Formulas for Effect Estimates

	Outcome	No Outcome	Total
(+)	a	b	$a+b$
(−)	c	d	$c+d$
Total	$a+c$	$b+d$	$a+b+c+d$

z = 1.959964 for 95% CI

Experimental Event Rate

$$\text{EER} = \frac{a}{a+b}$$

Control Event Rate

$$\text{CER} = \frac{c}{c+d}$$

Absolute Risk Reduction

$$\text{ARR} = \text{CER} - \text{EER}$$

Confidence Interval (Wilson score bounds with error propagation)

$$\text{Lower Bound: ARR} - z \cdot \sqrt{\frac{u_2(1-u_2)}{r_1} + \frac{w_1(1-w_1)}{r_2}}$$

$$\text{Upper Bound: ARR} + z \cdot \sqrt{\frac{u_1(1-u_1)}{r_2} + \frac{w_2(1-w_2)}{r_1}}$$

where...

$r_1 = a+b, \quad r_2 = c+d$

$$u_1 = \frac{2c + z^2 + z\sqrt{\frac{4(c \cdot d)}{r_2} + z^2}}{2r_2 + 2z^2}$$

$$u_2 = \frac{2a + z^2 + z\sqrt{\frac{4(a \cdot b)}{r_1} + z^2}}{2r_1 + 2z^2}$$

$$w_1 = \frac{2c + z^2 - z\sqrt{\frac{4(c \cdot d)}{r_2} + z^2}}{2r_2 + 2z^2}$$

$$w_2 = \frac{2a + z^2 - z\sqrt{\frac{4(a \cdot b)}{r_1} + z^2}}{2r_1 + 2z^2}$$

Risk Ratio

$$\mathrm{RR} = \frac{\mathrm{EER}}{\mathrm{CER}}$$

Confidence Interval (Log-normal, Zhou)

$$\text{Lower Bound: } \exp\left(\ln(\mathrm{RR}) - z \cdot \sqrt{\left(\frac{1}{a} - \frac{1}{a+b}\right) + \left(\frac{1}{c} - \frac{1}{c+d}\right)}\right)$$

$$\text{Upper Bound: } \exp\left(\ln(\mathrm{RR}) + z \cdot \sqrt{\left(\frac{1}{a} - \frac{1}{a+b}\right) + \left(\frac{1}{c} - \frac{1}{c+d}\right)}\right)$$

Relative Risk Reduction

$$\mathrm{RRR} = 1 - \mathrm{RR}$$

Confidence Interval (Derived from Log-normal RR)

$$\text{Lower Bound: } 1 - \exp\left(\ln(\mathrm{RR}) + z \cdot \sqrt{\left(\frac{1}{a} - \frac{1}{a+b}\right) + \left(\frac{1}{c} - \frac{1}{c+d}\right)}\right)$$

$$\text{Upper Bound: } 1 - \exp\left(\ln(\mathrm{RR}) - z \cdot \sqrt{\left(\frac{1}{a} - \frac{1}{a+b}\right) + \left(\frac{1}{c} - \frac{1}{c+d}\right)}\right)$$

Odds Ratio

$$\text{OR} = \frac{ad}{cb}$$

Confidence Interval (Log-normal)

$$\text{Lower Bound:}\ \exp\left(\ln(\text{OR}) - z \cdot \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}\right)$$

$$\text{Upper Bound:}\ \exp\left(\ln(\text{OR}) + z \cdot \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}\right)$$

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Formulas for Diagnostic Tests

	Outcome	No Disease	Total
(+)	a	b	$a+b$
(−)	c	d	$c+d$
Total	$a+c$	$b+d$	$a+b+c+d$

z = 1.959964 for 95% CI

Sensitivity

$$\text{Sensitivity} = \frac{a}{a+c}$$

Confidence Interval (Wilson Score)

$$\text{Lower Bound: } \frac{(2\cdot a)+z^2 - z\cdot\sqrt{\left(\frac{4ac}{a+c}\right)+z^2}}{2\cdot(a+c)+2z^2}$$

$$\text{Upper Bound: } \frac{(2\cdot a)+z^2 + z\cdot\sqrt{\left(\frac{4ac}{a+c}\right)+z^2}}{2\cdot(a+c)+2z^2}$$

Specificity

$$\text{Specificity} = \frac{d}{b+d}$$

Confidence Interval (Wilson Score)

$$\text{Lower Bound: } \frac{(2\cdot d)+z^2 - z\cdot\sqrt{\left(\frac{4db}{b+d}\right)+z^2}}{2\cdot(b+d)+2z^2}$$

$$\text{Upper Bound: } \frac{(2\cdot d)+z^2 + z\cdot\sqrt{\left(\frac{4db}{b+d}\right)+z^2}}{2\cdot(b+d)+2z^2}$$

Positive Likelihood Ratio

$$\text{LR}(+) = \frac{\text{Sensitivity}}{1 - \text{Specificity}} = \frac{a/(a+c)}{b/(b+d)}$$

Confidence Interval (Log-normal, Zhou)

$$\text{Lower Bound: } \exp\left(\ln\left(\frac{(b+d)\cdot a}{(a+c)\cdot b}\right) - z \cdot \sqrt{\left(\frac{c}{a\cdot(a+c)}\right) + \left(\frac{d}{b\cdot(b+d)}\right)}\right)$$

$$\text{Upper Bound: } \exp\left(\ln\left(\frac{(b+d)\cdot a}{(a+c)\cdot b}\right) + z \cdot \sqrt{\left(\frac{c}{a\cdot(a+c)}\right) + \left(\frac{d}{b\cdot(b+d)}\right)}\right)$$

Negative Likelihood Ratio

$$\text{LR}(-) = \frac{1 - \text{Sensitivity}}{\text{Specificity}} = \frac{c/(a+c)}{d/(b+d)}$$

Confidence Interval (Log-normal, Zhou)

$$\text{Lower Bound: } \exp\left(\ln\left(\frac{(b+d)\cdot c}{(a+c)\cdot d}\right) - z \cdot \sqrt{\left(\frac{a}{c\cdot(a+c)}\right) + \left(\frac{b}{d\cdot(b+d)}\right)}\right)$$

$$\text{Upper Bound: } \exp\left(\ln\left(\frac{(b+d)\cdot c}{(a+c)\cdot d}\right) + z \cdot \sqrt{\left(\frac{a}{c\cdot(a+c)}\right) + \left(\frac{b}{d\cdot(b+d)}\right)}\right)$$

Positive Predictive Value

$$\text{PPV} = \frac{a}{a+b}$$

Confidence Interval (Wilson Score)

$$\text{Lower Bound: } \frac{(2\cdot a) + z^2 - z \cdot \sqrt{\left(\frac{4ab}{a+b}\right) + z^2}}{2\cdot(a+b) + 2z^2}$$

$$\text{Upper Bound: } \frac{(2\cdot a) + z^2 + z \cdot \sqrt{\left(\frac{4ab}{a+b}\right) + z^2}}{2\cdot(a+b) + 2z^2}$$

Negative Predictive Value

$$\text{NPV} = \frac{d}{c+d}$$

Confidence Interval (Wilson Score)

$$\text{Lower Bound: } \frac{(2 \cdot d) + z^2 - z \cdot \sqrt{\left(\frac{4dc}{c+d}\right) + z^2}}{2 \cdot (c+d) + 2z^2}$$

$$\text{Upper Bound: } \frac{(2 \cdot d) + z^2 + z \cdot \sqrt{\left(\frac{4dc}{c+d}\right) + z^2}}{2 \cdot (c+d) + 2z^2}$$

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