R vs r in ecology

DISCRETE TIME

$$N_{t+1} = RN_t \quad (1)$$

$$\begin{aligned} N_{t+1} &= N_t + bN_t - dN_t \\ &= N_t + (b-d)N_t \\ &= N_t + rN_t \\ &= N_t(1+r) \\ &= RN_t, \text{ where } R = 1+r \end{aligned}$$

$$N_{t+1} = N_t + rN_t$$

$$\frac{N_{t+1} - N_t}{\Delta t} = rN_t$$

rate of change $\frac{\Delta N}{\Delta t} = rN_t$

$$\underbrace{\frac{\Delta N}{\Delta t}\frac{1}{N}}_{\text{per capita rate of change}} = r$$

CONTINUOUS TIME

$$\frac{dN}{dt} = rN \qquad \underbrace{\frac{dN}{dt}\frac{1}{N}}_{\text{per capita rate of change.}} = r$$

$$N(t) = e^{rt}N(0)$$

$t = 1$:

$$N(1) = e^{r}N(0), \text{ i.e. } N_{t+1} = e^{r}N_t \quad (2)$$

We see that eq (2) is the same as eq. (1) and so

$$R = e^r \quad \text{or} \quad r = \ln(R)$$

So, how can $R = e^r$ and $R = 1+r$ both be true?

$$e^r = \boxed{1 + \frac{r}{1!}} + \frac{r^2}{2!} + \frac{r^3}{3!} \dots$$ Maclaurin series (Taylor series centered at $r=0$)

So $1+r$ is a first order approximation to e^r.

e.g. $R = 1.1$ $\approx 1+r$ $\Rightarrow r \approx 0.1$

$$r = \ln(R) = \ln(1.1) = 0.0953 \approx 0.1$$