 1 0 0

pathogen prevalence = 0.2

Our scientific observation




True prevalence is 0.424

# Sampling distribution algorithm

repeat very many times law of large numbers

    sample n units from the population

    calculate the sample statistic

plot sampling distribution (histogram) of the sample statistic

## for pathogen prevalence

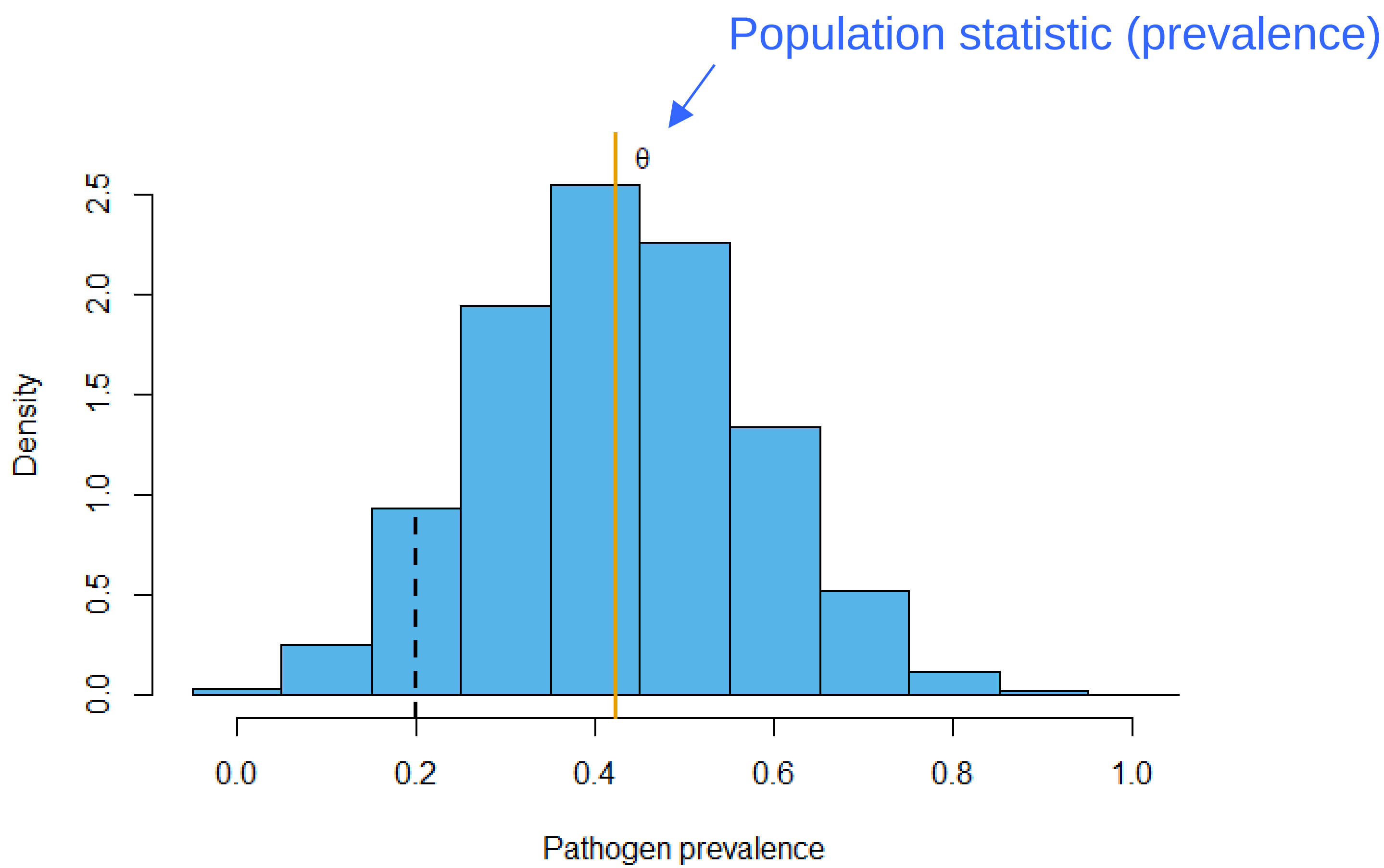
for a large number of repeated samples

    randomly sample 10 birds from the population

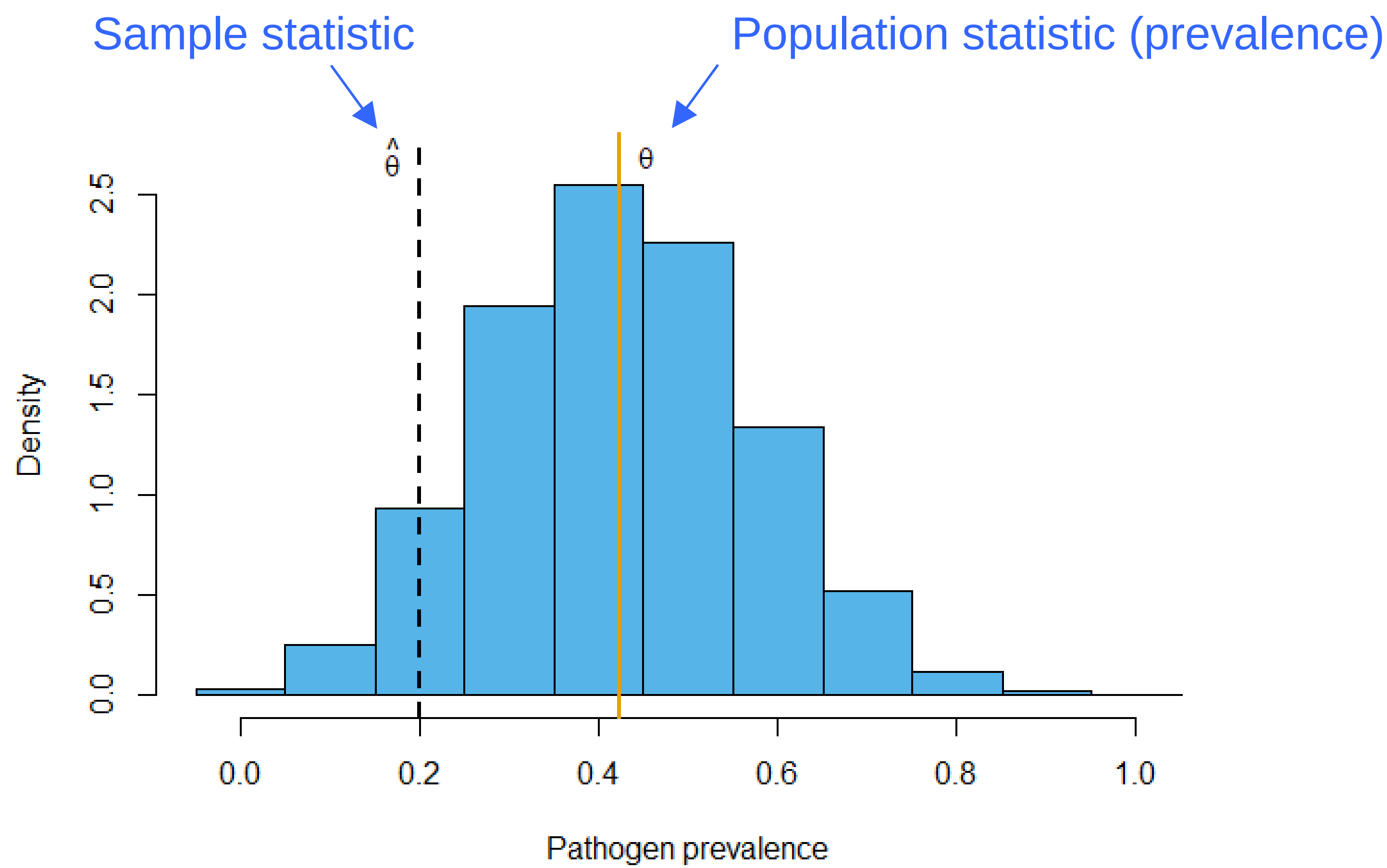
    calculate the prevalence in the sample

plot sampling distribution (histogram) of prevalence

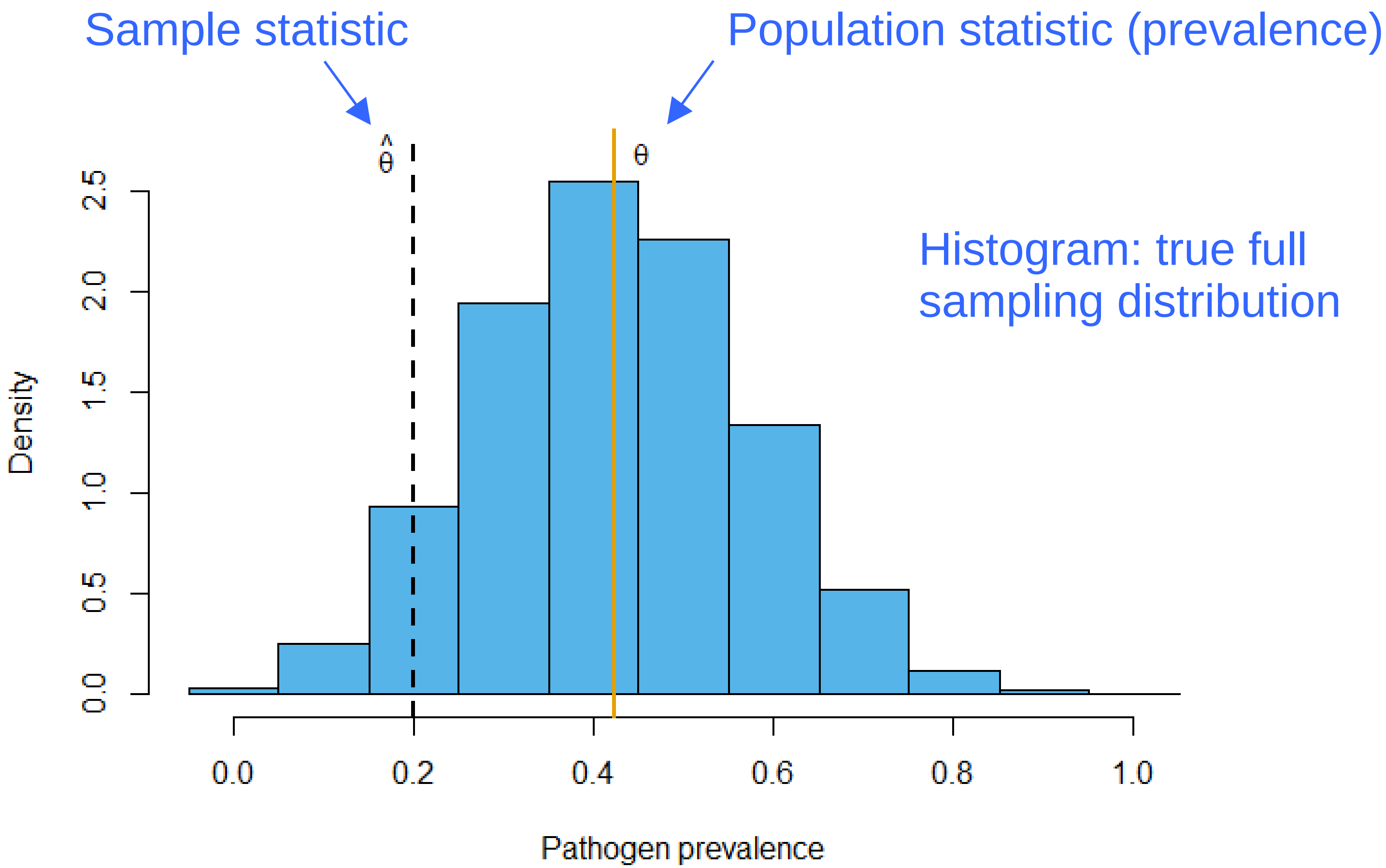
# The sampling distribution for prevalence



# The sampling distribution for prevalence



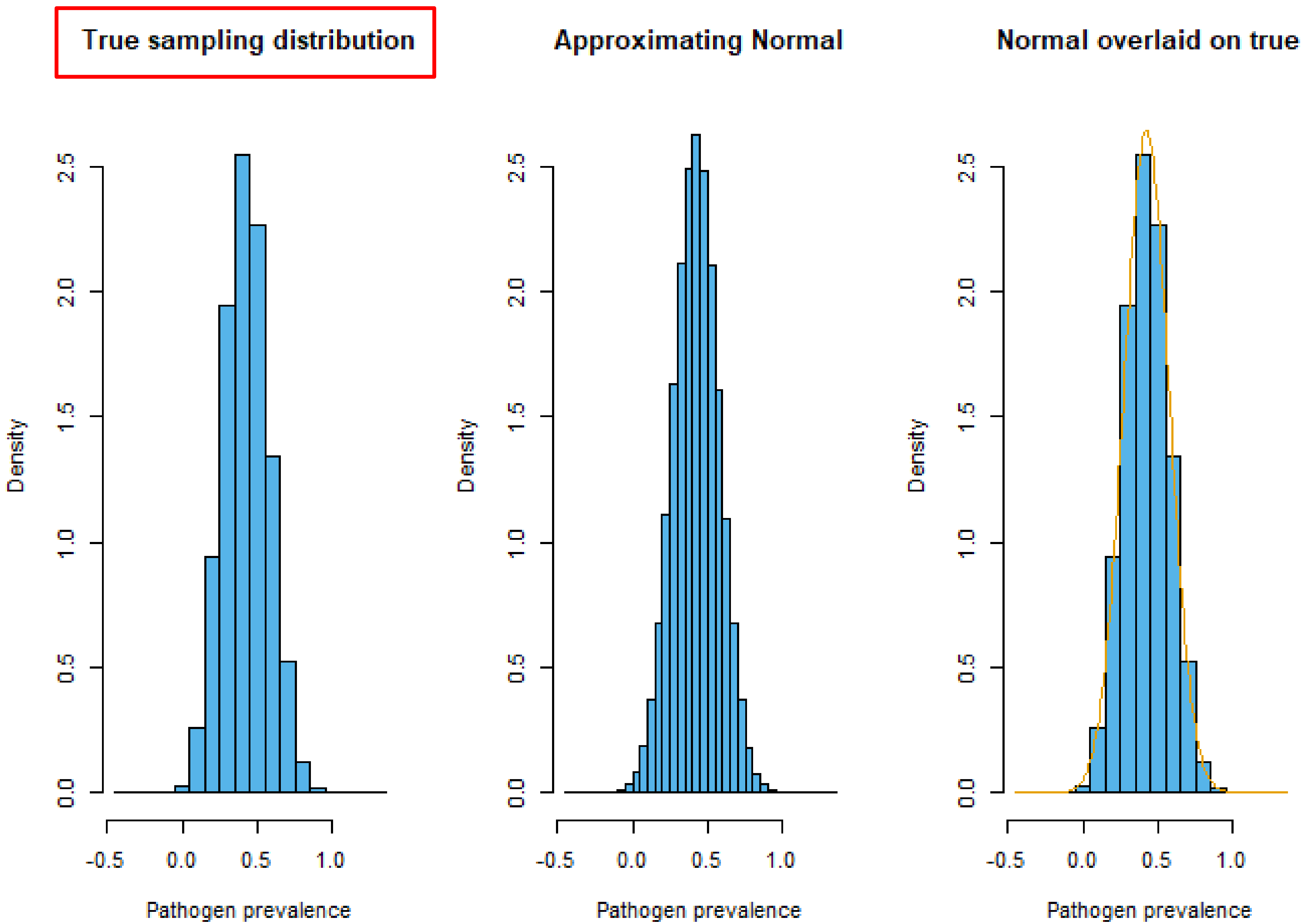
# The sampling distribution for prevalence



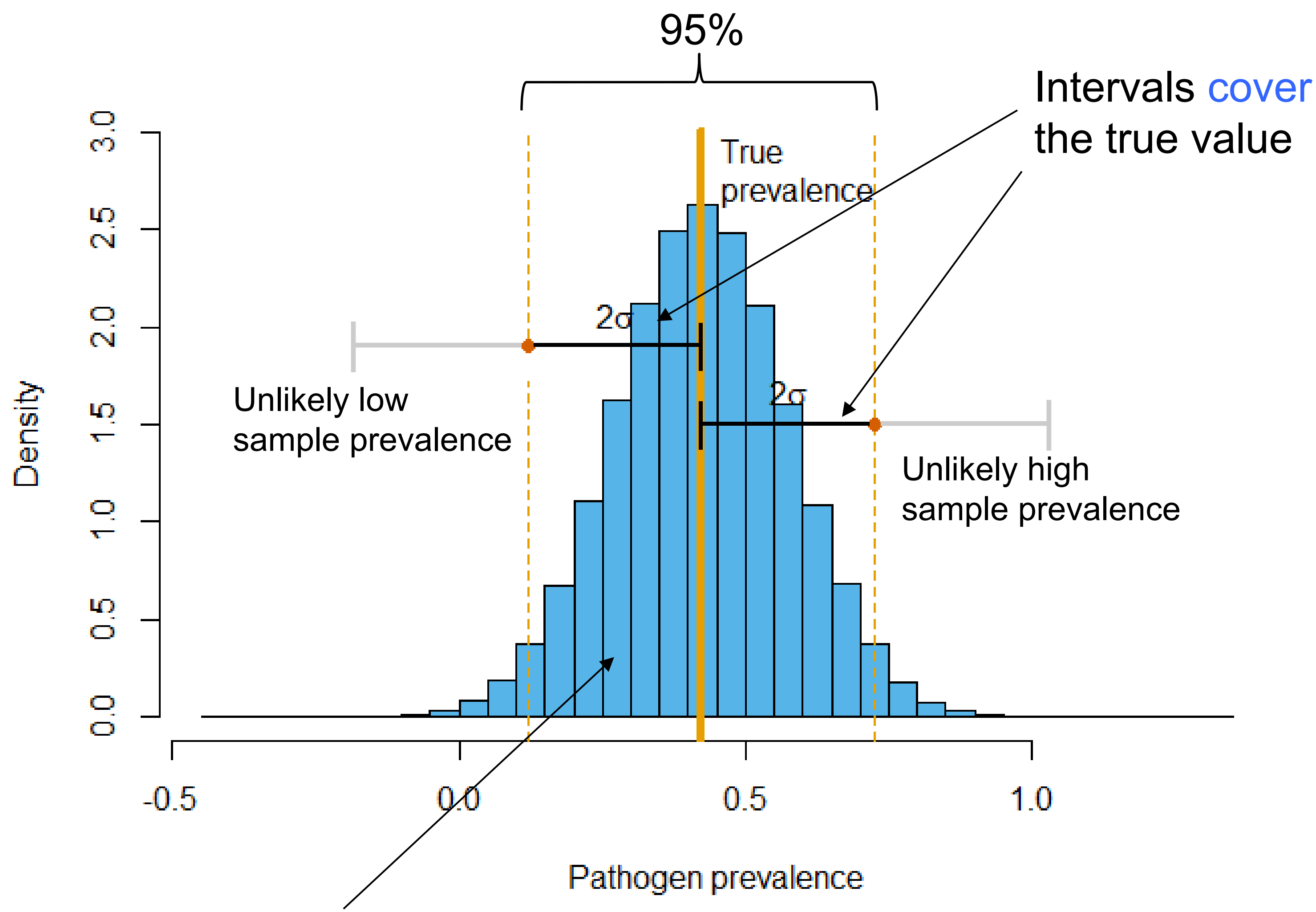
# Confidence interval

- An interval calculated by some procedure that would **contain** (or **cover**) the true population value 95% of the time, **if sampling and calculating an interval were repeated a very large number of times**

Confidence = reliability of the procedure


True sampling distribution
Approximating Normal
Normal overlaid on true
Density
0.0
0.5
1.0
1.5
2.0
2.5
-0.5
0.0
0.5
1.0
Pathogen prevalence

# Construct an interval to cover true value




Normal distribution approximating the true sampling distribution

# Plug in principle

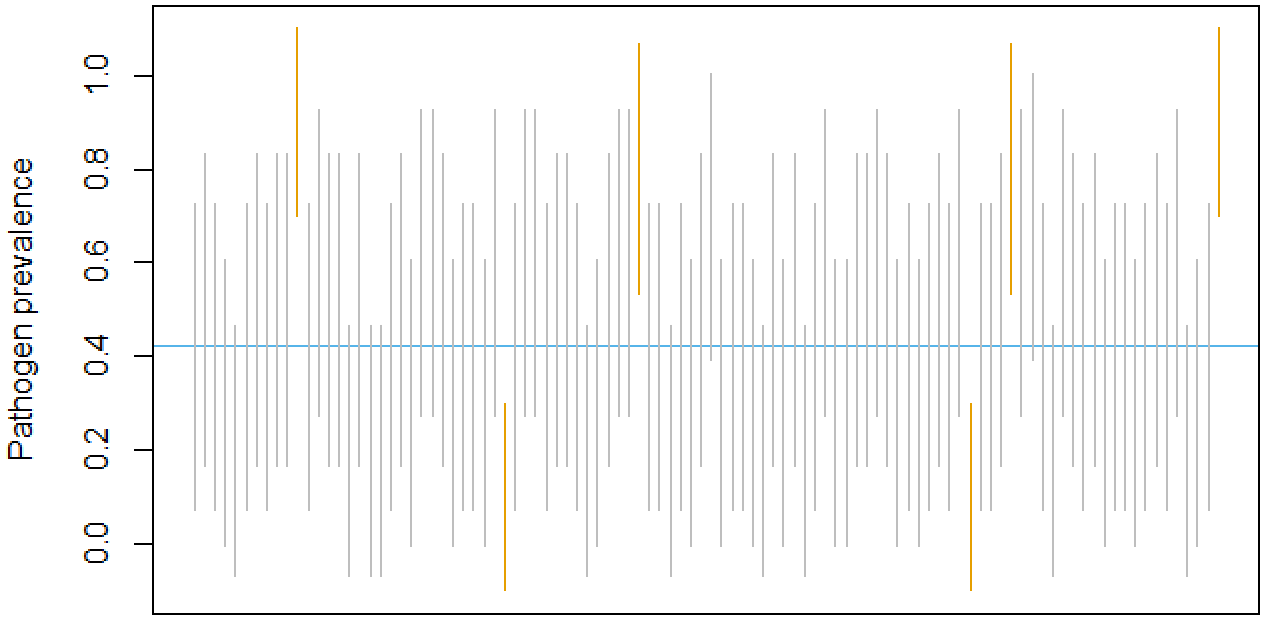
- We don't know the true sampling distribution or its parameters
- Plug in the sample instead as an estimate
  - in this example we can use the standard error of the sample as an estimate of the standard deviation of the sampling distribution

# Coverage

repeat very many times
    sample n units from the population
    calculate the sample statistic
    calculate the interval for the sample statistic
calculate frequency true value is in the interval

Calibrates the degree of confidence in the procedure

# First 100 95% confidence intervals




95.6% of the intervals cover the true value
In first 100, 6 do not cover the true value
(we expect about 5/100)